Crude Oil Short-Term Scheduling Algorithm with Multiple Charging Tanks Charging a Distiller Overlapingly Based on Priority-Slot Continuous-Time Formulation

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Abstract

The optimal scheduling of crude-oil operation in refineries has been researched by various groups during the past decade. Different mixed integer linear programming or mixed nonlinear programming formulations are derived. This paper proposes a novel MINLP formulation with multiple charging tanks charging a distiller overlapingly and oil residency time constraint that is based on multi-operation sequencing with synchronized start time (MOS-SST). It is different from previous formulations because of its consideration for oil residency time constraint and multiple charging tanks charging a distiller overlapingly. A branch and bound algorithm combined with the Lagrangian Relaxation Subgradient Optimization algorithm is proposed to solve this model, resulting in a satisfactory optimal result.

Keywords: oil refinery; scheduling algorithm; charging overlapingly; oil residency time

1. Introduction

A great challenge must exist during operating an oil refinery [23]. It is known that operating a plant well can increase profit by $10 per ton of product or more [16]. Thus, as a typical process industry, a refinery attracts many researchers to pay great attention to the development of efficient techniques for operating it.

Usually heuristics and meta-heuristics algorithms are used to solve a scheduling problem in discrete manufacturing process [1-2] and [12-21]. These techniques may not be able to find an optimal solution. Consequently, mixed integer programming-based techniques, especially mixed integer linear programming (MILP), have been preferred [4-7] and [14-20]. Based on time grids, a mixed integer programming formulation can easily model the capacity of a tank or production unit at the end of each time interval [3-14]. Through mixed integer programming models, the uniform time discretization formulations have been firstly successful in being applied to solve batch processes [9]. To reduce the number of discrete variables, continuous-time formulations have been brought in [10- 24].

Maravelias et al. [11] propose a new general continuous-time state-task network formulation named multi-operation sequencing with synchronized start time (MOS-SST) for short-term scheduling of multipurpose batch plants, whose advantage is that time-points can be applied to stalk the beginning time and finishing time of every operation. Mouret et al. [19] apply the above formulation to the crude oil operation scheduling problem. However, in [19], to make the problem solvable, they do not consider oil residency time and multiple charging tanks charging a distiller synchronously constraints which are solid and cannot be ignored in a real-life refinery. Hence, by using the model in [19], an infeasible solution may be obtained. We should notice that oil residency time and multiple charging tanks charging a distiller overlapingly make the scheduling problem much more complicated. To obtain a feasible schedule, oil residency time and multiple
charging tanks charging a distiller overlappingly should be taken into account. This motivates us to conduct the study on the crude oil operation scheduling problem of with multiple charging tanks charging a distiller overlappingly and oil residency time.

This paper proposes model based on MOS-SST, which considers multiple charging tanks charging a distiller overlappingly and oil residency time. To decrease the operation cost, the number of charging tanks overlappingly should be minimized, so the objective function of our model is to minimize the number of charging tanks overlappingly which can increase the power cost. This model is a MINLP model, so it cannot be solved using MINLP solver easily. According to the characteristic of our model, we use Lagrangian Relaxation combined with branch and bound to solve it and get a satisfactory feasible solution with small gap between the upper bound and the lower bound.

In the next section, refinery processes and the short-term scheduling problem addressed in this paper are briefly introduced. Section 3 develops multi-operation sequencing with synchronized start time (MOS-SST) formulation with multiple charging tanks charging a distiller overlappingly and oil residency time. Section 4 proposes an efficient approach to solve the problem. An industrial case study is used to illuminate the applications of the proposed method in Section 5. Finally, conclusion of this paper is addressed Section 6.

2. Processes and Their Scheduling Problem

Three stages are mainly contained in oil refinery processes as is illustrated in Figure 1: (1) crude oil operations; (2) production; and (3) product delivery. This paper aims at crude oil operation short-term scheduling, which is one of the hardest scheduling problems during operating a refinery.

![Figure 1. A Multiple Charging Tanks Charging Overlappingly Oil Refinery Process](image_url)

In summary, a short-term schedule is subject to the following resource and process constraints. The former includes: (1) the limited number of storage and charging tanks and capacity of each tank; (2) the limited flow rate of oil unloading and oil transportation through pipelines; and (3) the available amount of various crude oil types in storage and charging tanks, and in coming tankers. The latter includes: (1) a distiller should be kept in working all the time uninterruptedly unless a maintenance is needed; (2) at least one charging tank is dedicated to feed a distiller at any time; (3) a tank cannot be charged and discharged simultaneously; (4) RT must be met for any tank after charging; (5) at every time of whole scheduling horizon, the concentration of every kinds of composition should be within given ranges must be met; (6) crude oil is transported into only an empty
tank or one tank with the same type of crude oil. (7) multiple charging tanks are required to feed a distiller overlappingly as needed; (8) only one berth is available at the docking station for vessel unloading.

For the sake of describing a short-term schedule in detail, we define an operation decision OD = (S, D), where S is the source from which the crude oil comes, and D is the destination of crude oil to be delivered. For crude oil operations, resources include docks for crude vessels, storage and charging tanks, so there are three types of OD: 1) oil unloading OD from crude oil vessels to storage tanks; 2) oil transportation OD from storage tanks to charging tanks; and 3) oil feeding OD from charging tanks to distillers. The short-term scheduling problem for crude oil operations is to define all the ODs to be performed and then sequence them for a schedule horizon, at the same time answer the questions for every OD to occur which include the beginning time of an operation, and what and how it should be operated. At the beginning, one knows the initial state information of the system only. It includes 1) the current inventory of crude oil and types of crude oil in storage and charging tanks; 2) the arrival time of marine vessels, types and volume of crude oil in them; and 3) operation state of each production device.

Because of the limitation of resources, many operations should be performed one after another. Hence, one of the most average constraints arising in scheduling problems is the non-overlapping requirement for two ODs v and w, namely, if ODs v and w are to be performed in [t_{v1}, t_{v2}] and [t_{w1}, t_{w2}], respectively, and [t_{v1}, t_{v2}] \cap [t_{w1}, t_{w2}] = \emptyset. A short-term crude oil operation schedule consists of a sequence of ODs. By the MOS-SST method, to describe a schedule with a continuous-time formulation is to sequence these ODs by using a succession of priority slots. A priority slot i is defined as a position i on the time coordinate. Slot i is said to have a higher priority in scheduling than slot j with slots i and j being non-overlapping, if i is placed earlier than j on the time coordinate. Such a relation is denoted as j > i, or i < j. By the priority-slot-based method, to formulate a scheduling problem, every priority slot is assigned to more than one OD if they have the same beginning time. In this way, the priority slot number is less than the total number of ODs to be performed during the schedule horizon. The sequence of priority slots corresponds to the sequence of the ODs. By this means, the key is to decide the priority slot number that is required to be known in advance.

Assume that priority slots i and j accommodate two non-overlapping ODs v and w with i < j. Let S_i and S_j be the start time of slots i and j, and D_i and D_j be their operation durations, respectively. Since OD v has a higher priority than w, w is able to start only after the completion of v, i.e., we have S_i + D_i \leq S_j.

With this precedence relationship, given a sequence of ODs, a schedule obtained is feasible only if S_i + D_i \leq S_j is satisfied for any pair of non-overlapping ODs. In the meanwhile, with this priority-slot-based modeling strategy, different schedules can be obtained by ordering the ODs with respect to their start time.

3. Problem Formulation

This section presents the priority-slot-based continuous-time formulation for the short-term crude oil operation scheduling problem with oil residency time constraints, multiple charging tanks charging a distiller overlappingly being taken into account. For the purpose of readability, a crude oil operation process shown in Figure 2 is used to illustrate the modeling process. Here solid lines denote actual crude oil flow directions, and dashed lines are used to express simplified directions. For the sake of simplicity, some dashed lines are omitted. First, we present the notation for the model.
Sets and parameters:

\( T = \{1, 2, \ldots, n\} \): Set of priority-slots;

\( W \): Set of the \( n \) ODs for a schedule, \( W=W_U\cup W_T \cup W_D \) (\( W=\{1-13\} \) for the system shown in Figure 2);

\( W_U \subset W \): Set of unloading ODs (\( W_U = \{1, 2, 3\} \) for the system shown in Figure 2);

\( W_T \subset W \): Set of oil transportation ODs (\( W_T = \{4, 5, 6\} \));

\( W_D \subset W \): Set of oil feeding ODs (\( W_D = \{7-13\} \));

\( R = RV \cup RS \cup RC \cup RD \): Set of resources;

\( RV \subset R \): Set of vessels;

\( RS \subset R \): Set of storage tanks;

\( RC \subset R \): Set of charging tanks;

\( RD \subset R \): Set of CDUs;

\( I_r \subset W \): Set of inlet transportation ODs on resource \( r \);

\( O_r \subset W \): Set of outlet transportation ODs on resource \( r \);

\( C \): Set of crude oil types; \( K \): Set of crude oil properties (e.g., oil sulfur concentration);

\( H \): Scheduling horizon; \( RT \): Residency time;

\( S_r \): Arrival time of vessel \( r \);

\( f_r \): Refining velocity of CDU \( r \);

\( x_{rk} \): The volume percent limit of property \( k \) of crude oil type \( c \in C \);

\( f_{iv} \): The flow rate of OD \( v \) which priority-slot \( i \) accommodates;

\( [v_-, v_+] \) is the lower and upper bounds of the volume delivered by OD \( v \) and generally \( v_- = 0 \) for an OD except an unloading OD that is required to unload all the oil of a type in a vessel once, in this case, \( v_+ = v_- = V \);

\( [FR_-, FR_+] \) is the flow rate limit for OD \( v \);

\( [x_{rk}, x_{rk}] \) is the volume percent limit of property \( k \) of crude oil required by CDU \( r \);

\( [L_r, L_r] \) is the capacity limit of tank \( r \in RS \cup RC \);

\( [D_r, D_r] \) is the bound of the demand on crude oil to be delivered from charging tank \( r \in RC \) during the scheduling horizon;
Figure 2. The Refinery Configuration for Modeling Illustration

Variables:

\[ Z_{iv} \in \{0, 1\}, i \in T \text{ and } v \in W, \]  
\[ Z_{iv} = 1 \text{ if OD } v \text{ is assigned to priority-slot } i; \text{ and } Z_{iv} = 0 \text{ otherwise.} \]

\[ S_{iv} \geq 0 \text{ and } D_{iv} \geq 0, i \in T \text{ and } v \in W, \]  
\[ S_{iv} \text{ is the beginning time of OD } v \text{ if it is assigned to priority-slot } i, \text{ and } S_{iv} = 0 \text{ otherwise.} \]

\[ D_{iv} \text{ is the duration of OD } v \text{ if it is assigned to priority-slot } i, \text{ and } D_{iv} = 0 \text{ otherwise.} \]

\[ t_i \in [0, H], i \in T \]  
\[ t_i \text{ is positive synchronization time-points variable.} \]

\[ V_{ivc} \text{ is the volume of crude oil of type } c \in C \text{ delivered by OD } v \text{ if it is assigned to priority-slot } i, \text{ and } V_{ivc} = 0 \text{ otherwise.} \]

\[ L_{irc} \text{ is the accumulated volume of crude oil type } c \text{ in tank } r \in R_S \cup R_C \text{ at the beginning of slot } i. \]

With the notation given above, we can present our formulation for the short-term scheduling problem of crude oil operations with multiple charging tanks charging a distiller overlappingly and oil residency time being taken into account by using the priority-slot-based method given in [19]. First, we present the constraints as follows. Our formulation is adapted from the formulation in [19] by adding and dropping some constraints, so we only discuss residency time constraints and multiple charging tanks charging a distiller overlappingly constraints for lack of space, and other constraints can be referred to [19] in detail.

Storage tanks residency time constraint:

\[ \sum_{v \in I_r} (S_{iv} + D_{iv} + RT \times Z_{iv}) \leq \left( \sum_{v \in J_r} (S_{iv} + (H + RT) \times (1 - \sum Z_{ij})) \right), i \in T, j \in T, i < j, \text{ and } r \in R_S \tag{1} \]

Charging tanks residency time constraint:

\[ \sum_{v \in J_r} S_{iv} + H \times (1 - Z_{iv}) \geq \left( \sum_{v \in I_r} (S_{iv} + D_{iv} + RT \times \sum Z_{jv}) \right), i \in T, j \in T, i > j, \text{ and } r \in R_C \tag{2} \]

Multiple charging tanks charging a distiller overlappingly constraint:

A CDU must be charged at least one charging tank at any time. Constraint (4) restricts just one charging tank can feed a CDU at a time, so dropping this constraint permits charging operations overlapping. While, when more than one charging tank feed a CDU d, the sum of every charging tank’s charging flow rate must equal to the oil refining velocity of CDU d, and every component in the mixture from different charging tanks must be within given ranges according to the performance of CDU d at every time. We introduce a simple example to illustrate the following constraint. In MOS-SST
formulation, every operation can be assigned to the same priority-slot if they have the identical beginning time, so \( t_i \) is equal to \( S_{iv} \). \( S_{iv'} \leq t_i \leq (S_{iv'} + D_{iv'}) \) and \( t_i = S_{iv} \) can find all charging operations overlappingly between \( t_j \) and \( t_i \), \( t_j < t_i \). 

\[
\sum_{v' \in I_r} f_{jv'} x_{vk} + \sum_{v' \in I_r} f_{iv} x_{vk} \leq x_{vk} \]

is derived from

\[
\sum_{v' \in I_r} f_{jv'} x_{vk} + \sum_{v \in I_r} f_{iv} x_{vk} \leq x_{vk} \]

which assures that the concentration of every component is satisfied the given range between \( t_j \) and \( t_i \) according to performance of certain CDU. \( \sum_{v' \in I_r} f_{jv'} + \sum_{v \in I_r} f_{iv} = f_r \) implies that the mixed feeding flow rate must equal to the oil refining velocity of CDU.

\[
\begin{aligned}
x_{vk} &\leq \sum_{v' \in I_r} f_{jv'} x_{vk} + \sum_{v \in I_r} f_{iv} x_{vk} \\
S_{iv'} &\leq t_i \leq S_{iv'} + D_{iv'} \\
t_i &\leq S_{iv} \\
\sum_{v' \in I_r} f_{jv'} + \sum_{v \in I_r} f_{iv} &\leq f_r \\
f_{iv} &\leq \frac{v_{ivc}}{D_{iv}}, f_{jv'} = \frac{v_{jv'c}}{D_{jv'}}
\end{aligned}
\]

(3)

\[
\sum_{v \in I_r} (S_{iv} + D_{iv}) \leq \sum_{v \in I_r} H \times (1 - \sum_{v \in I_r} Z_{iv}) \quad i \in T, j \in T, i < j, \text{ and } r \in R_D
\]

(4)

Objective function: The objective is to minimize the number of switches of charging tanks in feeding the CDUs. Thus, the objective function is as follows.

\[
J = \sum_{i \in T} \sum_{v \in W_D} Z_{iv}
\]

(5)

Based on the discussion above, the short-term crude oil operation scheduling problem with multiple charging tanks charging a distiller overlappingly can be formulated as the following mathematical programming problem.

Problem P1: Minimize \( J = \sum_{i \in T} \sum_{v \in W_D} Z_{iv} \)

Subject to: constraints (1)-(3), and constraints in [19] excluding (4).

With the formulations for the scheduling problem developed above, we discuss how to solve the problem next.

4. Solution Method

For the mixed integer non-linear programming (MINLP) problem P1 developed in the last section, an outer-approximation method or spatial branch-and-bound search method can be used [18]. However, the former may converge to a poor suboptimal solution, while the latter is very time-consuming. Therefore Algorithm 1 is presented to find local optimal solution with an estimation of the optimality gap. In the following statement, problem P2 is MILP model and adapted from problem P1 by dropping composition constraint which includes bilinear item.

Algorithm 1: find a local optimal solution and estimate the optimality gap for Problem P1

Step 1: using Algorithm 3 given later to solve Problem P2;
Step 2: if optimal solution is not found for Problem P2, go step 7;
Step 3: if the optimal solution is satisfied composition constraint of problem P1, the optimal solution is the optimal one of problem P1; go step 7;

Step 4: the optimal solution is the lower bound of problem P1; Use the binary variables Ziv of problem P2 to fix corresponding ones of problem P1 and get problem P1*;

Step 5: apply outer-approximation method to solve problem P1*, get the optimal solution of P1* which is the upper bound of problem P1;

Step 6: calculate the gap between the upper bound and lower bound and output the gap, gap = \|\text{the upper bound of problem P1} - \text{the lower bound of problem P1}\| / \text{the upper bound of problem P1} ;

Step 7: there is no optimal solution for Problem P1;

Step 8: stop.

As presented above, Problem P2 is a mixed integer linear programming (MILP) model. Thus, if the number of priority slots is known, it can be solved by using Lagrangian relaxation based on a commercial solver such as CPLEX such that a satisfactory and feasible solution with a small gap between upper bound and lower bound can be obtained. In fact, to obtain such a model, the key is to postulate an appropriate number of priority slots a priori. Unfortunately, it is pretty difficult to predict the number of priority slots. When the priority slot number is set too small, there might be no solution at all. While a large priority slot number could make the problem impossible to solve because of the large binary variable number. Therefore, it is of great necessity to develop an effective strategy to determine the priority slot number.

By the definition of an OD, an operation may be executed more than one time, depending on the demands of CDUs. However, generally an operation should be executed at least once. Thus, although it is very difficult to guess the number of slots, it is reasonable to let the number of operations be the number of slots to start the solution process.

Based on the above guess, we propose branch and bound algorithm combined with the Lagrangian Relaxation Subgradient Optimization algorithm. Furthermore, we can get a satisfactory and feasible solution with a small gap between upper bound and lower bound.

The basic principle of Lagrange relaxation method is absorbing constraints causing problems hard into the objective function, at the same time keeping the objective function still linear, which makes the problem easy to solve.

The actual calculating results demonstrate that the lower bound derived from Lagrangian relaxation method is quite good, and the computation time is acceptable. In the meanwhile, we may further use the basic principles to construct heuristic Lagrangian relaxation algorithm based on Lagrangian relaxation as well.

Lagrangian relaxation is an upper bound of the original MILP problem. Our target is to find the lowest upper bound that is close to the optimal value of the original problem. So we need to solve the dual problem of the original MILP problem (LD).

We adopt Subgradient Optimization algorithm in the process of solving (LD), which is analogous to nonlinear programming gradient. Assuming that (LP) is the linear relaxation of the original problem MILP, and that \( Z_{LP} \) is the optimal value of (LP), we have \( Z_{LD} \geq Z_{LP} = \min_{x} \{ C^T x | Ax \leq b, Cx \leq d \} \).

Usually Branch and Bound Algorithm uses linear relaxation to get lower bound. Lagrangian relaxation can get more tight lower bound than linear relaxation does, which account for our usage of Lagrangian relaxation method in branch and bound algorithm.

Algorithm 2: Subgradient Optimization algorithm

Step 1: Choose an arbitrary initial Lagrangian multiplier \( \lambda^1, t = 1; \)
Step 2: In regard to $\lambda^t$, arbitrarily choose a subgradient $\xi^t$ from $\partial q(\xi^t)$. If $||\xi^t|| \leq \varepsilon$, $\lambda^t$ gets the approximate optimal value, the computation can be stopped. Otherwise, 

$$\lambda^t = p^+(\lambda^t + s_t \xi^t / ||\xi^t||)$$

where $s_t > 0$ is the step size, $p^+$ is projection function. Repeat step 2.

Sophisticated step size rules for $s_t : s_t = \rho \frac{w_t - d(\lambda^t)}{||\xi^t||}$, $0 < \rho < 2$, where $w_t$ is an approximation to the optimal value of $Z_{I, T}, w_t \geq d(\lambda^t)$ and $\xi^t \neq 0$.

Algorithm 3: Branch and bound algorithm combined with Subgradient Optimization algorithm

The rule and method using in Algorithm 2: We use $x_i \leq \lfloor x_i^* \rfloor, x_i \geq \lfloor x_i^* \rfloor$, where $x_i^*$ is a fractional variable as branching method; branching the most fractional variable is branching rule; Hybrid Strategies composed of Depth-First and Best First the strategy for selecting the next subproblem to be processed.

Step 1: Set $T = \{1\}$.

Step 2: Solve the linear relaxation of the problem $P_2$ (LP2) using CPLEX solver. If (LP2) is infeasible, let $T = T \cup \{\max(T) + 1\}$, max (T) denote the greatest element of T. Repeat step 2. Otherwise set $Z = \min (LP_2)$. If all assignment variables of the solution are integer, then we get the optimal solution, go to Step 8;

Step 3: create two new subproblems by branching on a fractional variable.

Step 4: solve the Lagrange relaxation of two nodes using Subgradient Optimization algorithm (Algorithm 2), and get two low bounds.

Step 5: a node (subproblem) is not active and to be pruned when any of the following occurs:

(1) the low bound of the node is great than $Z$;
(2) the solution is integral. Compare the optimal value with $Z$. If $Z$ is great than the optimal value, assign the value to $Z$, and record this node; otherwise, prune this node;
(3) the subproblem is infeasible. Prune this node;

Step 6: ascertain whether there are node, which have not yet been fathomed. If there are, go step 3, otherwise go to Step 8;

Step 7: if there is integral solution node recorded; otherwise, let $T = T \cup \{\max(T) + 1\}$ and Go to Step 2.

Step 8: get the optimal solution and output it;

Step 9: calculations are stopped;

5. Industrial Case Study

In this section, an industrial example is given to show the applications of the algorithm obtained in this paper. This system presented as problem 4 in [18] contains four charging tanks, six storage tanks and three distillers. Every distillation processes crude oil that has particular specification on a certain component, such as sulfur. In order to testify multiple charging tanks charging a CDU overlappingly, we modify the refinery configuration of data such that one crude oil in charging Tank r2 is not in the given rage. The other data are the same as problem 4 in [18]. If there is no satisfied specification crude oil to distill, it has to use several charging tanks to feed a CDU in order to get satisfied mixed crude oil. The crude oil in charging tank r2 is not in the permitted range, it must cooperate with another charging tank in charging the same CDU when distilled. Every crude oil else meets the requirement that does not need being mixed.
Using our proposed algorithm, we get a detailed schedule for distiller feeding as shown in Figure 3. The bars have the same texture charge an identical distiller. From Figure 3 we can find that due to violating of characteristic of crude oil in charging tank r2, it has to couple with charging tank r1 together to charging distiller d1. After the substandard crude oil charging is finished and the one meeting the specification has been transported, charging tank r2 starts to feed distiller d2 independently.

**Figure 3. The Detailed Scheduling for Charging Distiller**

6. Conclusions

Because of its combinatorial character and complicated scheduling requirements, short-term crude oil operation scheduling problem in an oil refinery is NP-hard. A variety of resource and process constraints have to be met that bring about an enormous difficulty to discover even a feasible scheduling. The presented problem in this work is more complicated than the previous system due to multiple charging tanks charging a distiller overlapping and oil residency time because they are hard to be expressed accurately. One new objectives is introduced due to the cost consideration, i.e., to minimize the sum of charging operations. As far as we have known, this work is the first one to formally research the scheduling problem with multiple charging tanks charging a distiller overlapping systems using mathematical programming model. It uses mathematical programming methodology to get a feasible scheduling. This model is MINLP model, so it cannot be solved effectively by CPLEX which is one of the most popular and efficient commercial software for mathematical programming. According to the characteristics of the model, we propose branch and bound algorithm combined with the Lagrangian Relaxation Subgradient Optimization algorithm. And it can get a satisfactory and feasible solution with a small gap between upper bound and lower bound.

It reveals that when the characteristic of crude oil in a charging tank violates the given range it must cooperate with another charging tank in charging the same CDU overlappingly when distilled. This situation happening greatly depends on the system’s initial state. Therefore, we must deliberately arrange the initial conditions. The results of case study testify the effectiveness of the proposed approach.

A very important thing should be noticed that this situation happens only in extremity or as a last resort when there is no appropriate crude oil in charging tanks at some time, because it will occupy more charging tanks and result in unschedulability of the system. But if this extreme situation is not considered, the unschedulability of the system yet happens.

References


