Two-Stage Flowshop Scheduling with Outsourcing Allowed

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Abstract

In this paper we study the production-scheduling problem for a two-stage flow shop with outsourcing options. The maintenance of in-house machine and job-related discount for outsourcing cost are considered in model, in which total cost and makespan are regarded as bi-objective for the scheduling problem. Based on the analysis of model, we introduce a concept-packaging to reduce the complexity of problem and then develop heuristic algorithm to solve the problem. Computational experiments using different groups of data are conducted to test the algorithm.

Keywords: Production-Scheduling, Outsourcing, Maintenance, Cost discount, Heuristic Algorithm

1. Introduction

Nowadays, driven by the trend of economic globalization, a large number of modern companies are seeking for transformations, in particularly, production outsourcing is becoming common phenomenon during this transformation. The benefits incurred by outsourcing are quite significant. Many manufacturers outsource some jobs to subcontractors so as to focus on their core business and gain profits as much as possible. Outsourcing can relieve the burdens of their own production factory, spare more time for them to concentrate on the core business-selling or marketing for example-rather than the traditional production activities.

On the other sides, with the subcontractor involved in the production activities, subsequent costs incurred by outsourcing are inevitable, which contains both processing costs and transportation costs. Moreover, as we all know, the profit for manufacture is largely depends on the efficiency of its production-scheduling plan. With the outsourcing resources, the optimal production scheduling is even more complicated, but also prospective and valuable. It is the reason why many researchers have delved into this problem under different outsourcing models.

During the outsourcing models, some manufacturer outsourced part of production operations of jobs to the subcontractor, but the other one that part of jobs entirely are outsourced to subcontractor is more common in practice, especially for raw products. Unlike the certain outsourcing cost assumed in previous models, the outsourcing cost is more likely to be announced and charged according to the number of jobs and related processing time. Furthermore, the objective of scheduling problem is usually regarded as the production time (e.g., makespan). However, the scheduling problems with outsourcing allowed in this paper do not only consider the time, but also balance the production time and production cost,

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which is the key objective to make two decisions: whether adopt outsourcing and which jobs are outsourced.

In the rest of this paper, we will describe the problem in detail, followed by the literature review. After introducing the basic mathematic model and related assumptions, we will study the relations between parameters and find out an objective function to coordinate those new elements. Then by developing a heuristic algorithm, we will conduct related computational experiments and compare their results respectively, then to reach a conclusion in the practical level.

2. Literature Review

Outsourcing under machine scheduling models has become a hot issue in manufacturing fields. At the beginning, most such researches are focused on the single-stage problems. Chen and Li (2008) [2], and Lee and Sung (2008a,b) [5-6] have studied the scheduling problem with outsourcing options under the assumption that the subcontractor has virtually unlimited capacity. Therefore they do not need to schedule the outsourced jobs. Qi (2008) [14] considered a single-machine scheduling problem that allows some jobs to be outsourced to a single outsourcing machine, with a transportation delay and a transportation cost. Based on it, Qi(2009) [15] considered the problem that only stage-one operation is the outsourced stage and stage-two operations is started in-house. He developed an optimal algorithm and a heuristic algorithm to solve the problems. In addition to transportation time delay and cost, Mokhtari(2013) [11] believe the outsourcing cost is also the important factor affecting the scheduling. He develop an integer programming formulation to solve the joint scheduling of both in-house and outsourced jobs simultaneously with the objective of minimization for sum of the total weighted completion time and total outsourcing cost.

Since Johnson’s seminal paper of 1954, which analyzed the makespan minimization in a two-machine flow shop, many researchers have studied flow shop problems. Hou and Hoogeveen (2003) [9] and Choi et. al., (2007) [3] considered a three-machine flow shop and showed that the problem is NP-hard in the ordinary sense. For surveys of flow shop scheduling research, the reader is referred to Hejazi and Saghafian (2005) [8] and Cheng et. al., (2000) [1]. It has been shown that minimizing the makespan in a flow shop is strongly NP-hard, even for the three-machine case. Ruiz-Torres et. al., (2008) [13] presented a bi-objective model to deal with the problem of finding outsourcing strategies and considered trade-offs between the following measures. Similar bi-objective job shop scheduling problem was studied by Guo(2014) [10], in which the total tardiness and the outsourcing cost are considered. The lexicographic approach is used to handle these objectives simultaneously and an effective two-phase neighbourhood search (TPNS) is presented to solve this complicated problem.

In terms of objective in scheduling problem with outsourcing allowed, many previous researches considered the weighted sum of time and cost (Qi (2011)) [16]. Lee and Choi (2011) [7] considered a two-stage production problem in which objective is to minimize sum of makespan $C_{\text{max}}(\bar{O})$ associated with sequence and total outsourcing cost, where weight of processing time for different operations are given. In the two-machine ordered flow shop problem in Chung (2013) [4], a schedule is constructed and its performance is measured by the makespan for in-house jobs. Jobs processed by subcontractors require paying an outsourcing cost. The objective is to minimize the sum of the makespan and the total outsourcing cost. Since this problem is NP-hard, an approximation algorithm is presented and three special cases were analyzed. Moreover, Morteza(2013) [12] addressed the production and delivery scheduling integration problem, in which the objective is to
minimize the sum of the total weighted number of tardy jobs and the delivery costs. 
And special cases of the two-machine flow shop problem are investigated and used 
to set up a new branch and bound algorithm.

3. Problem Description

In this paper, the two-stage flowshop scheduling problem with both in-house 
machines and outsourcing machines are studied (See Figure 1), in which second 
operation in-house is the bottleneck (longer processing time and machine 
maintenance). A subcontractor, kilometers away from the in-house factory, serves as 
an outsourcing choice for the manufacturer. Part operations of jobs are not allowed 
to outsourced to subcontractor. Instead, jobs are outsourced with both operations, 
that is, they are entirely outsource. Once the outsourcing producing is completed, all 
the outsourced jobs are transported toward the in-house factory in one batch.

![Figure 1. Two-Stage Flowshop Scheduling Problem with Subcontractor](image)

With outsourcing included in the production activities, the subcontractors supply 
their working machines that are parallel to the in-house machines, which 
complicates the scheduling problem compared with production situation without 
outsourcing. Apart from transportation delay and its costs which many previous 
researchers has already considered, several elements from practice in real world are 
briefly outlined as follows:

- Different processing time for jobs in-house and outsourced
- Job-related discounts on outsourcing cost
- Bi-objective in terms of cost and time

In actual production activities, the processing speed differs from job to job. Due 
to disparate machine qualities, machines of the subcontractor process jobs a little 
fastier than those of in-house machines.

Under the modern in-batch production actualities, the more jobs manufacturers 
outsources to the subcontractors, the more discounts in processing costs that 
manufacturers can enjoy. The subcontractors providing packaging service for 
example, you can find their service price on their website. The more naked products 
the manufacturers outsourced to it, the lower of its price of package service.

Unlike the objective in most previous research, we believe that production time 
and costs cannot be added together for their different units, even though they are 
both the index to scheduling problem.

4. Mathematic Modeling

Based on the problem definition, we suppose that n jobs are to be processed by a 
two-stage flow shop, which has only one machine at each stage. At any time, one 
job can only be processed by one machine, and one machine can only process one 
job. All jobs are available at time zero, and there is no job preemption. Notations are 
as follows:
Parameters:

- $C_{\text{max,}}$: the maximum completion time of in-house and outsourced jobs, $j=1$ suggests operations are processed in-house and $j=2$ for outsourced jobs;
- $C_{\text{max,}}$: the maximum completion time if all jobs are processed in-house without outsourcing;
- $q_{ij}$: the processing time for the second operation of job $i$ on machine $j$.

Decisions variables ($i \leq n$):

- $O_1$: the job sets. $O_1$ is the outsourced job-set, $O_2$ is for the in-house;
- $X_i$: the job allocation. $X_i = 1$ notes that job $i$ is outsourced, otherwise, is in-house processed;
- $S_i$: the starting time for the first operation of job $i$;
- $T_i$: the starting time for the second operation of job $i$.

Other parameters ($i \leq n$):

- $F_i$: the completion time for the first operation of job $i$;
- $C_i$: the completion time for the second operation of job $i$;
- $a_i$: the job set processed before the job $i$;
- $q_i$: the job set next to the job $i$.

Consequently, job $i$ begins its first-stage and second-stage operation satisfying

$$
F_i = S_i + p_{i1} (1 - X_i) + p_{i2} X_i
$$

$$
C_i = T_i + q_{i1} (1 - X_i) + q_{i2} X_i
$$

The job set before job $i$ is denoted as $\mathcal{E}_i = \{ k | S_k < S_i, X_k = 0 \}$, $i,k \leq n$, and $\mathcal{E}_i$ suggests that job $i$ is the first job to be processed in-house or outsourced, in which case, $F_i = 0$; otherwise, $\mathcal{E}_i = \{ k | S_k < S_i, X_k = 1 \}$.

Due to the maintenance for the in-house machine of stage-two operation, the starting time of the second operation is

$$
T_i = \max \left\{ F_i, C_i + (1 - \xi_i) \left[ 1 + \text{sgn}(\text{mod}(j_{il}, 6)) \right] \right\}
$$

With the outsourcing, the production might be saved compared to the case with bottleneck machine in house and no outsourcing resources allowed. The saved time can be viewed as reduced cost, which is related to the avoidance of punishment for delayed production. Thus, without losing generality and accuracy, we consider two indexes:

$$
\alpha = \frac{M}{C_{\text{max,}}}, \quad \beta = \frac{|O_1|}{|O_1| + |O_2|}
$$
Define the cost-reduction percentage brought by saved time as function $g_1$, by customer’s awards as function $g_2$, by subcontractor as $g_3$. Function $g_1$ and $g_2$ are solely related to the variable $\eta$, while $\lambda$ is the only variable to function $g_3$.

Consequently, the objective function—the whole costs—is described as follows:

$$\min f = [1 - g_1(\eta)] \cdot [1 - g_2(\eta)] \cdot \left( \mu - e_2 \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) + [1 - g_2(\eta)] \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) \right)$$

s.t. $s_i \leq F_i$

$X_i \in \{0,1\}$

$s_i \geq 0$

$T_i \geq 0$

(1)

The time and costs are coordinated in formula (1) with the three functions of $g_1, g_2, \text{and } g_3$. In actual production activities, manufacturers can roughly calculate the costs of each order according to the contracts and determine whether a scheduling plan is taken or not. Formula (2) is another “occupation limit” for the operations of stage-one, that is, one machine only processes one job at the same time. Formula (3) suggests that one job which consists of two processing operations must be processed in-house or outsourced entirely, operation-division not allowed. Formula (4) and (5) limit the value for the starting time into a reasonable range.

With the assumption that processing speed in subcontractor’s shop goes faster than that in in-house factory and unit time costs also lower, it is well anticipated that when the transportation costs can be negligible when the number of jobs is large, production activities with outsourcing are more economic than those of non-outsourcing. Then, we will have a deep view on the quality of our objective function.

Define the function of the costs of non-outsourcing as $f'$

$$f' = \left[ \sum_{i=1}^{n} (p_{i1} + q_{i1}) \right] \cdot \left[ 1 - g_1(\eta) \right] \cdot \left[ 1 - g_2(\eta) \right] \cdot \left( \mu - e_2 \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) + \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) \right)$$

(6)

Machine of stage-one/two in the subcontractor processes the first/second operations for the same job faster that those of in-house, that is, $p_{i1} < p_{i2}, q_{i1} < q_{i2}$ It is obvious that $\left\{ 1 - \left[ \left[ 1 - g_1(\eta) \right] \cdot \left[ 1 - g_2(\eta) \right] \right] \cdot \left( \mu - e_2 \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) + \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) \right) \right\} \geq 0$

Thus,

$$f = \left[ \sum_{i=1}^{n} (p_{i1} + q_{i1}) \right] \cdot \left[ 1 - g_1(\eta) \right] \cdot \left[ 1 - g_2(\eta) \right] \cdot \left( \mu - e_2 \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) + \sum_{i=1}^{k_2} (p_{i2} + q_{i2}) \right)$$

(7)

Define $k_1 = \frac{\sum_{i=1}^{n} \left( p_{i1} + q_{i1} \right)}{n}$ and $k_2 = \frac{\sum_{i=1}^{n} \left( p_{i2} + q_{i2} \right)}{n}$, it signifies the cost-discount incurred by reduced makespan. $n = k_1, k_2 \cdot \left[ 1 - g_2(\eta) \right]$. In most situations,
the processing speed for each job of in-house machines is almost a constant ratio to that of subcontractor’s machines, so that the index \(k_1\) and \(k_2\) can be viewed as constant. The index \(l\) and function \(g_3(h_2)\) has the same effect on the costs-value. Consequently, the index \(n\) signifies the effects of the quantity of outsourced jobs on the eventual costs.

Then, \(f = \left[1 - m(1 - l + n)\right] \sum_{i=1}^{n} (p_{i2} + q_{i2}) \quad m \approx \left[1 - m(1 - l + n)\right] \sum_{i=1}^{n} (p_{i2} + q_{i2})\) (8)

Generally speaking, the index \(l\) and \(n\) has the same effect on the costs-value, but from formula (8) it is hard to say the costs-value must be fewer if the manufacturer outsources more jobs to the subcontractor in that on the one hand, there is some balance between the index \(l\) and \(n\) and on the other hand, more jobs that are outsourced to the subcontractor may also raise the whole makespan, resulting in the increasing of the costs-value.

It is well anticipated that the optimal result may lie in the situation that the quantity of outsourced jobs is near to half of the number of the whole jobs and the makespan of in-house factory is near to that of the subcontractor’s shop.

5. Algorithm Design

It is obvious that the problem in this paper is an NP hard problem, with those new elements attached on the basic F2|P: F2 model and there is no certain precise algorithm for this improved problem in this paper, so approximation algorithm is our choice for this problem. In order to get a production plan with less program-running time and fewer costs, we conceive a heuristic algorithm to achieve this goal.

5.1. Package Method and Time Complexity

Given the fact that the jobs in an order may be enormous in production practice, there is no need and necessity for the manufacturers to carry out his production activities under an optimal production-scheduling schema. Otherwise, time complexity will soar in the way of exponential growth as the number of jobs increase. In other words, computer quality and program-running time are two main limitations for such searching. In order to get a relatively optimal scheduling plan with less program-running time, we develop a heuristic algorithm and introduce a new approach for this problem---package.

Packaging is to include a certain number of jobs into a group as a pack. Instead of dealing with jobs one by one, package is the unit for sequencing and outsourcing, in which way the time complexity and program-running time is largely reduced as the number of jobs in a pack increase. However, as the program-running time lessens, the results under this package-algorithm will be further from the optimal results. Consequently, we need to manage to strike a balance between the production costs and the program-running time, which are two equally important aspects for the making of scheduling plan.

Furthermore, we define an index \(D_{\text{max}}\) here as the max difference that we control in the program between the pack numbers of in-house and outsourced. There are two reasons for this definition:

- The application of \(D_{\text{max}}\) can further reduce the program-running time when the number of jobs is great and pack-numbers are large.
- As formula (8), the whole makespan and outsourced-job-scale are influential on the costs so that controlling the pack-number difference between the in-house and outsourced packages into a range can ensure that the whole makespan is not
very long and the costs-discount from the subcontractor is not very small. Consequently, the eventual costs are relatively satisfactory.

Combined with Johnson’s rule which is to provide an optimal sequence for jobs consisting of two independent operations in one process factory, we devise the program procedures shown in Section 5.2.

5.2. Program Procedures

Step 1: Sequence the n jobs to be processed in accordance with the Johnson’s rule.

Step 2: Package the sequenced jobs every percentage, making these packs as an array A.

- If \( \frac{100}{x} \notin N_i \), the left number of jobs is \( n \times \left\lfloor \frac{1}{x} \right\rfloor \), which is less than \( \frac{nx}{100} \); then they are also treated as one pack.

Step 3: Determine the pack in array A one by one that whether it should be processed in-house or outsourced and then wipe it off from array A. If array A is empty, calculate the eventual costs according to the objective function Min f.

- When the program is running, define the dynamic jobs set which is processed in-house/outsourced as \( B_2/B_1 \), at the end of the program, \( B_1=O_1, B_2=O_2 \).

Step 4: Calculate the value of \( d = |B_1| - |B_2| \). If \( |d| < D_{max} \), distribute that pack into the in-house factory or the subcontractor’s shop, then go back to step 3. If \( |d| = D_{max} \), go to step 5.

Step 5: If \( d = D_{max} \), it signifies that there are too many outsourced packs to be processed, then distribute that pack to be processed in-house; if \( d = D_{max} \), the in-house packs are too many, then distribute that pack to the subcontractor shop. Go to step 3.

Step 6: Output the jobs in \( O_1 \) and \( O_2 \) in order respectively and calculate the value of the objective function, compare them and take the minimum result and its corresponding plan of pack allocation.

6. Computational Results and Comparison

We will firstly aim at the costs change with different \( D_{max} \); therefore, the objective function Min f is a function of two variables: \( \text{Min } f_i = f_i(x, D_{max}) \).

Then we will compare the effects of disparate rules on the costs with same index of \( D_{max} \), in which case, \( \text{Min } f_i = f_i(x, \text{Rule}) \).

After determining \( D_{max} \) and the sequencing rule according to the previous results, we move forward to study whether the index \( a \), the job number between twice maintenances, does effects on our conclusion. Under this case, \( \text{Min } f \) can be signifies as \( \text{Min } f_i = f_i(x, a) \).

6.1. Variable \( D_{max} \)

Suppose that the sequencing rule is in accordance with the Johnson’s Rule and the index is valued 1, 2, 3 respectively. The simulation-results are shown in Table 1 and Figure 2.
Table 1. Results and Running Time Under Different $D_{max}$

<table>
<thead>
<tr>
<th>Jobs number in each package</th>
<th>x %</th>
<th>max difference=3</th>
<th>max difference=2</th>
<th>max difference=1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>function value</td>
<td>running time</td>
<td>function value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3%</td>
<td>53145</td>
<td>74753. 1 5</td>
<td>53545</td>
</tr>
<tr>
<td>20</td>
<td>4%</td>
<td>52952</td>
<td>265. 22</td>
<td>53597</td>
</tr>
<tr>
<td>25</td>
<td>5%</td>
<td>53025</td>
<td>12.09</td>
<td>53033</td>
</tr>
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<tr>
<td>35</td>
<td>7%</td>
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<td>40</td>
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<td>45</td>
<td>9%</td>
<td>53093</td>
<td>0.09</td>
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<tr>
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<td>18%</td>
<td>53636</td>
<td>0.00</td>
<td>53636</td>
</tr>
</tbody>
</table>

Figure 2. Function Value Under Different $D_{max}$

From the Figure 2, and Table 1, we can have a clear reorganization that both costs-value and figure-fluctuation when $D_{max}=1$ are inferior to the cases of $D_{max}=2,3$.

For the cases of $D_{max}=2$ and $D_{max}=3$, there is not large difference between the costs-value but the program-running time of $D_{max}=2$ is always less than that of $D_{max}=3$ when the percentage x is the same. And it is within our anticipation that the costs-value will not change much as increases other than the running time soaring manyfold.

Consequently, we choose $D_{max}=2$ as a constant for the following experiments.
6.2. Variable

In order to certify that job-number between twice maintenance dose little effect on the costs-value, we conduct this experiment with \( n=1000 \), and \( \alpha=50,75,100,125 \) respectively.

From the following figure and table, we can see that four figure lines almost overlap together, that is, under the same cases, the fluctuation of \( \alpha \) does little effect on the eventual costs-value.

![Figure 3. Results of the influence of Different \( \alpha \)](image)

7. Conclusions

In this paper, we consider a specific model with several new practical elements involved in, besides the transportation costs and delay, such as costs discount, different processing speed, and periodical maintenance. We introduce a concept of package to deal with the enormous jobs to be processed and choose an index \( D_{\text{max}} \) to balance the whole makespan and the scale of outsourcing jobs. Having mathematically attested that the outsourcing model can help the manufacturer save much money, we conduct several experiments under different sequencing rules and through comparing the costs-value, we finally find out a relatively satisfactory production plan for the manufacturer in practice.

However, there are many other outsourcing models and unpredictable but practical factors that are not considered in this paper, such as policy, road circumstances at the time of transmitting and the extra time delay incurred by that, etc., Nevertheless, our practical level results would provide important guidance for the strategic decisions on the whole.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (NSFC) for the projects of “Production Strategy and Scheduling Problem for Hybrid Production System with Outsourcing Allowed” (No. 71202053).

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