A Novel Multiple Attribute Decision Making Method and its Application in Sustainable Assessment

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Abstract

It is a key point for an enterprise in the long run whether it has an ability to achieve sustainability development. Therefore, it is important to assess this ability from the perspective of an enterprise. In this paper, a new multiple attribute decision making method in the dual hesitant fuzzy context is proposed to deal with this problem. Dual hesitant fuzzy set, as a combination of intuitionistic fuzzy set and hesitant fuzzy set, can describe the complex preferences of the decision maker or experts more accurately where membership, non-membership and uncertainty is included simultaneously. Distance measure is introduced in TOPSIS to measure the difference between two alternatives in this assessment problem, which forms the multiple attribute decision making method. Finally, an illustrative example is demonstrated to verify the applicability of the proposed method.

Keywords: sustainable development, multiple attribute decision making (MADM), dual hesitant fuzzy set, distance measure.

1. Introduction

The enterprise of combining with sustainable development ability is of great importance to improving the quality and efficiency of it, optimizing resource allocation, facilitating industrial and national further development[1-2]. In other words, the sustainable development about an enterprise is a key part of it about a country. Based on enterprise life cycle theory, as enterprises have their life cycle in the fierce market competition, it is essential to study their sustainable development abilities [3]. In most studies, assessment of sustainable development abilities is the first step to complete the research of them, which should be considered as a multiple decision making problems [4-5].

Multiple attribute decision making (MADM) problems are widely applied in real life. The target of it is to find a best compromise solution from all feasible alternatives assesses on multiple attributes, both quantitative and qualitative [6-7].

With other MADM methods, fuzzy MADM method can solve uncertainty and complex problems. Since fuzzy set was defined by Zadeh, it has been a useful method to represent the decision maker's preference [8-9]. There are several famous extensions which have been developed, such as intuitionistic fuzzy set, hesitant fuzzy set, type 2 fuzzy set, type N fuzzy set, interval-valued fuzzy set and so on[10-15]. Hereinto, intuitionistic fuzzy set and hesitant fuzzy set as two most popular method is used widely. Recently, In fact, when evaluating some problems, experts have memberships and non-memberships and experts’ possible memberships and non-memberships may be not only crisp values in[0,1], but also interval values. So to solve these problems, Zhu et al defined dual hesitant set (DHFS), which is a new extension of fuzzy set. Combing of intuitionistic fuzzy set and hesitant fuzzy set, the main characteristic of it is that: the membership function assigns to each element x in a universe of discourse X a membership in interval [0, 1], the non-membership function also assigns to each element x in a universe of discourse X a non-
membership in interval \([0,1]\) and the uncertain degree equals one minus the membership degree and the non-membership degree\([16]\).

In this paper, a new multiple decision making method is proposed under dual hesitant fuzzy context. Firstly, the concept the dual hesitant fuzzy set is introduced and compared with hesitant fuzzy set and intuitionistic fuzzy set. Secondly, distance measure between two dual hesitant fuzzy sets is defined to measure the difference between them. According to the distance measure, TOPSIS is developed to aggregate the information of each alternative to obtain a final result. As mentioned above, a procedure of decision making is demonstrated. The main contributions of this paper include the following: (1) the construction of the sustainable development ability assessment model; (2) the introduction of dual hesitant fuzzy set; (3) the design of distance measure in TOPSIS of dual hesitant fuzzy set; (4) the application of the proposed method. The rest of this paper is organized as follows. In Section 2, we introduce some method of dual hesitant fuzzy set to complete this assessment framework. A case study is demonstrated in Section 3. Finally, Section 4 concludes this paper and develops the further direction of the research.

2. An Evaluation Model based on Dual Hesitant Fuzzy Sets

In this section, we introduce dual hesitant fuzzy multiple decision making methods to evaluate the sustainable development of an enterprise. Based on the basic concepts of dual hesitant fuzzy set and TOPSIS approach, we define new distance measure and discuss the parameter of the distance measure.

2.1 Dual Hesitant Fuzzy Sets

Zhu et al. proposed the concept of dual hesitant fuzzy set which combine the advantage of intuitionistic fuzzy set with hesitant fuzzy set.

Definition 1\([16]\). Let \(X\) is a universe of discourse, and then a dual hesitant fuzzy set \(M\) over \(X\) is defined as

\[
M = \left\{ (x, h_M(x), g_M(x)) | x \in X \right\},
\]

where \(h_M(x)\) and \(g_M(x)\) are sets of some different values in \([0, 1]\), symbolizing the possible membership degrees and non-membership degrees of the element \(x\) to \(M\), respectively, with the condition:

\[
0 \leq \gamma, \eta \leq 1, \ 0 \leq \gamma^* + \eta^* \leq 1
\]

where for all \(x \in X\), \(h_M(x) = \bigcup_{\gamma \in \gamma(x)} \{\gamma\}\), \(g_M(x) = \bigcup_{\eta \in \eta(x)} \{\eta\}\), \(\gamma^* \in h_M^* = \bigcup_{x \in X} \max_{\gamma \in \gamma(x)} \{\gamma\}\), and \(\eta^* \in g_M^* = \bigcup_{x \in X} \max_{\eta \in \eta(x)} \{\eta\}\).

For convenience, we call the pair \(d(x) = (h_M(x), g_M(x))\) a dual hesitant fuzzy element (DHFE).

The complement of a DHFE \(M\), denoted by \(M^c\) is defined in form as follow:

\[
M^c = \left\{ (x, g_M(x), h_M(x)) | x \in X \right\}.
\]

It is obvious that the complement of a DHFS is symmetrical, that is, \((M^c)^c = M\).

Definition 2\([16]\). Let \(\alpha_1 = (h_1, g_1, f_1)\) and \(\alpha_2 = (h_2, g_2, f_2)\) be two dual hesitant fuzzy numbers, their basic operations are defined as

\[
\begin{align*}
(1) \alpha_1 \oplus \alpha_2 & = (\bigcup_{\gamma_1 \in \gamma_1(x_1)} \gamma_1 + \gamma_2 - \gamma_2, \bigcup_{\eta_1 \in \eta_1(x_1)} \eta_1 + \eta_2, \bigcup_{\eta_1 \in \eta_1(x_1)} (1 - \gamma_1)(1 - \gamma_2) - \eta_1 \eta_2) \\
& = (h_1 + h_2, g_1 + g_2, f_1 + f_2).
\end{align*}
\]
(2) \[ a_i \otimes a_j = \bigcup_{y_h, y_j \in X} \bigcup_{\eta_i, \eta_j \in \{0, 1\}} \{ (1 - \eta_i)(1 - \eta_j) \} \]

(3) \[ \lambda a_i = (\bigcup_{y_h} (1 - (1 - \eta)^i), \bigcup_{\eta_i} \{\eta^i\}, \bigcup_{y_h, \eta_i} \{ (1 - \eta^i) \}) \]

(4) \[ a_i^c = (\bigcup_{y_h} \{\eta^i\}, \bigcup_{\eta_i} \{1 - \eta_i \}) \]

(5) \[ \lambda a_i^c = (\bigcup_{y_h} \{1 - \eta_i \}, \bigcup_{\eta_i} \{\eta^i\}) \]

Here, \( a_i^c \) represents the complement of the dual hesitant fuzzy number \( a_i \).

In addition, Zhu et al. also defined some aggregation principles for dual hesitant fuzzy elements as follows:

Definition 3 [16]. Let \( a_i = (h_i, g_i, f_i) \) and \( a_j = (h_j, g_j, f_j) \) be two dual hesitant fuzzy numbers, their basic operations are defined as

(1) \[ a_i \oplus a_j = a_{i,j} \]

(2) \[ a_i \otimes a_j = a_j \otimes a_i \]

(3) \[ (a_i \oplus a_j)^c = a_i^c \oplus a_j^c \]

(4) \[ (a_i \otimes a_j)^c = a_i^c \otimes a_j^c \]

(5) \[ \lambda(a_i)^c = (a_{i,c})^c \]

(6) \[ (a_i^c)^c = \lambda a_i \]

As similar as hesitant fuzzy number, the length of the membership and non-membership of different dual hesitant fuzzy elements does not mostly equal. Many studies have been done to deal with this problem in hesitant fuzzy number. Xu and Xia suggested that we should extend the shorter one depending on the decision maker’s risk preferences until both of them have the same length [17]. Optimists expect desirable results and should add the maximum value, while pessimists anticipate unfavorable outcomes and should add the minimal value.

2.2 Distance Measures

Distance measure is fundamentally important in a variety of scientific fields such as decision making, pattern recognition, machine learning and marker prediction [18-20], lots of studies have been done on this issue. Li and Cheng [21] generalized the Hamming distance and the Euclidean distance by adding a parameter for intuitionistic fuzzy sets only based on the membership degrees and non-membership degrees. Xu and Xia [22] proposed distance and similarity measures for hesitant fuzzy sets.

By taking into account the three parameter characterizations of dual hesitant fuzzy sets, and following the basic lines of reasoning on which the definition of distances between intuitionistic fuzzy sets and hesitant fuzzy sets are based, we define the two basic distances between the dual hesitant fuzzy sets.

Definition 4 \( M = \{ (x, h_u(x), g_u(x)) | x \in X \} \) and \( N = \{ (x, h_v(x), g_v(x)) | x \in X \} \) are two dual hesitant fuzzy sets in \( X = \{ x_1, x_2, ..., x_n \} \), we define the normal weighted Hamming distance between \( M \) and \( N \) as follows:

\[
d_h(M, N) = \frac{1}{2} \sum_{i=1}^{n} \left[ \sum_{j=1}^{k} \left( 1 - \frac{h_u^{(i)}(x_j)}{h_v^{(i)}(x_j)} \right) + \left( 1 - \frac{g_u^{(i)}(x_j)}{g_v^{(i)}(x_j)} \right) \right]
\]

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Definition 5 $M = \{(x, h_x(x), g_x(x)) | x \in X\}$ and $N = \{(x, h_x(x), g_x(x)) | x \in X\}$ are two dual hesitant fuzzy sets in $X = \{x_1, x_2, ..., x_n\}$, we define the normal weighted Euclidean distance between $M$ and $N$ as follows:

$$d_e(M, N) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ w_i \frac{1}{h_{ij}^M(x_i) - h_{ij}^N(x_i)} + \frac{1}{g_{ij}^M(x_i) - g_{ij}^N(x_i)} \right]^{1/2}$$

(3)

Based on the normal weighted Euclidean distance measure and the normal weighted Hamming distance measure, we defined the extended normal weighted distance considering the decision maker’s risk preferences.

Definition 6 $M = \{(x, h_x(x), g_x(x)) | x \in X\}$ and $N = \{(x, h_x(x), g_x(x)) | x \in X\}$ are two dual hesitant fuzzy sets in $X = \{x_1, x_2, ..., x_n\}$, we define the generalized weighted distance measure between $M$ and $N$ as follows:

$$d_s(M, N) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ w_i \frac{1}{h_{ij}^M(x_i) - h_{ij}^N(x_i)} + \frac{1}{g_{ij}^M(x_i) - g_{ij}^N(x_i)} \right]^{1/2}$$

(4)

where $\lambda > 0$, indicating the decision makers’ risk attitude.

Especially, When $\lambda = 1$, $d_s$ reduces to $d_h$; When $\lambda = 2$, $d_s$ reduces to $d_e$.

Definition 7. Let $M$ and $N$ be two dual hesitant fuzzy sets on $X = \{x_1, x_2, ..., x_n\}$, then the distance measure between $M$ and $N$ is defined as $d(M, N)$, which satisfies the following properties:

1. $0 \leq d(M, N) \leq 1$;
2. $d(M, N) = 0$, if and only if $M = N$;
3. $d(M, N) = d(N, M)$.

2.3 TOPSIS Method

TOPSIS (technique for order preference by similarity to an ideal), proposed by Hwang and Yoon[23], whose basic principle is to choose the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). It has been acquired great attention in MADM problems. Jahanshahloo[24] extended the TOPSIS method to decision-making problems with fuzzy data, where the rating of each alternative and the weight of each criterion are expressed in triangular fuzzy numbers. Tan[25] developed an extension of TOPSIS method to deal with a multi-criteria interval-valued intuitionistic fuzzy group decision making problems.

Under dual hesitant fuzzy environment, the dual hesitant fuzzy PIS, denoted by $A^+$, and the dual hesitant fuzzy NIS, denoted by $A^-$ can be defined as follows:

$$A^+ = \left\{ x_j, \min_i \left\langle h_{ij}^+ \right\rangle, \max_i \left\langle g_{ij}^+ \right\rangle | j = 1, 2, L, n \right\}$$

(5)

$$A^- = \left\{ x_j, \max_i \left\langle h_{ij}^- \right\rangle, \min_i \left\langle g_{ij}^- \right\rangle | j = 1, 2, L, n \right\}$$

(6)

In order to simplify, we use the Eq. (7) and Eq. (8) instead of the Eq. (5) and Eq. (6).

$$A_j = \{1, 0, 0 | j = 1, 2, L, n\}$$

(7)
\[ A_j = \{0,0,1,1,2,\ldots,n\} \]  

(8)

The relative closeness coefficient of an alternative \( A_i \) with respect to the dual hesitant fuzzy PIS \( A_{+}^{d} \) and \( A_{-}^{d} \) is expressed as follows:

\[ CC_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}}, \]

(9)

where \( 0 \leq CC_i \leq 1, i = 1, 2, \ldots, m \).

Obviously, when an alternative \( A_i \) is closer to the dual hesitant fuzzy PIS and farther from the dual hesitant fuzzy NIS, \( CC_i \) will be closer to 1. Hence, according to the closeness coefficient \( CC_i \), the ranking-order of all alternatives can be determined and the best alternative can be found.

2.4 Procedure of Assessment Model

According to this framework showed in section 2, we can propose a procedure to solve this problem, where attribute values take the form of dual hesitant fuzzy numbers. For an assessment problem, we firstly construct a decision matrix \( D = \left[a_{ij}\right]_{m \times n} \), where all the arguments \( a_{ij} \) are dual hesitant fuzzy numbers, given by the decision maker. As for every alternative \( A_i \) (\( i = 1, 2, \ldots, m \)), the decision maker is invited to express evaluation or preference according to each attribute \( C_j \) (\( j = 1, 2, \ldots, n \)) by a dual hesitant fuzzy number \( a_{ij} \) and specifies the relative weights of the \( n \) attributes denoted as \( w = (w_1, w_2, \ldots, w_n)^T \) with \( 0 \leq w_j \leq 1 \) (\( j = 1, 2, \ldots, n \)) and \( \sum_{i=1}^n w_j = 1 \). Then, the procedure includes the following steps:

Step 1. According to preference of the decision maker, the decision making matrix is obtained as follow:

\[ D_{wxx} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix} \]

(10)

Step 2: Utilize Eq. (7) and Eq. (8) to determine the corresponding dual hesitant fuzzy PISA+ and the dual hesitant fuzzy NISA-.

Step 3: Based on Definition 6, similarity of each alternative with dual hesitant fuzzy PISA+ and the dual hesitant fuzzy NISA- can be obtained as follows:

\[ d_i^{+} = \frac{1}{2} \left[ \sum_{j=1}^{m} w_j \left( \frac{1}{\delta_{ij}} \sum_{k=1}^{n} \tilde{a}_{ij} \tilde{h}_{ij}^{(\alpha)}(x_j) - 0 \right) \right] \]

(11)

\[ d_i^{-} = \frac{1}{2} \left[ \sum_{j=1}^{m} w_j \left( \frac{1}{\delta_{ij}} \sum_{k=1}^{n} \tilde{a}_{ij} \tilde{g}_{ij}^{(\alpha)}(x_j) - 0 \right) \right] \]

(12)

Step 4: According to Eq. (9), the closeness coefficient of each alternative is obtained as follow:
\[ CC_i = \frac{d_i^{-}}{d_i^{-} + d_i^{+}}, \quad i = 1, 2, ..., m \]  

Step 5: Rank the entire alternative Ai according to the closeness coefficient \( CC_i \), where the greater the value \( CC_i \), the better the alternative \( A_i \).

Step 6: End.

3. Illustrative Example

In this section, the proposed method is applied to assess sustainable development ability about a company. We invited an expert as the decision maker. The decision maker chooses four enterprises as the alternatives including \( (A_1) \), \( (A_2) \), \( (A_3) \), \( (A_4) \) and four attributes denoted as brand \( (C_1) \), quality \( (C_2) \), history \( (C_3) \), and revenue \( (C_4) \).

Then, firstly the decision maker with other experts gives the weight vector of these four attributes denoted as \( w = (0.2, 0.25, 0.25, 0.3) \). Secondly, they give their preference of every alternative on each attribute, respectively. Therefore, the decision maker combines the opinions of these experts to provide a dual hesitant fuzzy decision matrix \( D = \tilde{u}_{\alpha=1}^{l=4} \) demonstrated in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Original Dual Hesitant Fuzzy Decision Matrix</th>
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</thead>
<tbody>
<tr>
<td>( A_i )</td>
</tr>
<tr>
<td>( C_1 )</td>
</tr>
<tr>
<td>( C_2 )</td>
</tr>
<tr>
<td>( C_3 )</td>
</tr>
<tr>
<td>( C_4 )</td>
</tr>
</tbody>
</table>

Based on Eqs (11) and (12), it can be obtained distance of each alternative and the dual hesitant fuzzy PISA+ and the dual hesitant fuzzy NISA- in Figures 1-2. When \( l = 1 \), \( l = 2 \), \( l = 6 \), and \( l = 8 \), we can obtain different closeness coefficients according to Eq. (13) showed in Figure 3, respectively.

<table>
<thead>
<tr>
<th>Table 2. Distance Measure between Each Alternative and the Dual Hesitant fuzzy PISA^*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
</tr>
<tr>
<td>( l = 1 )</td>
</tr>
<tr>
<td>( l = 2 )</td>
</tr>
<tr>
<td>( l = 6 )</td>
</tr>
<tr>
<td>( l = 8 )</td>
</tr>
</tbody>
</table>
Table 3. Distance Measure between Each Alternative and the Dual Hesitant Fuzzy NISA

<table>
<thead>
<tr>
<th></th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l=1$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.7028</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.6025</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.7037</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.5088</td>
</tr>
</tbody>
</table>

Figure 1. Distance of Each Alternative to Positive Ideal Solution
In order to simplify, we select \( \lambda \in [1, 8] \) to demonstrate the rule between parameter \( \lambda \) with the rank-order. It is obvious that different parameter \( \lambda \) has different rank. So, we interview these experts to realize their preference. Because most preference is risk-aversion, we select \( \lambda = 2 \).

Then, based on Eqs 11-13 and parameter \( \lambda \), the closeness coefficient and rank-order can be obtained in Table 4.
An adaptable decision making model for sustainable enterprise

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4. Conclusion and Further Study

Recently, the spreading of globalization, technological change and an increasing demand for specialization has led to a new economic activities, new business model and new value propositions. If an enterprise wants to integrate in this trend, sustainable development ability is a tool to help them achieve it. Multiple decision making method with dual hesitant fuzzy set, which is a way of combining hesitant fuzzy set with intuitionistic fuzzy set, is introduced to solve the assessment problem of sustainable development ability of an enterprise. There, the main contributions of this paper include the following: (1) the construction of the sustainable development ability assessment model; (2) the introduction of dual hesitant fuzzy set; (3) the design of distance measure in TOPSIS of dual hesitant fuzzy set; (4) the application of the proposed method.

References


Table 4. Closeness Coefficient and Rank-order

<table>
<thead>
<tr>
<th></th>
<th>Closeness coefficient</th>
<th>rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.7694</td>
<td>1</td>
</tr>
<tr>
<td>A₂</td>
<td>0.5928</td>
<td>2</td>
</tr>
<tr>
<td>A₃</td>
<td>0.561</td>
<td>4</td>
</tr>
<tr>
<td>A₄</td>
<td>0.5892</td>
<td>3</td>
</tr>
</tbody>
</table>

From Table 4, the rank-order is demonstrated as A₁ ≥ A₂ ≥ A₄ ≥ A₃. It is obviously to select A₁ is the optimal alternative. Obviously, this result can cover preferences of the decision maker. It is reasonable and objective in a multiple attribute decision making process. Based on this result, the further research can be implemented better.


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