

Stiffness computation for optimization of shape of elastomer shock isolator

Rohit, Sunil Nimje, Ankit Varshney
Defense Institute of Advanced Technology

Abstract

This paper explains about the analytical method of calculating the stiffness of the Shock isolator subjected to half sine shock pulse. The value of stiffness obtained can be further used for deciding the shape and dimensions of the Elastomer based shock isolator. The method used utilizes the concept of shock response spectrum.

Keywords-*shock isolation; Half sine shock pulse; Shock response spectrum*

I. INTRODUCTION

Shock applied to a mechanical system causes to change the response of the system to change in very small period of time. It may be defined in the term of displacement, velocity, variation in force applied or acceleration shock pulse applied to a point in the system. The reduction in the effects of the shock can be achieved by the use of isolators between the system and the source of the shock.

The essential features of an isolator are stiffness and damping. Stiffness restricts the deflection of the system to a certain amount. Damping restrains the vibratory motion by dissipation of energy.

The use of elastomer based shock isolators is preferred over metal springs as isolators because of their high energy storing capacity and their tendency to be easily molded into any shape. The stiffness of elastomer based shock isolator depends on the shape and the chemical composition of the rubber used. Keeping the chemical composition to be constant we can vary the shapes of the isolator and try to find the best possible design which can fulfill our purpose. For determining the shape we are first required to calculate the stiffness of the isolator which keeps the displacement and isolated acceleration of the system within permitted range. Based on the stiffness obtained the basic shape for the elastomer based isolator can be estimated. The relation between the stiffness and the basic shape for isolator can be given as follows[3];

$$\text{For Cylinder, } K = \frac{E \pi a^4}{4l} \quad (1)$$

$$\text{For ring, } K = \frac{E \pi (a_o^4 - a_i^4)}{4l} \quad (2)$$

$$\text{For block, } K = \frac{E w t^3}{12l} \quad (3)$$

where, E= Dynamic modulus of elastomer

a = Disk radius

a_o = outer radius of ring

l = length of block

w = width of block

t = thickness

II. LITERATURE REVIEW

Jerome E.Ruzicka et.al; [1] has discussed the methods of isolation from mechanical shock, particularly with regard to providing protection from aircraft and aerospace shock environments. Idealized forms of shock excitation are employed, and the performance capabilities of shock-isolation systems are emphasized. For impulsive shock excitation, which is defined, completely by the velocity change imposed upon the isolation system, The maximum stress created in the isolator and the clearance requirements for the isolation system are both a function of the maximum relative displacement.

Tom Irvine[2] has derived equations for relative displacement between base and mass of the system and the equation for absolute acceleration of the mass of system subjected to a shock pulse of half sine applied to the base of the system.

III. PROBLEM STATEMENT

Elastomer based shock isolator is required to be designed, consisting of a cylinder weighing 10 tonnes and 9 meters in length which is concentrically inside another hollow cylinder. The diametrical clearance between the two cylinders is 100mm. A half sine shock pulse of amplitude 10g for 18ms is given to the outer cylinder. The shock isolators are required to be put in between the two cylinders within the diametrical clearance. The isolated output of the mass should be within 3g in amplitude and the maximum allowable compression for the isolator is 50%. The arrangement for the setup is shown in the fig

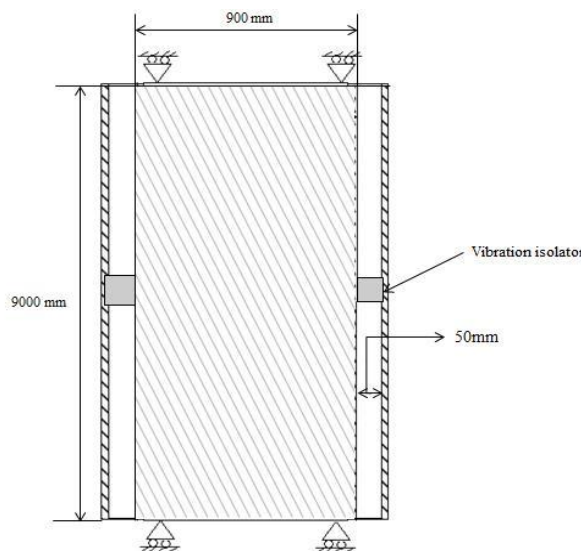
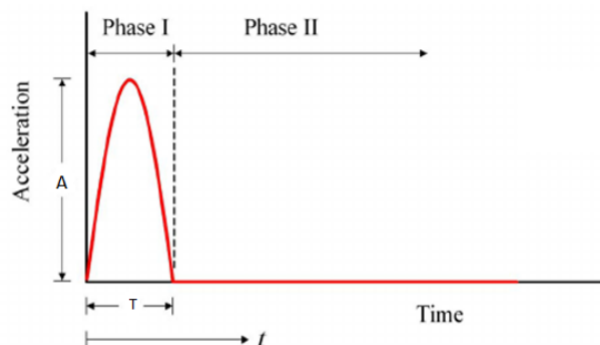


Fig 1: Problem statement representation

A. Half sine Pulse

A half-sine pulse of acceleration of duration T is shown in Fig.2; the corresponding velocity and displacement time-histories also are shown, for the initial conditions $u'' = u''' = 0$ when $t = 0$.



Where, $T = 18\text{ms}$
 $A = 10\text{g}$

Fig 2: Half sine shock pulse

$$\square \quad \square \left(\frac{t}{T} \right) \quad 0 < t < T \quad (4)$$

$$\square \quad \square \quad t < 0 \quad (5)$$

IV. ANALYTICAL METHOD TO PLOT SHOCK RESPONSE SPECTRUM

In the shock response spectrum we are basically required to plot the response of the system for an excitation. The excitation in this particular case is in the form of half sine shock pulse with an amplitude of 10g and for 18ms. The half sine shock pulse for this case can be represented as shown in fig 2. We will assume the system to be a SDOF spring mass system, with the shock pulse being applied to the base.

The relative displacement of the system for $0 < t < T$ the above shown excitation can be given by the following equations[2]

$$z(t) = \exp(-\xi\omega_n t) \left\{ z(0) \cos(\omega_d t) + \left\{ \frac{\dot{z}(0) + (\xi\omega_n)z(0)}{\omega_d} \right\} \sin(\omega_d t) \right\}$$

$$+ \frac{A}{\left[(\omega^2 - \omega_n^2)^2 + (2\xi\omega\omega_n)^2 \right]} \left[(2\xi\omega\omega_n) \cos(\omega t) + (\omega^2 - \omega_n^2) \sin(\omega t) \right]$$

$$- \frac{\frac{A\omega}{\omega_d} [\exp(-\xi\omega_n t)]}{\left[(\omega^2 - \omega_n^2)^2 + (2\xi\omega\omega_n)^2 \right]} \left[(2\xi\omega_n\omega_d) \cos(\omega_d t) + (\omega^2 - \omega_n^2 (1 - 2\xi^2)) \sin(\omega_d t) \right]$$

The relative displacement for $t > T$

$$z(t) = \exp(-\xi\omega_n (t - T)) \left\{ z(T) \cos(\omega_d (t - T)) + \left\{ \frac{\dot{z}(T) + (\xi\omega_n)z(T)}{\omega_d} \right\} \sin(\omega_d (t - T)) \right\} \quad (6)$$

Where, z is the relative displacement of the mass with respect to the base;

$$\omega_n \text{ is the } \sqrt{\frac{1}{m}} \text{ natural frequency.} \quad (7)$$

It can also be observed from the above equations that the maximum relative displacement for phase 1 occurs at time t , where

$$t = \left[\frac{\pi}{\omega_d} \right] \quad (8)$$

Also the relative displacement in phase 2 i.e.; for $t > T$ occurs at time, t , where

$$t = * \frac{\pi}{\omega_d} + T \quad (9)$$

From the above equations it can be concluded that for zero damping, first maximum in response for natural frequency less than π/T occurs during residual response, i.e; after $t = T$ and the magnitude of each succeeding response peak is same as that of first maximum.

The absolute acceleration of the mass for $0 < t < T$ is given by,

$$\begin{aligned} \ddot{x}(t) = & -\omega_n \exp(-\xi\omega_n t) \left\{ [\omega_n z(0) + 2\xi \dot{z}(0)] \cos(\omega_d t) + \frac{\omega_n}{\omega_d} \left[-\xi\omega_n z(0) + (1 - 2\xi^2) \dot{z}(0) \right] \sin(\omega_d t) \right\} \\ & + \frac{A\omega^2}{\left[(\omega^2 - \omega_n^2)^2 + (2\xi\omega\omega_n)^2 \right]} \left[-(\omega^2 - \omega_n^2) \sin(\omega t) - (2\xi\omega\omega_n) \cos(\omega t) \right] \\ & + \frac{A\omega\omega_n \left[\exp(-\xi\omega_n t) \right]}{\left[(\omega^2 - \omega_n^2)^2 + (2\xi\omega\omega_n)^2 \right]} \left\{ \left\{ 2\xi\omega^2 \right\} \cos(\omega_d t) + \left\{ \frac{\omega_n}{\omega_d} \right\} \left\{ -\omega_n^2 + \omega^2 (1 - 2\xi^2) \right\} \sin(\omega_d t) \right\} \\ & + A \sin(\omega t) \end{aligned} \tag{10}$$

The absolute acceleration of the mass for $t > T$

$$\begin{aligned} \ddot{x}(t) = & -\omega_n \exp(-\xi\omega_n (t - T)) \left\{ [\omega_n z(T) + 2\xi \dot{z}(T)] \cos(\omega_d (t - T)) \right\} \\ & - \omega_n \exp(-\xi\omega_n (t - T)) \left\{ \frac{\omega_n}{\omega_d} \left[-\xi\omega_n z(T) + (1 - 2\xi^2) \dot{z}(T) \right] \sin(\omega_d (t - T)) \right\} \end{aligned} \tag{11}$$

V. RESULT

Let us assume the system to be undamped. Then $\xi = 0$ and $\omega_n = \omega_d$. Calculating the absolute acceleration of the mass and relative displacement for different natural frequencies and plotting the obtained results. Other set of calculations is done for damped system with a damping of 5%. The two set of data obtained are plotted, and the graph as shown in fig 3 is obtained.

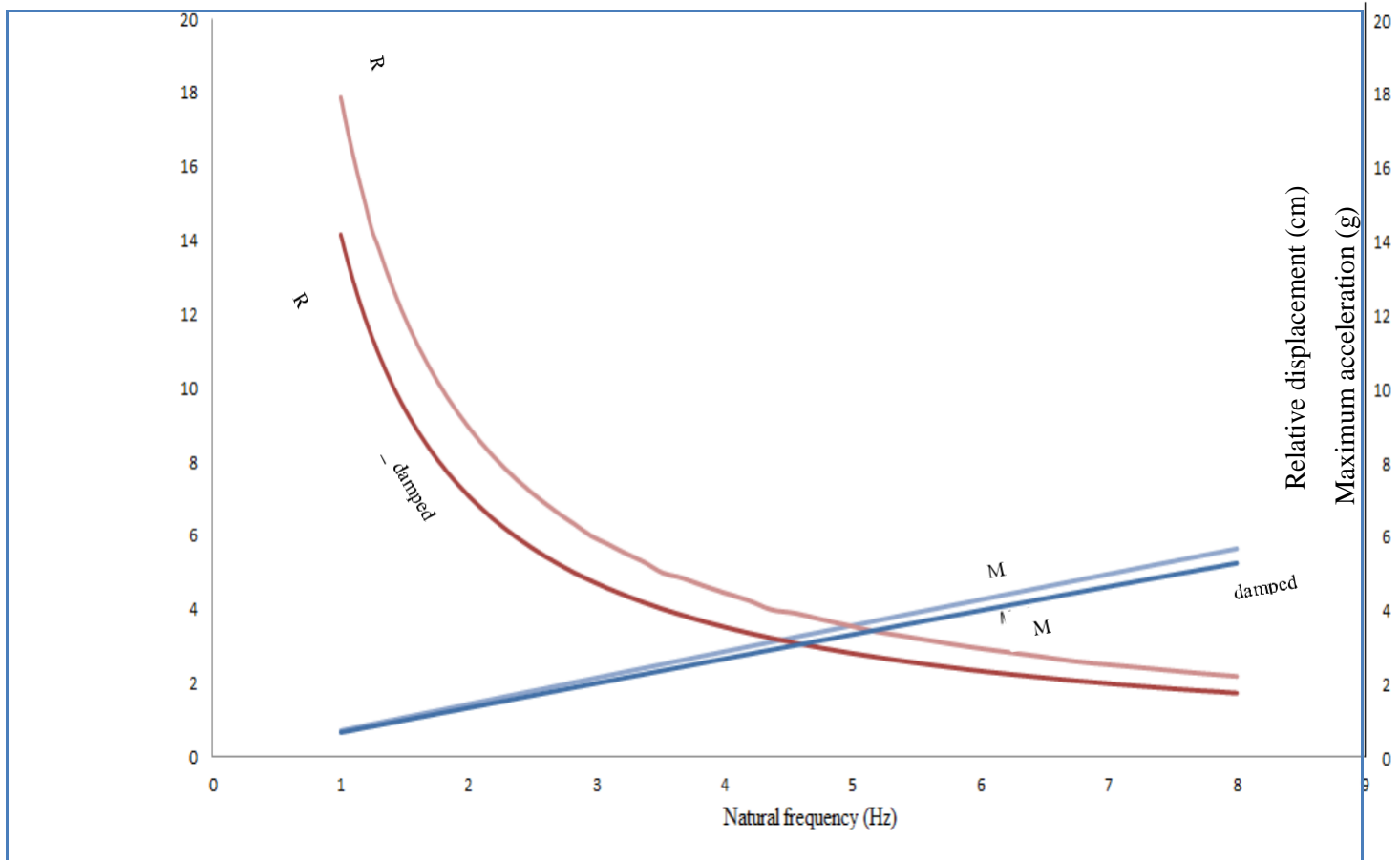


Fig 3: Maximum acceleration and relative displacement Vs natural frequency

VI. CONCLUSION

From the SRS plot and the plot for relative displacement as in fig 3, it can be noted that for an undamped system a natural frequency of 4 Hz, the absolute acceleration of the mass is 2.866g and the corresponding relative displacement obtained is 44.5mm. From the same graph it can be seen that for a system with damping of 5%, the maximum acceleration and relative displacement for a natural frequency of 3.22 Hz are 43.84mm and 2.15g respectively. Unlike spring and damper system, the elastomer based shock isolator shows non linear properties. Therefore a range of 3.22Hz to 4Hz can be used for the estimation of the shape of the shock isolator.

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