The Application of Intuitionistic Fuzzy Set TOPSIS Method in Employee Performance Appraisal

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Abstract

With the rapid development of knowledge economy, human resources have been one of the most important strategies of all walks of life. The role of performance appraisal of human resource management in the modern enterprise is becoming increasingly prominent, and has become the core issue of human resource management. There are two basic methods, qualitative and quantitative appraisal, in the employee performance appraisal. However, there are a large number of uncertainty fuzzy concepts in the human resource appraisal, which make lots of indexes are difficult to direct quantify. So the article utilizes the intuitionistic fuzzy set TOPSIS method to evaluate the employee performance. Through the case study, the method can evaluate the employee performance more comprehensive and effective. Therefore, the method can provide strong support in decision-making, such as training, promotion, rewards and punishments in human resource management. And it is an effective performance appraisal method.

Keywords: the intuitionistic fuzzy set, the TOPSIS method, the employee performance appraisal

1. Introduction

With the rapid development of economy and the advent of big data era, the competition among enterprises is intensified. However, the talent is the most critical asset in enterprises, and the role and status of talent are becoming more and more prominent in economic activities. To some extent, human resources have been the first core competitiveness of enterprises. Therefore, human resource management has a pretty important significance to the enterprise [1]. Performance appraisal is an important part of human resource management, and it has a vital role in improving the employee productivity and motivation and retaining talent. The purpose of employee performance appraisal is shown in Figure 1. For the performance evaluation, the most critical part is to select the appropriate appraisal method. Although some companies have established a performance appraisal system, the overall effect of the performance appraisal is not ideal due to some indexes are sometimes difficult to quantify. Therefore, human resource managers need to continue to explore the performance evaluation methods, and improve the performance appraisal system [2-4].

Performance appraisal is one of the focuses of management theory research in recent decades, so scholars and managers make useful exploration to the performance appraisal from different point of view. The key issue is how to build a suitable enterprise performance appraisal system, which scholars and managers have a considerable debate on, and they also present a variety of performance appraisal systems [5]. Key performance indicators (KPI) are an evaluation tool for human resources and business performance management. It emphasizes
the managers must select 2-6 job content items as appraisal indexes which are critical and are more closely with the organizational objectives, so that the employees can prioritize the work. Li Yiquan applied the KPI to the performance appraisal of middle managers in universities, and established a performance appraisal system of middle-level managers in universities [6]. Zhou Yucheng utilized the KPI method to construct the performance appraisal system of employee in the functional department from the customer perspective [7]. 360 Appraisal Method, also known as the all-round appraisal method, appraises the employee’s work utilizing the different entities together, such as the employees themselves, the superior, colleagues, subordinates, customers, etc. Yang Chunying used the 360-degree appraisal method to the performance appraisal of core staff [8]. Zhao Fengchang applied the 360-degree performance appraisal method to the middle managers in the enterprises [9]. The Balanced Score Card (BSC), based on the traditional financial indicators, establishes the four-dimensional rating dial from four basic dimensions which are creative learning, internal processes operating, external customer service and economic benefit, and then sets the appropriate performance evaluation indexes system based on the above framework. Therefore, managers can reflect the performance of the organization and staff comprehensively and can conduct the dynamic monitor. Yang Yuqiang applied the BSC to the performance evaluation of the sales team [10]. Sui Qingli uses the BSC to evaluate the civil servants’ performance [11]. The forced distribution method is a method to revise and adjust the results of the performance evaluation. Li Yunmei and Song Nairui applied this method to the staff performance appraisal study in the state-owned enterprises [12]. Management by Objectives (MBO), proposed by Peter Drucker, means employees establish their personal goals in consultation with superiors, which are based on the company's strategic goals and objectives of the relevant department, and these goals should be as consistent as possible. Liang Zhendong used the MBO to evaluate the performance of managers [13]. Feng Guohua evaluated the performance of the middle managers in the state-owned business, combining the KPI and the 360-degree method [14]. Qi Jing evaluated the performance of the middle managers in the state-owned enterprises, combining the BSC and 360-degree method [15].

In the application research of performance appraisal, the foreign scholar Cleveland believed that performance appraisal can provide related information for personnel decision-making, including salary, job recommendation, transfer, training programs, staff development and performance feedback [16]. Spangenberg and Herman proposed a performance evaluation model with strategic integration, emphasizing the incentive and constraint to the results of performance appraisal, to encourage the employee with high performance and punish the employees with poor performance [17]. Boswell and Boudreau considered that the value orientation and development orientation were the two goal orientation of the performance appraisal. The value orientation focused on evaluating the previous performance of employees, regarding the results of the performance appraisal as the basis to employees’ reward or punishment. Whereas the development orientation considered the results of appraisal as the basis to improve the performance of staff in the future [18]. Pettijohn thought that the staff performance appraisal facilitated other human resource management activities in the actual economic management [19].

In summary, each performance appraisal method has its advantages and disadvantages. However the human resource appraisal is a multi-level, multi-factor comprehensive evaluation, there are a lot of fuzzy uncertainty in employee performance appraisal, not only objective factors, for example, many performance indexes are difficult to quantify, but also lots of subjective factors, such as human resource managers’ experience, knowledge and values. Therefore, lots of indicators of performance appraisal are difficult to analyze by
classical mathematical methods [20]. So the article uses the theory and methods of intuition fuzzy set and the TOPSIS method, trying to establish TOPSIS method based on intuitive fuzzy set to evaluate the employee performance. The structure of the article is as follows: in section two, the article describes the relevant knowledge of intuitive of fuzzy sets, and builds the employee performance appraisal index system; in section three, the article establishes the evaluation model of intuition fuzzy set TOPSIS; there is case study in the fourth section.

Figure1. The Purpose of Employee Performance Appraisal

2. The Theory of Intuitionistic Fuzzy Set and the Establishment of Employee Performance Appraisal Index System

2.1 The Theory of Intuitionistic Fuzzy Set

In 1983, the Bulgarian scholar K.T.Atanassow proposed the concept of intuitionistic fuzzy set. Intuitionistic fuzzy set is a successful promotion of fuzzy set, which utilizes two scales (that is membership and non-membership) to describe the ambiguity. It can indicate three states, namely support, opposition and neutrality at the same time. Thus, it can describe the uncertainty of objective phenomenon more delicate and comprehensive.

2.1.1 The Intuitionistic Fuzzy Set and Operations: Let $X$ is a domain. If there are two maps in the domain $X$ : $\mu_A : x \rightarrow [0,1]$ and $\gamma_A : x \rightarrow [0,1]$, making

$\mu_A \in \{x, \mu_A(x), \gamma_A(x)\} \in [0,1]$  

And  

$\gamma_A \in \{x, \mu_A(x), \gamma_A(x)\} \in [0,1]$  

In addition satisfy the condition

$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$  

So $\mu_A$ and $\gamma_A$ determine an intuitionistic fuzzy set $A$ in the domain $X$ : 

$A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$

In the definition above, $\mu_A$ and $\gamma_A$ are the membership function and non-membership function respectively. $\mu_A(x)$ and $\gamma_A(x)$ are the degree of membership and non-membership of element $x$ belonging to $A$ respectively [21]. The intuitionistic fuzzy set $A$ can be denoted as $A = (x, \mu_A, \gamma_A)$. The set consisting of all the intuitionistic fuzzy sets in the domain $X$ is recorded as $IFS[X]$. 
For the intuitionistic fuzzy set $A$ in the domain $X$, $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ is the vague index or hesitancy degree of element $x$ in $A$. It is a measurement of hesitation degree whether $x$ belongs to intuitionistic fuzzy set $A$ or not. Obviously, for $\forall x \in X$, there is $0 \leq \pi_A(x) \leq 1$. When $\mu_A + \gamma_A = 1$ or $\pi_A = 0$, the intuitionistic fuzzy set degenerates into fuzzy set. Clearly, for any fuzzy set in the domain $X$, there is $\pi_A = 1 - \mu_A - (1 - \mu_A) = 0$.

For the intuitionistic fuzzy set $A$, the membership $\mu_A(x) \in [0,1]$, non-membership $\gamma_A(x) \in [0,1]$ and the hesitation degree $\pi_A(x) \in [0,1]$ express support, opposition and neutrality respectively. Thus, intuitionistic fuzzy set can describe "neither this nor that".

Let $A$ and $B$ are any two intuitionistic fuzzy sets in the domain $X$, $\lambda > 0$ is any real number. The definition of intuitionistic fuzzy set operations is as follows:

1. Containment relationship: For any $x \in X$, if and only if there are $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$, so $A \subseteq B$. Then again, we can define $A \supseteq B$.

2. Equality relationship: For any $x \in X$, if and only if there are $\mu_A = \mu_B$ and $\gamma_A = \gamma_B$, so $A = B$.

3. Complementary set: $A^c = \{(x, \gamma_A(x), \mu_A(x)) \mid x \in X\}$.

4. Intersection set: $A \cap B = \{(x, \mu_A(x) \land \mu_B(x), \gamma_A(x) \lor \gamma_B(x)) \mid x \in X\}$, among which, the symbol $\land$ and $\lor$ indicate the operations of choosing small and choosing big respectively, namely $\min$ and $\max$ operators.

5. Union set: $A \cup B = \{(x, \mu_A(x) \lor \mu_B(x), \gamma_A(x) \land \gamma_B(x)) \mid x \in X\}$.

6. Sum: $A + B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \gamma_A(x)\gamma_B(x)) \mid x \in X\}$.

7. Product: $AB = \{(x, \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x)) \mid x \in X\}$.

8. Scalar multiplication: $\lambda A = \{(x, 1 - (1 - \mu_A(x))\lambda, (\gamma_A(x))^\lambda) \mid x \in X\}$.

9. Power: $A^\lambda = \{(x, (\mu_A(x))^{\lambda\omega}, 1 - (1 - \mu_A(x))^{\lambda\omega}) \mid x \in X\}$.

2.1.2 The Distance between Intuitionistic Fuzzy Sets

The distance is a pretty important concept in the intuitive fuzzy set theory. It can reflect the difference between two intuitive fuzzy sets.

Let $A$ and $B$ are two intuitive fuzzy sets in the domain $X$, $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$, $B = \{(x, \mu_B(x), \gamma_B(x)) \mid x \in X\}$, $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ and $\pi_B(x) = 1 - \mu_B(x) - \gamma_B(x)$. Then the Euclidean normalized distance between $A$ and $B$ is:

$$d(A, B) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

Its weighted distance is:

$$d'(A, B) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} \omega_i [(\mu_A(x_i) - \mu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}$$

In the definition above, $\omega_i$ is the weight of $x_i$ and satisfies $0 \leq \omega_i \leq 1$ and $\sum_{i=1}^{n} \omega_i = 1$.

It is easy to prove that the distance has the following properties:

1. $d(A, B) \geq 0$
2. $d(A, B) = 0$ if and only if $A = B$
3. $d(A, B) = d(B, A)$
4. $d(A, B) \leq d(A, C) + d(C, B)$
5. $C$ is closer to $A$ than $B$ if and only if $d(A, C) \leq d(A, B)$
2.2 The Establishment of Employee Performance Appraisal Index System

In this part, we emphasize the design of the index system. In all the performance evaluation systems, the most difficult part is to design an index system matching the evaluation method. The matching degree between the index system and evaluation method determines the outcome of the appraisal whether in line with the true performance of employee or not.

2.2.1 The Principles of Establishing the Employee Performance Appraisal Index System

The index system is a scientific and complete system consisting of a series of mutually connected and restricted index. Indexes can reflect some characteristics of the research object, while combining a number of mutually connected and restricted indexes can meet and expound the features and regularity of complex phenomena from several aspects and multiple dimensions. The principles of designing this evaluation index system are as follows:

First, the principle of scientific and rational. The design of index system should be able to fully reflect the connotation and propose of the employee performance appraisal, and can describe the characteristics of performance appraisal reasonably;

Second, the principle of practical and comparability. The index should be easy to be understood and have wide applicability. It also can distinguish the employee's behavior. The basic data is easy to collect and can be compared with historical data;

Third, the principle of combining the qualitative and quantitative indexes. The indexes of performance appraisal should be as quantitative as possible. For some indexes which are hard to quantify but have great significance can also be described by the qualitative indexes;

Fourth, the principle of comprehensiveness and emphasis. The index system should reflect the whole situation of employee and consider the various factors influencing the performance appraisal comprehensively, whereas highlight the key factors and priorities.

2.2.2 The Establishment of Employee Performance Appraisal Index System

 Keeping to these principles above and referring related literature, the article establishes the evaluation index system of performance appraisal from the following four aspects, which are shown in table 1 below:

<table>
<thead>
<tr>
<th>The index system of employee performance</th>
<th>the first class index</th>
<th>the second class index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talent quality $U_1$</td>
<td>Enterprise $U_{11}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Responsibility $U_{12}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Integrity $U_{13}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Professional responsibility $U_{14}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Team spirit $U_{15}$</td>
<td></td>
</tr>
<tr>
<td>Personal ability $U_2$</td>
<td>Professional skills $U_{21}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coordination ability $U_{22}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Learning ability $U_{23}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Decision-making ability $U_{24}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Interpersonal skills $U_{25}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The quantity of job $U_{31}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The quality of work $U_{32}$</td>
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</table>
Nowadays, talent quality has become the first index to evaluate employees in various enterprises. It can be divided into five second indexes which are the enterprise, responsibility, integrity, professional responsibility and team spirit. The enterprise emphasizes the efforts of employee in improving efficiency, creating value and shaping corporate image, and the disposing of enterprise interests and personal gains or losses. The responsibility evaluates whether employees do their work by heart. Integrity evaluates whether employees get the work they promised done timely and correctly. Team spirit is proposed based on the complexity of the modern work, using to evaluate the information sharing and cooperation intention of employee. It can be measured by whether employee can actively cooperate within the department or among departments in the past.

Personal ability evaluates employee performance from five aspects which are the professional skills, coordination ability, learning ability, decision-making ability, interpersonal skills. The professional skills mainly examine the professional knowledge of employee. The coordination ability evaluates the management and communication skills, and the ability of integration and coordination of resources and information, which can be measured by the employee cooperation among various departments. The learning ability evaluates staff whether they can obtain accurate knowledge and information in a quick, simple and effective way, and convert them into their own capabilities. It can be evaluated by whether employee can learn new knowledge efficiently and the ability of understanding, analysis and comprehension of work in the past. The decision-making ability evaluates the synthetic abilities of making decisions and direction. The interpersonal skills examine the ability of handling the relationships within and outside the organization properly, including establishing extensive connection with the surrounding environment and the ability of absorption and integration of the external information, and the ability of dealing with the relationship between superior and subordinate.

For employee performance appraisal, the work performance can be divided into five aspects, such as the quantity of job, the quality of work, the efficiency of work, the effect of work and the innovation of work. The quantity of job refers to the workload the employee completing in a certain period of time. The quality of work considers the degree of realizing the employee work. The efficiency of work can be measured by the number of days on average to completing a particular work. The effect of work considers the contribution of stuff’s work to the company. The innovation of work evaluates the ability of proposing new theory and method at work.

Work attitude is divided into six indexes, which are active learning and training, the enthusiasm of work, execution, attendance rate, discipline and the rate of extra work. The attendance rate and the rate of extra work are easy to quantify. The enthusiasm of work considers the completion status of the employee’s work as well as whether employees

<table>
<thead>
<tr>
<th>appraisal</th>
<th>Work performance $U_3$</th>
<th>The efficiency of work $U_{3e}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>The effect of work $U_{3f}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The innovation of work $U_{3i}$</td>
</tr>
<tr>
<td>Work attitude $U_4$</td>
<td>Active learning and training $U_{4a}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The enthusiasm of work $U_{4e}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Execution $U_{4x}$</td>
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<tr>
<td></td>
<td>Attendance rate $U_{4t}$</td>
<td></td>
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<td></td>
<td>Discipline $U_{4d}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The rate of extra work $U_{4w}$</td>
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</table>
cooperate with other colleagues to finish the work actively. Execution refers to whether employees carry out the command of superior well. Discipline can be evaluated by the execution of the company rules and regulations in the past.

With regard to the above indexes, some are easy to quantify while some are not. For the qualitative indexes in the employee performance appraisal, the human resource managers can use three methods to determine: (1) the employee self-evaluation; (2) the evaluation from the company performance appraisal team; (3) the evaluation from the colleagues in the same and related department.

The employee performance appraisal system is shown in the figure below:

![Figure 2. The Enterprise Employee Performance Appraisal System](image)

3. The Intuition Fuzzy Set TOPSIS Method

3.1 Introduction to TOPSIS Method

In 1981, Huang Qinglai first put forward the TOPSIS method (Technique for Order Preference by Similarity to an Ideal Solution), to solve the multi-objective problem of individual decision makers, namely the sorting method of closing to the ideal solution. It sorts according to the proximity degree of the evaluated object to the ideal target. It has been widely used since it was proposed and it is an effective method to solving the multi-objective decision.

The principle of TOPSIS method: Its basic principle is sorting according to comparing the distance of the evaluated objects to the optimal and the worst scheme respectively. If the evaluated object is closest to the optimal scheme and furthest away from the worst one, then it is the optimal solution, or is the worst. The basic idea of TOPSIS method is as follows: Based on the weighted decision matrix, we can find out the optimal scheme and the worst one in the decision matrix, and then calculate the distance of the evaluated object to the optimal and the worst scheme respectively. Then we can obtain the relative proximity degree of the evaluated
object to the optimal scheme, in order to evaluate the advantages and disadvantages of each evaluation object.

3.2 The Intuition Fuzzy Set TOPSIS Method

Let \( \{A_1, A_2, ..., A_n\} \) is a scheme set consisting of \( n \) evaluated objects. The evaluation indexes of the evaluated objects are \( U = \{U_1, U_2, ..., U_m\} \). The weight of evaluation index \( U_j \) is \( \omega_j \), which satisfies \( \omega_j \geq 0 \) and \( \sum \omega_j = 1 \).

(1) Establish the intuitionistic fuzzy set evaluation matrix

The evaluation of scheme \( A_i \) in regard to the evaluation index \( U_j \) can be expressed by the intuitionistic fuzzy set: \( F_{ij} = (\mu_{ij}, \gamma_{ij}, \pi_{ij}) \), in which \( \mu_{ij} \in [0,1] \) denotes the importance of the evaluation index \( U_j \) to the scheme \( A_i \) and \( \gamma_{ij} \in [0,1] \) means the unimportance of the evaluation index \( U_j \) to the scheme \( A_i \), satisfying \( 0 \leq \mu_{ij} + \gamma_{ij} \leq 1 \) and \( \pi_{ij} = 1 - \mu_{ij} - \gamma_{ij} \). The evaluation value of scheme \( A_i \) in regard to \( n \) indexes can be denoted by:

\[
(F_{i1}, F_{i2}, ..., F_{im}) = (\langle \mu_{i1}, \gamma_{i1}, \pi_{i1} \rangle, \langle \mu_{i2}, \gamma_{i2}, \pi_{i2} \rangle, ..., \langle \mu_{in}, \gamma_{in}, \pi_{in} \rangle)
\]

We can obtain the evaluation value of scheme \( A_i \) about the index \( U_j \) through the expert scoring method. Its intuitionistic fuzzy set evaluation matrix is:

\[
F = (\langle \mu_{ij}, \gamma_{ij}, \pi_{ij} \rangle)_{n \times m} = \begin{pmatrix}
\langle \mu_{11}, \gamma_{11}, \pi_{11} \rangle & \langle \mu_{12}, \gamma_{12}, \pi_{12} \rangle & \cdots & \langle \mu_{1n}, \gamma_{1n}, \pi_{1n} \rangle \\
\langle \mu_{21}, \gamma_{21}, \pi_{21} \rangle & \langle \mu_{22}, \gamma_{22}, \pi_{22} \rangle & \cdots & \langle \mu_{2n}, \gamma_{2n}, \pi_{2n} \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle \mu_{m1}, \gamma_{m1}, \pi_{m1} \rangle & \langle \mu_{m2}, \gamma_{m2}, \pi_{m2} \rangle & \cdots & \langle \mu_{mn}, \gamma_{mn}, \pi_{mn} \rangle 
\end{pmatrix}
\]

(2) Determine the weight of each evaluation index by intuitionistic fuzzy entropy method

There are plenty of methods to calculate the weight of index, such as analytic hierarchy process and the expert scoring method. Considering the characteristics of the evaluation index data, the article uses the intuitionistic fuzzy entropy weight method to calculate the weight of each evaluation index.

Given an intuitionistic fuzzy set \( A = (x, \mu_A, \gamma_A) \), in which \( \mu_A \) is the degree of membership, \( \gamma_A \) is the degree of non-membership, \( \pi_A = 1 - \mu_A - \gamma_A \) is the hesitancy degree and \( \lambda_A = 1 - | \mu_A - \gamma_A | \) is the fuzzy degree. Then the intuitionistic fuzzy entropy of evaluation index \( U_j \) is:

\[
E(U_j) = \frac{1}{m} \sum_{i=1}^{m} \left( \frac{\pi_{ij}^2 + \lambda_{ij}^2}{2} \right), \quad i = 1, 2, ..., m, \quad j = 1, 2, ..., n
\]

The bigger the intuitionistic fuzzy entropy \( E(U_j) \) is, the higher the uncertainty of \( U_j \) will be, namely the fuzzy degree of index \( U_j \) is higher, then the index weight of \( U_j \) should be smaller. The weight of index \( U_j \) is:

\[
\omega_j = \frac{1 - E(U_j)}{\sum_{j=1}^{n} (1 - E(U_j))}, \quad j = 1, 2, ..., n
\]

Then we can obtain the weight vector \( \omega = (\omega_1, \omega_2, ..., \omega_n)^T \) of evaluation index.

(3) Determine the intuitionistic fuzzy set positive ideal solution (the best solution) and the negative ideal solution (the worst solution).

The intuitionistic fuzzy set positive ideal solution \( A' \) and the intuitionistic fuzzy set negative ideal solution \( A^- \) are:
\[ A^+ = (\langle \mu^+_1, y^+_1, \pi^+_1 \rangle, \langle \mu^+_2, y^+_2, \pi^+_2 \rangle, \ldots, \langle \mu^+_n, y^+_n, \pi^+_n \rangle) \]
\[ A^- = (\langle \mu^-_1, y^-_1, \pi^-_1 \rangle, \langle \mu^-_2, y^-_2, \pi^-_2 \rangle, \ldots, \langle \mu^-_n, y^-_n, \pi^-_n \rangle) \]

in which, \( \mu^+_j = \max \{ \mu^+_i \} \), \( \gamma^+_j = \min \{ \gamma^+_i \} \), \( \mu^-_j = \min \{ \mu^-_i \} \), \( \gamma^-_j = \max \{ \gamma^-_i \} \), \( \pi^+_j = 1 - \mu^+_j - \gamma^+_j \).

(4) Calculate the weighted Euclidean normalized distance of each scheme \( A_i \) to the intuitionistic fuzzy set positive and negative ideal solution:

\[
d(A_i, A^+) = \sqrt{\frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu^-_j - \mu^-_i)^2 + (\gamma^-_j - \gamma^-_i)^2 + (\pi^-_j - \pi^-_i)^2]} \]
\[
d(A_i, A^-) = \sqrt{\frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu^-_j - \mu^-_i)^2 + (\gamma^-_j - \gamma^-_i)^2 + (\pi^-_j - \pi^-_i)^2]} \]

(5) Calculate the relative proximity degree of scheme \( A_i \) to the intuitionistic fuzzy set positive ideal solution:

\[
C_i = \frac{d(A_i, A^+)}{d(A_i, A^+) + d(A_i, A^-)}, \quad i = 1, 2, \ldots, m
\]

We sort according to the relative proximity degree of the scheme \( A_i \) to the intuitionistic fuzzy set positive ideal solution. The bigger the \( C_i \) ( \( 0 \leq C_i \leq 1 \) ) is, the closer the scheme \( A_i \) to the positive ideal solution. Therefore the scheme \( A_i \) is better and vice versa.

4. Case Study

Assuming an enterprise chooses five employees to evaluate their performance. The employees are \( A_1, A_2, A_3, A_4, A_5 \) respectively, denoted by \( A = \{A_1, A_2, A_3, A_4, A_5\} \). According to the characteristics of the employee performance appraisal, the article chooses four first class indexes, which are talent quality \( U_1 \), personal ability \( U_2 \), work performance \( U_3 \), and work attitude \( U_4 \), denoted by the attribute set \( U = \{U_1, U_2, U_3, U_4\} \), to assess the employees performance. Each index can be divided into several second class indexes. Following the steps of the intuition fuzzy set TOPSIS method above, the article sets up the intuitionistic fuzzy evaluation matrix of schemes firstly, then determines the weight of each index by the intuitionistic fuzzy entropy weight method, defining the optimal scheme and the worst one, and calculates the distance of each scheme to the optimal solution and the worst one. Finally, the article calculates the relative proximity degree of each scheme to the intuitionistic fuzzy positive ideal solution.

(1) We can obtain the degree of membership, the degree of non-membership and the hesitancy degree of scheme \( A_i \) about the attribute \( U_j \in U \), according to the existing knowledge, experience and investigation of the human resource managers. Therefore we can obtain the intuitionistic fuzzy evaluation matrix of scheme \( A_i \) about the evaluation indexes:

<table>
<thead>
<tr>
<th></th>
<th>( U_1 )</th>
<th>( U_2 )</th>
<th>( U_3 )</th>
<th>( U_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( 0.6,0.3,0.1 )</td>
<td>( 0.4,0.3,0.3 )</td>
<td>( 0.3,0.5,0.2 )</td>
<td>( 0.6,0.3,0.1 )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( 0.5,0.3,0.2 )</td>
<td>( 0.5,0.3,0.2 )</td>
<td>( 0.4,0.1,0.5 )</td>
<td>( 0.4,0.1,0.5 )</td>
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<tr>
<td>( F = A )</td>
<td>( 0.8,0.1,0.1 )</td>
<td>( 0.3,0.2,0.5 )</td>
<td>( 0.6,0.1,0.3 )</td>
<td>( 0.5,0.4,0.1 )</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>( 0.7,0.2,0.1 )</td>
<td>( 0.1,0.6,0.3 )</td>
<td>( 0.5,0.3,0.2 )</td>
<td>( 0.2,0.5,0.3 )</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>( 0.6,0.2,0.2 )</td>
<td>( 0.5,0.1,0.4 )</td>
<td>( 0.3,0.4,0.3 )</td>
<td>( 0.6,0.2,0.2 )</td>
</tr>
</tbody>
</table>
(2) Determine the weight of each evaluation index by the intuitionistic fuzzy entropy method

\[ E(U_i) = \frac{1}{m} \sum_{i=1}^{m} \sqrt{\frac{\pi_i^2 + \lambda_i^2}{2}} = \frac{1}{5} \left( \sqrt{\frac{0.1^2 + 0.7^2}{2}} + \sqrt{\frac{0.2^2 + 0.8^2}{2}} + \sqrt{\frac{0.1^2 + 0.3^2}{2}} + \sqrt{\frac{0.1^2 + 0.5^2}{2}} + \sqrt{\frac{0.2^2 + 0.6^2}{2}} \right) = 0.423 \]

Then again, we can obtain:

\[ E(U_2) = 0.581, E(U_3) = 0.571, E(U_4) = 0.547. \]

Thus it can be seen that the intuitionistic fuzzy entropy \( E(U_i) \) is the smallest, and the fuzzy degree of \( U_i \) is the smallest, so the weight of \( U_i \) should be the biggest. While \( E(U_2) \) and \( E(U_3) \) are bigger, and there is not much difference in their values, then the weight of \( U_2 \) and \( U_3 \) should be smaller and the weight of \( U_4 \) may be near to the value of \( U_3 \). We can obtain the weight of each index by the intuitionistic fuzzy entropy method:

\[ \omega_i = \frac{1 - E(U_i)}{1 - E(U_1) + (1 - E(U_2)) + (1 - E(U_3)) + (1 - E(U_4))} \]

\[ \omega_1 = \frac{1 - 0.423}{1 - 0.423 + 0.581 + 0.571 + 0.547} = 0.307 \]

\[ \omega_2 = \frac{1 - 0.581}{1 - 0.423 + 0.581 + 0.571 + 0.547} = 0.223 \]

\[ \omega_3 = \frac{1 - 0.571}{1 - 0.423 + 0.581 + 0.571 + 0.547} = 0.229 \]

\[ \omega_4 = \frac{1 - 0.547}{1 - 0.423 + 0.581 + 0.571 + 0.547} = 0.241 \]

Therefore, the weight vector of the evaluation indexes is \( \omega = (0.307, 0.223, 0.229, 0.241)^\top \).

As the article says in making the index system, the talent quality has become the first index to evaluate employees in various enterprises. The weight \( \omega_1 = 0.307 \) is the biggest in the weight vector, fully reflecting this discourse. Work attitude reflects the employees' motivation and the cognition of the enterprise, and enterprises attach bigger importance to it, so its weight is a little bigger. The fuzzy degree of personal ability and work performance is larger, and the difference between the two fuzzy degrees is not big, so their weights are smaller and the values are very close.

(3) Calculate the intuitionistic fuzzy set positive ideal solution and negative ideal solution

The intuitionistic fuzzy set positive ideal solution is:

\[ A^+ = (\langle \mu_1^+, \gamma_1^+, \pi_1^+ \rangle, \langle \mu_2^+, \gamma_2^+, \pi_2^+ \rangle, \ldots, \langle \mu_n^+, \gamma_n^+, \pi_n^+ \rangle) = ((0.8, 0.1, 0.1), (0.5, 0.1, 0.4), (0.6, 0.1, 0.3), (0.6, 0.1, 0.3)) \]

The intuitionistic fuzzy set negative ideal solution is:
(4) Calculate the weighted Euclidean normalized distance of each scheme $A_i$ to the intuitionistic fuzzy set positive and negative ideal solution:

The weighted Euclidean normalized distance of scheme $A_i$ to the intuitionistic fuzzy set positive ideal solution is:

$$d(A_i, A^+) = \sqrt{\frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu_{ij} - \mu_i^+) + (\gamma_{ij} - \gamma_i^+)^2 + (\pi_{ij} - \pi_i^+)^2]}$$

$$d(A_i, A^+) = \frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu_{ij} - \mu_i^+) + (\gamma_{ij} - \gamma_i^+)^2 + (\pi_{ij} - \pi_i^+)^2]$$

$$= \left[\frac{1}{2} \times 0.307[(0.6 - 0.8)^2 + (0.3 - 0.1)^2 + (0.1 - 0.1)^2] + \frac{1}{2} \times 0.223[(0.4 - 0.8)^2 + (0.3 - 0.1)^2 + (0.3 - 0.1)^2] + \frac{1}{2} \times 0.229[(0.3 - 0.8)^2 + (0.5 - 0.1)^2 + (0.2 - 0.1)^2] + \frac{1}{2} \times 0.241[(0.6 - 0.8)^2 + (0.3 - 0.1)^2 + (0.1 - 0.1)^2]\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{1}{2} (0.025 + 0.054 + 0.096 + 0.019)} = \sqrt{0.097} = 0.311$$

Then again, we can obtain $d(A_1, A^+) = 0.221$, $d(A_4, A^+) = 0.153$, $d(A_5, A^+) = 0.309$ and $d(A_3, A^+) = 0.18$.

Therefore it can be seen that the scheme $A_3$ is nearest to intuitionistic fuzzy set positive ideal solution. $A_4$ and $A_5$ are nearer to the positive ideal solution while $A_1$ and $A_4$ are far away from the ideal solution relatively.

The weighted Euclidean normalized distance of scheme $A_i$ to the intuitionistic fuzzy set negative ideal solution is:

$$d(A_i, A^-) = \sqrt{\frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu_{ij} - \mu_i^-)^2 + (\gamma_{ij} - \gamma_i^-)^2 + (\pi_{ij} - \pi_i^-)^2]}$$


\[ d(A_i, A^-) = \frac{1}{2} \sum_{j=1}^{n} \omega_j [(\mu_j - \mu_i)^2 + (\gamma_j - \gamma_i)^2 + (\pi_j - \pi_i)^2] \]

\[ = \frac{1}{2} \times 0.307[(0.6 - 0.5)^2 + (0.3 - 0.3)^2 + (0.1 - 0.2)^2] \]

\[ + \frac{1}{2} \times 0.223[(0.4 - 0.1)^2 + (0.3 - 0.6)^2 + (0.3 - 0.3)^2] \]

\[ + \frac{1}{2} \times 0.229[(0.3 - 0.3)^2 + (0.5 - 0.5)^2 + (0.2 - 0.2)^2] \]

\[ + \frac{1}{2} \times 0.241[(0.6 - 0.2)^2 + (0.3 - 0.5)^2 + (0.1 - 0.3)^2] \]

\[ = \sqrt{\frac{1}{2} (0.006 + 0.04 + 0.06)} = \sqrt{0.053} = 0.23 \]

Then again, we can obtain \( d(A_2, A^-) = 0.296 \), \( d(A_3, A^-) = 0.308 \), \( d(A_4, A^-) = 0.136 \) and \( d(A_5, A^-) = 0.289 \).

Therefore it can be seen that the scheme \( A_i \) are farthest away from the intuitionistic fuzzy set negative ideal solution. \( A_1, A_2 \) and \( A_3 \) are farther away from the negative ideal solution while \( A_4 \) is the nearest to the negative ideal solution.

It can be seen from the result of the calculation that the scheme \( A_3 \) is nearest to intuitionistic fuzzy set positive ideal solution and farthest away from the intuitionistic fuzzy set negative ideal solution, satisfying the definition of the optimal solution in the article. In order to determine the ranking of the employee performance, it is necessary to calculate relative proximity degree of each scheme to the intuitionistic fuzzy set positive ideal solution.

The distance of each employee to the intuitionistic fuzzy set positive and negative ideal solution is shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( A_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A^- )</td>
<td>0.311</td>
<td>0.221</td>
<td>0.153</td>
<td>0.309</td>
<td>0.18</td>
</tr>
<tr>
<td>( A^+ )</td>
<td>0.23</td>
<td>0.296</td>
<td>0.308</td>
<td>0.136</td>
<td>0.290</td>
</tr>
</tbody>
</table>

Calculate the relative proximity degree of each scheme to the intuitionistic fuzzy set positive ideal solution:

\[ C_1 = \frac{d(A_1, A^-)}{d(A_1, A^+) + d(A_1, A^-)} = \frac{0.23}{0.311 + 0.23} = 0.425 \]

\[ C_2 = \frac{d(A_2, A^-)}{d(A_2, A^+) + d(A_2, A^-)} = \frac{0.296}{0.221 + 0.296} = 0.573 \]

\[ C_3 = \frac{d(A_3, A^-)}{d(A_3, A^+) + d(A_3, A^-)} = \frac{0.308}{0.153 + 0.308} = 0.668 \]

\[ C_4 = \frac{d(A_4, A^-)}{d(A_4, A^+) + d(A_4, A^-)} = \frac{0.136}{0.136 + 0.309} = 0.306 \]
By the results of the calculation, according to the relative proximity degree of each scheme to the intuitionistic fuzzy set positive ideal solution, we can obtain the ranking of the employee performance: \( C_1 > C_3 > C_5 > C_4 > C_2 \). The performance of employee \( A_1 \) is the highest in these five employees, conforming to the calculation results of distance before. That is to say the employee \( A_1 \) is nearest to intuitionistic fuzzy set positive ideal solution and farthest away from the intuitionistic fuzzy set negative ideal solution. The enterprise can give priority to employee \( A_1 \) when giving promotion and reward. While employee \( A_4 \) is nearest to the negative ideal solution and further away from the positive ideal solution, his performance is the lowest in the five employees. The employee and human resource managers should work together to improve his self-quality and the level of work, so as to improve his performance as a whole.

Through the case analysis above, the value of each index of employee \( A_3 \) is larger on average, and when determining the intuitionistic fuzzy evaluation matrix, the experts give a higher score to the personal qualities, work performance, work attitude, of which the weights are higher, of employee \( A_3 \). The experts scoring are consistent with the results of the calculation, showing that the model has the availability in the employee performance appraisal and can provide the objective fact for the decision-making of the human resource managers.

5. Conclusion

The effective employee performance appraisal plays a pretty important role in perfecting the internal management, improving the market competitiveness of the enterprise and attracting and retaining the excellent employees. The article establishes a set of employee performance appraisal index system according to the characteristics of the enterprise employee performance appraisal, describing the performance appraisal of employees comprehensively and systematically, then applies the index system to the employee performance appraisal. The article proposes the intuitionistic fuzzy set TOPSIS method based on intuitionistic fuzzy set and TOPSIS method, then establishes the intuitionistic fuzzy evaluation matrix of each evaluation index, and calculates the distance of each employee to the intuitionistic fuzzy set positive and negative ideal solution using the weight of each evaluation index and the distance intuitionistic fuzzy set. Therefore we can obtain the comprehensive evaluation index of employee performance appraisal and the ranking of each employee performance. It can provide the objective fact for the decision-making for human resource managers. The case study shows that the intuitionistic fuzzy set TOPSIS method is effective and feasible in the employee performance appraisal. Through the above analysis, the intuitionistic fuzzy TOPSIS method is beneficial to evaluate the performance of human resources more comprehensively and reflect the quality of employee as a whole. The intuition fuzzy set TOPSIS method can avoid the problem that the qualitative indexes are difficult to quantify of the traditional performance appraisal methods, making the evaluation results relatively objective and fair, and has high accuracy and operability. It can provide strong support for the decision-making, such as training, promotion, rewards and punishments, of human resources management and it is an effective method for performance appraisal. The principle of the method is simple and is easy to implement. The method can provide a new thought for the complex multiple attribute decision-making problems in actual economic management, and is expected to be applied in more similar fields.
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