Application of Weighted Markov Chain in Stock Price Forecasting of China Sport Industry

Qing-xin Zhou

School of Basic Science, Harbin University of Commerce, Harbin
E-mail: zhouqingxin1981@126.com

Abstract

The stock's price of China Sport Industry was studied in this paper, and the theory of Weighted Markov Chain was applied to forecast the stock's price. The main contents are as follows:

Firstly, based on the historical data of China Sport Industry's stock, Spss software was used to conduct fuzzy clustering analysis, and the known data was divided into six categories.

Secondly, For the application of Weighted Markov Chain, the historical data of the stock's closing price was tested about "Markov Property". After testing that the stock's price of China industry satisfies "Markov Property", state transition matrix was constructed using data. The weight values of every state were calculated with the method of Weighted Markov Chain theory and prediction intervals of the industry's future stock prices were obtained.

This article bases on the markov property of the stock price sequence and uses weighted markov model to predict the stock price change block, providing certain reference value for individual or collective investment. And this method has strong applicability.

Keywords: Weighted Markov Chain, "Markov Property" Test, Transition Matrix, Stock Price Forecast

1. Introduction

During the period of 1906-1912, the Russian mathematician Markov proposed and studied a random process to study the natural process by mathematics analysis method, this is the known Markov process.

The application of Weighted Markov Chain in the financial sector has been a hot and key point in the research of scholars at home and abroad, and the group of scholars in this field is relatively more. When carrying on the forecast, the weighted Markov forecasting method does not require continuous historical data; it needs only recent data to predict the future.

In short, Weighted Markov Chain can be used to describe a lot of dynamic system problem in economic and social phenomenon [1-3]. From the perspective of the methods to predict, some directly use Weighted Markov Chain transition probability for prediction; some combine it with fuzzy mathematics to predict and some combine it with the linear time series model; furthermore, various kinds of the Weighted Markov Chain are also used to predict. From the point of the predicted results, the accuracy and practicability of the Weighted Markov Chain are relatively high and clear. As it is objective and easy to understand, simple to calculate, accurate and reliable, it is worthy of our study and promotion [4].

Taking the stocks of China Sports Industry as an example, this paper used the Weighted Markov Chain theory to forecast the stock price.
2. Basic Theory

2.1. The Basic Conception of Markov

Definition 1. Assuming whenever the process \( \{ X_n \} \) is in \( i \), the probability with which the process \( \{ X_n \} \) will be in \( j \) at the next time is a fixed value \( P_{ij} \), i.e., whatever \( n \) is, the following formula is workable:

\[
P(X_{n+1} = j \mid X_n = i) = P_{ij}
\]
	hen, we call \( P_{ij} \) one-step transition probability of Markov Chain.

Markov Chain is a particular stochastic process and the definition is as follows:

Definition 2. Provided that when any state of process \( \{ X_n \} \) in the past is a nonnegative integer \( i_0, i_1, ..., i_{n-1} \in E \) and \( n \geq 0 \), the following formula is workable:

\[
P((X_{n+1} = j, X_n = i_n, ..., X_1 = i_1, X_0 = i_0) = P(X_{n+1} = j \mid X_n = i)
\]

then this stochastic process is named Markov Chain.

2.2. "Markov Property" Inspection of Random Process

To test whether random variables sequences have "Markov Property" is the necessary premise to use the Weighted Markov Chain model to analyze and solve practical problems. The statistics of \( \chi^2 \) is generally used as for the Weighted Markov Chain of discrete sequence. The specific method is as follows:

\( f_{ij} \) represents the frequency of the process of a step through which the starting state \( i \) transfers to the state \( j \) in the sequence \( X_1, X_2, X_3, ..., X_n, ... \); the matrix involving \( f_{ij} \) as its elements is shift frequency matrix. The marginal frequency is gained by the sum of each column in shift frequency matrix dividing the sum of all the columns and all the lines, respectively, and it could be taken as the marginal probability estimates to be indicated as \( p_i \).

Where

\[
p_i = \frac{\sum f_{ij}}{\sum \sum f_{ij}}
\]

(assuming that the sequence includes \( m \) states)

When \( n \) is sufficiently large, the statistics is constructed: statistics is

\[
\chi^2 = 2 \sum_{i=1}^{m} \sum_{j=1}^{m} f_{ij} \ln \frac{P_{ij}}{P_i}
\]

and it obeys \( \chi^2 \) distribution with the degree of freedom of \( (m-1)^2 \), therein \( P_{ij} \) represents the transition probability matrix. As significance level \( \alpha \) is known, look up the table to get quantile values and obtain the value of the statistic \( \chi^2 \) by calculation. If \( \chi^2 > \chi^2_{\alpha}(m-1)^2 \), the sequence \( \{ X_n \} \) can be thought to conform to the markov property, otherwise this sequence would not to be dealt with as a Markov Chain [5].
2.3. The Basic Theory of Weighted Markov Chain

Weighted Markov process is the theory mainly to study the state of things and state transition rules, and describe a dynamic change process of random time series. The main difference between Weighted Markov Chain prediction method and Markov Chain prediction method lies in the weight of the initial state. The weight of the initial state predicted by the Weighted Markov Chain is not just 1 or 0, but every state is analyzed according to the reasonable calculation formula and the weight is recalculated [13]. The changing trend of the states is identified through analyzing the initial probability of different states and the relationship of transition probability between the states, to achieve the purpose of predicting the future. The above is the basic idea of Weighted Markov Chain prediction.

Stock's closing price is a random sequence changing over time, so we can analyze the stock's closing price and predict the change range of stock's closing price in the next state. Now the stock price changes (yuan) in the stock market is a random sequence, collect the stock's closing price sorted according to the time to form a random sequence \( X_n \), and carry out the forecast in certain steps according to the basic idea of weighted markov prediction [6].

3. The Application of the Weighted Markov Chain in Forecasting the Stock Price of China Sports Industry

Now taking the stock price changes of China Sports Industry in 70 trading weeks from February 6, 2012 to June 11, 2013 for example, use the Weighted Markov Chain model for prediction. Figure 1 is the broke line chart of the stock's closing prices of 70 weeks using Excel software(data resource: big wisdom):

![Figure 1. The Trend Chart of Stocks Closing Prices of China Sports Industry from February 6, 2012 to June 11, 2013](image)

Now we use the Weighted Markov Chain prediction theory to analyze the stock prices [7]:

Step 1. Calculate the mean value \( \bar{x} \) and mean square error of the sequence \( X_n \)

The average weekly closing price for 70 weeks is \( \bar{x} = 9.3879 \) and variance is \( s = 1.3698 \) by Calculation.

Step 2. The sequence index is divided into six levels according to the mean value \( \bar{X} \) and mean square error \( S \).
Now the changes of the stock's closing prices of China Sports Industry will be divided into six state blocks, according to the mean square error of the sample, as plunge, flat plunge, downward flat, upward flat, rise, soar and so on, the corresponding state space is $E = \{1, 2, 3, 4, 5, 6\}$. The specific dividing results are shown in Table 1:

**Table 1. State Division Table of Stocks Closing Prices of China Sports Industry**

<table>
<thead>
<tr>
<th>State</th>
<th>Grades</th>
<th>Standards for Grading</th>
<th>Blocks of the stock prizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Plunge</td>
<td>$x &lt; \bar{x} - 1.0s$</td>
<td>$x &lt; 8.02$</td>
</tr>
<tr>
<td>2</td>
<td>Flat Plunge</td>
<td>$\bar{x} - 1.0s \leq x &lt; \bar{x} - 0.5s$</td>
<td>$8.02 \leq x &lt; 8.70$</td>
</tr>
<tr>
<td>3</td>
<td>Downward Flat</td>
<td>$\bar{x} - 0.5s \leq x &lt; \bar{x}$</td>
<td>$8.70 \leq x &lt; 9.39$</td>
</tr>
<tr>
<td>4</td>
<td>Upward Flat</td>
<td>$x \leq x &lt; \bar{x} + 0.5s$</td>
<td>$9.39 \leq x &lt; 10.07$</td>
</tr>
<tr>
<td>5</td>
<td>Rise</td>
<td>$\bar{x} + 0.5s \leq x &lt; \bar{x} + 1.0s$</td>
<td>$10.07 \leq x &lt; 10.76$</td>
</tr>
<tr>
<td>6</td>
<td>Soar</td>
<td>$x \geq \bar{x} + 1.0s$</td>
<td>$x \geq 10.76$</td>
</tr>
</tbody>
</table>

Step 3. Carrying out the "Markov property" inspection of the known stock's closing price sequence.

Inspect the "Markov property" of the stock's closing price series, in order to make use of the Weighted Markov Chain to forecast the stock price. The data of the closing prices of 70 weeks from February 6, 2012 to June 11, 2012 was calculated, and the frequency matrix $f_{ij}$ of each stock prize state was concluded as follows:

$$
\begin{bmatrix}
    7 & 3 & 1 & 0 & 0 & 0 \\
    2 & 3 & 4 & 0 & 0 & 0 \\
    1 & 3 & 7 & 5 & 1 & 0 \\
    0 & 0 & 4 & 6 & 1 & 1 \\
    0 & 0 & 1 & 1 & 4 & 3 \\
    0 & 0 & 0 & 1 & 3 & 7
\end{bmatrix}
$$

The statistic value $\chi^2 = 124.3377$ was calculated by the formula

$$
\chi^2 = 2 \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{f_{ij}}{P_{ij}} \ln \frac{P_{ij}}{P_{ij}}
$$

As the given significance level was $\alpha = 0.05$, by looking up the table, the quantile was $\chi^2_{0.05}(6-1)^2 = \chi^2_{0.05}(25) = 37.652$.

so the stock's closing prize sequence conformed to the markov property as $\chi^2 > \chi^2_{0.05}(25)$ [8]. Therefore the Weighted Markov Chain prediction theory could continue to be used to forecast the stock price.

Step 4. Calculate each order autocorrelation coefficient $r_k$ of the sequence parameter values and the weight of Markov Chain with various steps.

$$
\begin{align*}
    r_k &= \frac{\sum_{j=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \\
    &= \frac{\sum_{j=1}^{n-k} x_i x_{i+k}}{\sum_{i=1}^{n} (x_i - \bar{x})^2} - \frac{1}{n}
\end{align*}
$$

and
\[ w_k = \frac{|r_k^*|}{\sum_{k=1}^n |r_k^*|} \]

where \( r_k \) is the \( k \) order autocorrelation coefficient (the retention time involves \( k \) periods of time); \( x_i \) is the stock's closing price of the \( i \) week; \( \bar{x} \) is the mean stock's closing price; \( n \) is the length of the reference sample sequence; \( w_k \) is the weight of Markov Chain with a step of \( k \) orders. Calculate each order autocorrelation coefficient and the weight of each step, as shown in the table below.

**Table 2. Each Order Autocorrelation Coefficient and the Weight of each Step**

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_k )</td>
<td>0.8015</td>
<td>0.6737</td>
<td>0.5704</td>
<td>0.3792</td>
<td>0.2245</td>
</tr>
<tr>
<td>( w_k )</td>
<td>0.3025</td>
<td>0.2543</td>
<td>0.2153</td>
<td>0.1431</td>
<td>0.0847</td>
</tr>
</tbody>
</table>

Step 5. The prediction of the price range

We forecast the state of the stock price in the future, if taking the stock's closing prices in the 66th, 67th, 68th, 69th and 70th weeks as the initial state and the corresponding state transition probability matrix to forecast the stock's closing price in the 71th week, from the formula

\[ P_i = \sum_{k=1}^n w_k P_i^{(k)} ; i \in E \]

and Table 3 was obtained.

**Table 3. The Prediction about Stock's Closing Price in the 71th Week**

<table>
<thead>
<tr>
<th>Initial week</th>
<th>state retention time</th>
<th>State of weight</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>3 1</td>
<td>0.302 5</td>
<td>0.1111</td>
<td>0.1111</td>
<td>0.5556</td>
<td>0.1667</td>
<td>0.0556</td>
<td>0</td>
</tr>
<tr>
<td>69</td>
<td>3 2</td>
<td>0.254 3</td>
<td>0.1176</td>
<td>0.0588</td>
<td>0.4118</td>
<td>0.3529</td>
<td>0.0588</td>
<td>0</td>
</tr>
<tr>
<td>68</td>
<td>3 3</td>
<td>0.215 3</td>
<td>0.0625</td>
<td>0.0625</td>
<td>0.4375</td>
<td>0.1875</td>
<td>0.2500</td>
<td>0</td>
</tr>
<tr>
<td>67</td>
<td>3 4</td>
<td>0.143 1</td>
<td>0.1333</td>
<td>0.0667</td>
<td>0.2667</td>
<td>0.2000</td>
<td>0.3333</td>
<td>0</td>
</tr>
<tr>
<td>66</td>
<td>2 5</td>
<td>0.084 7</td>
<td>0</td>
<td>0.2857</td>
<td>0.4286</td>
<td>0</td>
<td>0.2857</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ P_i = \sum_{k=1}^n w_k P_i^{(k)} ; i \in E \]

Table 3 indicated \( max \{ p_i ; i \in E \} = 0.4414 \) at this point, namely the stock's closing status of China Sports Industry in the 71th week was 3 (that is downward flat), and the closing prize met the block 8.70 \( \leq x < 9.39 \), with probability of 44.14%. We got known
from the history data that the actual closing price in 71th week was 8.70, which was consistent with the prediction block.

In the following the state in the 71th week was involved in the initial state, namely the state in the 67th, 68th, 69th, 70th and 71th weeks was served as the initial state for forecasting the stock's closing price in the 72th weeks, as seen in Table 4.

<table>
<thead>
<tr>
<th>Initial week</th>
<th>state</th>
<th>retention time</th>
<th>State of weight</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>3</td>
<td>1</td>
<td>0.3025</td>
<td>0.111</td>
<td>0.111</td>
<td>0.5556</td>
<td>0.1667</td>
<td>0.0556</td>
<td>0</td>
</tr>
<tr>
<td>70</td>
<td>3</td>
<td>2</td>
<td>0.2543</td>
<td>0.117</td>
<td>0.0588</td>
<td>0.4118</td>
<td>0.3529</td>
<td>0.0588</td>
<td>0</td>
</tr>
<tr>
<td>69</td>
<td>3</td>
<td>3</td>
<td>0.2153</td>
<td>0.062</td>
<td>0.0625</td>
<td>0.4375</td>
<td>0.1875</td>
<td>0.2500</td>
<td>0</td>
</tr>
<tr>
<td>68</td>
<td>3</td>
<td>4</td>
<td>0.1431</td>
<td>0.133</td>
<td>0.0667</td>
<td>0.2667</td>
<td>0.2000</td>
<td>0.3333</td>
<td>0</td>
</tr>
<tr>
<td>67</td>
<td>3</td>
<td>5</td>
<td>0.0847</td>
<td>0.133</td>
<td>0.0667</td>
<td>0.2667</td>
<td>0.2000</td>
<td>0.3333</td>
<td>0</td>
</tr>
</tbody>
</table>

predicted results in the two weeks above were consistent with actual situation.

| $P_i$ | 0.107 4 | 0.0772 | 0.4221 | 0.2205 | 0.1615 | 0.011 3 |

Table 4. The Prediction About Stock's Closing Price in the 72th Week

The stock's closing prices in the 73th, 74th, 75th and 76th weeks predicted by the above method met $8.70 \leq x < 9.39$ and $9.39 \leq x < 10.07$, respectively. The actual closing prices were 9.20, 9.20, 9.38, 9.63, respectively, indicating that the predicted results were in conformity with the actual result.

4. Comparison of the Weighted Markov Prediction Result and Markov Prediction Result

Aiming at the problem, Markov chain theory could also be used to forecast the stock price. In the following the prediction error of the two kinds of forecasting method for the stock's closing price was compared:

<table>
<thead>
<tr>
<th>Forecasting Method</th>
<th>Median of Prediction Range (the actual value was 8.70)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>weighted Markov chain prediction</td>
<td>9.045</td>
<td>0.345</td>
</tr>
<tr>
<td>Markov chain prediction</td>
<td>9.73</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 5 showed that the error in the prediction for the 71th week by the weighted markov chain prediction method was 0.345, the weighted markov chain prediction
method was more accurate and the prediction value was closer to the actual value as $0.345 < 1.03$. Whereas the error of markov chains forecasting method was larger.

**Table 6. Comparison of the Prediction Results in the 72th Week by Two Kinds of Forecasting Method**

<table>
<thead>
<tr>
<th>Forecasting Method</th>
<th>Median of Prediction Range</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>weighted Markov chain prediction</td>
<td>9.045</td>
<td>0.145</td>
</tr>
<tr>
<td>Markov chain prediction</td>
<td>9.045</td>
<td>0.145</td>
</tr>
</tbody>
</table>

The results in Table 6 by the two forecasting methods were the same. It was observed in combination with the prediction results in the two weeks above that the weighted Markov chain prediction method is more accurate and the error is relatively smaller. Comparison chart of the predicted results by the two methods and the actual values of the stock price were as follows:

![Figure 3. Comparison Chart of the Prediction Results by the Two Methods with the Actual Value](image-url)

In the Figure 3, curve 1 represented the median of the stock price range predicted by weighted Markov, curve 2 represented the actual closing price, and curve 3 represented the median of the stock price range predicted by Markov. From the predictions by the two methods for forecasting stock's closing prices of the next several weeks we could see the prediction results by weighted Markov chain were more close to the actual value, while the error of Markov chain prediction was relatively larger.

In the engineering of Markov chain forecasting, it is generally assumed that external economic condition is stable, namely the range predicted will not arise the message of good-interest or bad-interest, leading to roaring or plunging of stock prices. But it is totally possible in the real stock market, so that the two methods is suitable for short-term forecasting and not very ideal for medium-term and long-term forecasting [17].

This paper only selected 70 data in the application model, which is far from enough for the prediction of higher precision. In addition, how to better divide blocks of each state to improve the prediction accuracy is worthy of further study.
5. Conclusion

This article bases on the markov property of the stock price sequence and uses weighted markov model to predict the stock price change block, providing certain reference value for individual or collective investment. Due to the high risk of stock and investment options and the complex factors that influence investment payback period, the change of stock prices, stock payback period and the option price could be further enriched and perfected after accumulating enough data about stock price and option price, indicating that the analysis results are consistent with the actual data. In addition, with the increase of representation of the data, autocorrelation coefficient, state transition matrix and weight will come about some changes, and this change will further perfect the forecast and further improve the prediction precision.

The Chinese financial market is an emerging market, and there are numerous conclusions of researches for all types of securities. Investors should maintain a healthy state of mind toward the caprice and unpredictability of stock and options market. There is a lot of development space for the future prospects of stock and options market, we need to keep trying to studying and innovating.

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References