Advertising and Order Quantity Decision Based on the Newsvendor Model

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Abstract

This paper studied optimal order quantity and advertising level of newsvendor. According to the concepts of unit opportunity shortage penalty and unit opportunity expected excess loss, analyzed the relationship between optimal order quantity and the loss aversion coefficient. On this basis, from two aspects of the impact of advertising on the market, we studied the retailer optimal advertising and order quantity decision problems, got the optimal level of advertising and the optimal order quantity, finally examples have shown the impacts of advertising on the retailer.

Keywords: newsvendor, advertising, loss aversion

1. Introduction

With the continuous development of market economy, advertising become an important marketing tool for the supply chain. In the supply chain, enterprises use advertising to describe product features, price, quality and services, provide the product or service for the target audience. Enterprises not only can motivate consumers to buy their products and services, improve competitiveness by advertising, but also can establish a good market image and create a good market environment with advertising. Advertising and pricing decisions on the enterprise and supply chain management have important significance. The newsvendor problem actually reflect enterprise management. Classical newsvendor problem doesn't consider shortage penalty, its optimal solution is obtained under the risk neutral. Because the market demand is uncertain, random, then how many the newsvendor's optimal order quantity can make the best profit with fixed the costs and sales price of products? If the newsvendor advertising, the optimal advertising and the optimal order quantity how to determine? This paper tries to solve the problems.

newsvendor model. Liu Wei and Song Shiji (2014) studied a single-period inventory problem with random yield and demand, where the loss-averse preferences are adopted to describe the retailer's (newsvendor's) decision-making behavior. Guo Peijun and Ma Xiuyan (2014) considered products whose life cycles are expected to be smaller than the procurement lead times, determining optimal order quantities of such products is a typical one-shot decision problem for a newsvendor. Kki and Anssi (2015) derived the optimal order quantity for interdependent demand and supply and provide a closed-form solution for a specific copula-based dependence structure, analyze the impact of supply uncertainty on newsvendor decisions. Lee and Chung-Yee (2015) shown how the newsvendor's loss aversion behavior affects his ordering decision, and propose an efficient algorithm to compute his optimal solution in the general case with n options.

Unlike most models in the literature, in this paper, based on previous research work, we will analysis the loss aversion newsvendor optimal order quantity based on shortage penalty and advertising.

Paper is divided into two parts. Firstly we discuss the loss of risk-averse newsvendor problem without considering shortage penalty. Based on this, secondly, we discuss the loss of risk-averse newsvendor problem with considering shortage penalty by two concepts, which are 1) unit opportunity shortage penalty, it refers to opportunity cost of unit shortage plus with the probability of negative returns caused by the shortage of product; 2) unit opportunity expected excess loss, it means when the order is greater than the demand, the opportunity cost of the unit of remains plus with the probability of negative returns caused by the order exceed. At last we will discuss the risk neutral newsvendor advertising and order quantity optimal decision problem.

2. Model Framework

Newsboy orders newspapers with unit price \( b \) in the morning and sells the newspapers with retail price \( a \), he can returns the unsold newspapers with unit discount price \( c \), in the night. Suppose \( a > b > c \), so when the newsboy sell a newspaper, he can earn \( a - b \), but if he return a paper, he will loss \( b - c \). When the newspaper cannot meet the market demand leading to shortage, unit shortage penalty of the newsboy is \( p \). Suppose market demand \( r \) is a nonnegative random variable, the density function is \( f(r) \), cumulative density function is \( F(r) \) which continuous and monotone increasing in the interval \( I \), and there are upper bound \( \sup I \) and lower bound \( \inf I = 0 \).

The newsboy daily order newspaper quantity is \( \theta \), which can be more than or less than demand \( r \). The newsboy problem is how to determine the optimal order quantity goal makes his expected utility maximum, while the utility function for the newsboy is:

\[
v(x) = \begin{cases} 
  x^\beta & \text{if } x \geq 0 \\
  -\hat{\lambda}(-x)^\beta & \text{if } x \leq 0
\end{cases}
\]

When \( \alpha = \beta = 1 \), \( v(x) \) is called pure loss aversion utility function. In this paper, the utility function of newsboy is the pure loss aversion utility function and \( \hat{\lambda} > 1 \).
3. Loss Aversion Newsvendor Model without Shortage Penalty

In this case, newsvendor’s payoff function is:

\[ \pi_+(r, \theta) = (a - c)r - (b - c)\theta \quad r \leq \theta \]  

\[ \pi_-(r, \theta) = (a - b)\theta \quad r > \theta \]

Let \( \pi_+(r, \theta) = (a - c)r - (b - c)\theta = 0 \), we can get \( r = \frac{b - c}{a - c} \). Because of \( b < a \), so \( r < \theta \). When demand quantity \( r \) in the interval \([0, r_0]\), the newsvendor income is negative. When demand quantity \( r \) in the interval \([r_0, \theta]\), the newsvendor income is positive.

From above analysis, newsvendor expected utility function is:

\[ E[U(\pi(r, \theta))] = E[\pi(r, \theta)] + (\lambda - 1) \int_{0}^{r_0} \pi_-(r, \theta) f(r) \, dr \]  

First derivative, second derivative of the function (3) about \( \theta \), respectively,

\[ \frac{\partial E[U(\pi(r, \theta))]}{\partial \theta} = (a - b)(1 - F(\theta)) - (b - c)F(\theta) - (\lambda - 1)(b - c)F(r_I) \]

\[ \frac{\partial^2 E[U(\pi(r, \theta))]}{\partial \theta^2} = -(a - c)f(\theta) - (\lambda - 1)\frac{(b - c)^2}{a - c} f(r_I) < 0 \]

Therefore, \( E[U(\pi(r, \theta))] \) is a concave function, so exists \( \theta = \theta_\lambda \) when the expected utility’s first derivative equal to 0, that is,

\[ (a - b)(1 - F(\theta_\lambda)) - (b - c)F(\theta_\lambda) - (\lambda - 1)(b - c)F\left(\frac{b - c}{a - c}\theta_\lambda\right) = 0 \]  

(4)

The function \( E[U(\pi(r, \theta))] \) gets max.

Theorem 1: \( \theta_\lambda \) is risk-averse newsvendor optimal order quantity, \( \theta_\lambda \) is the risk-neutral newsvendor optimal order quantity, then \( \theta_\lambda < \theta_\lambda \), and \( \lim_{\lambda \to 1} \theta_\lambda = \sup I \), \( \lim_{\lambda \to 0} \theta_\lambda = \inf I \),

\[ \lim_{\lambda \to 0} \theta_\lambda = \inf I \]  

Proof: When the newsvendor is risk neutral, that is say \( \lambda = 1 \), the optimal order quantity \( \theta_\lambda \) meet to \((a - b)(1 - F(\theta_\lambda)) - (b - c)F(\theta_\lambda) = 0\). When the newsvendor is risk aversion, because \( \lambda > 1 \), so \((\lambda - 1)(b - c)F\left(\frac{b - c}{a - c}\theta_\lambda\right) > 0\) permanent establishment, therefore \( F(\theta_\lambda) < F(\theta_\lambda) \). As \( F(x) \) is a monotone increasing function, so \( \theta_\lambda < \theta_\lambda \). Don’t considered out of stock losses, the risk aversion of the newsvendor’s optimal order quantity is less than the risk-neutral newsvendor order quantity.

From (4), when \( b - c \to 0 \), then \( F(\theta_\lambda) \to 1 \), newsvendor tend to order quantity \( \theta_\lambda \to \sup I \).

When \( b - c \to +\infty \), \( F(\theta_\lambda) \to 0 \), newsvendor tend to order quantity \( \theta_\lambda \to \inf I \);

When \( a - b \to 0 \), \( F(\theta_\lambda) \to 0 \), newsvendor tend to order quantity \( \theta_\lambda \to \inf I \);

When \( a - b \to +\infty \), \( F(\theta_\lambda) \to +\infty \), newsvendor tend to order quantity \( \theta_\lambda \to \sup I \).

From theorem 1, we can know that risk-averse newsvendor optimal order quantity is smaller than the risk-neutral newsvendor optimal order quantity. Unit newspaper earns
more profit, newsvendor order more quantity. The less the loss of each newspaper, the greater quantity the newsvendor order.

First derivative of the function (4) about \( \lambda \) for \( \theta \):
\[
\frac{\partial \theta}{\partial \lambda} = -\frac{(b - c)F(r_i)}{f(\theta')(a - c) + (\lambda - 1)\frac{(b - c)^2}{a - c}} < 0
\]

Order quantity \( \theta \) increases with the risk factor \( \lambda \) reduction.

First derivative of the function (4) about \( a \) for \( \theta \):
\[
\frac{\partial \theta}{\partial a} = \frac{1 - F(\theta_j) + (\lambda - 1)f(r_i)\frac{(b - c)^2}{a - c}}{(b - c)F(r_i) + (\lambda - 1)f(r_i)\frac{(b - c)^2}{a - c}} > 0
\]

Order quantity \( \theta \) increases with price \( a \) increasing.

First derivative of the function (4) about \( b \) for \( \theta \):
\[
\frac{\partial \theta}{\partial b} = \frac{-[1 + (\lambda - 1)F(r_i) + (\lambda - 1)\frac{b - c}{a - c}f(r_i)\theta_j]}{(b - c)F(r_i) + (\lambda - 1)f(r_i)\frac{(b - c)^2}{a - c}} < 0
\]

Order quantity \( \theta \) increases with wholesale price \( b \) reduction.

First derivative of the function (4) about \( c \) for \( \theta \):
\[
\frac{\partial \theta}{\partial c} = \frac{F(\theta') + (\lambda - 1)F(r_i) + (\lambda - 1)\frac{(a - b)(b - c)\theta_j}{a - c}}{(a - c)f(\theta_j) + (\lambda - 1)(b - c)^2f(r_i)\frac{(b - c)^2}{a - c}} > 0
\]

Order quantity \( \theta \) increases with unit salvage value \( b \) reduction.

From above analysis, we get theorem 2.

**Theorem 2**: \( \frac{\partial \theta}{\partial \lambda} < 0; \frac{\partial \theta}{\partial a} > 0; \frac{\partial \theta}{\partial b} < 0; \frac{\partial \theta}{\partial c} > 0 \).

### 4. Loss Aversion Newsvendor Model with Shortage Penalty

Because of downtime lost sales opportunities, and cause the loss and is unable to perform the contract and pay the fines, these are the shortage penalty, therefore shortage penalty cannot be ignored. When considering the shortage penalty, from Section 3, the income function of newsboy is:

\[
\Pi_-(r, \theta) = (a - c)r - (b - c)\theta \quad r \leq \theta \tag{5}
\]

\[
\Pi_+(r, \theta) = (a - b + p)\theta - pr \quad r > \theta \tag{6}
\]

Let \( \Pi_-(r, \theta) = (a - c)r - (b - c)\theta = 0 \), we can get \( r_1(\theta) = \frac{(b - c)\theta}{(a - c)} \). Because of \( b < a \), hence \( r_1(\theta) < \theta \), which mean that the newsboy’s income is negative if \( r \in [0, r_1(\theta)] \), or positive if \( r \in [r_1(\theta), \theta] \). Let \( \Pi_+(r, \theta) = (a - b + p)\theta - pr = 0 \) can get \( r_2(\theta) = \frac{(a - b + p)\theta}{p} \), so when \( r > \theta \), breakeven point is \( r_2(\theta) \).
From Figure 1, we can get the newsvendor’s expected profit function:

\[ E[U(\pi(r, \theta))] = E[\pi(r, \theta)] + (\lambda - 1)\int^{r_1}_{r} \pi_+(r, \theta)f(r)dr + \int^{\infty}_{r_1} \pi_-(r, \theta)f(r)dr \]  
(7)

From function (7), we can get

\[ \frac{\partial E[U(\pi(r, \theta))] }{\partial \theta} = (a - b + p)(1 - F(\theta)) - (b - c)F(\theta) + (\lambda - 1)((a - b + p)(1 - F(r_1)) - (b - c)F(r_1)) \]

\[ \frac{\partial^2 E[U(\pi(r, \theta))] }{\partial \theta^2} = -(a + p - c)f(\theta) - (\lambda - 1)((a + p - b)^2 f(r_1) + p + (b - c)^2 f(r_1)(a - c) < 0 \]

So \( E[U(\pi(r, \theta))] \) is a concave function. And set \((0, +\infty)\) is convex, so exist \( \hat{\theta} \in (0, +\infty) \)

\[ (a - b + p)(1 - F(\hat{\theta})) - (b - c)F(\hat{\theta}) + (\lambda - 1)((a - b + p)(1 - F(r_1)) - (b - c)F(r_1)) = 0 \]
(8)

The value of function \( E[U(\pi(r, \theta))] \) can get max.

For the function (8) derivative on both sides with \( p \), there is

\[ \frac{\partial^2 \theta}{\partial p^2} = (a - b + p)(1 - F(\hat{\theta})) + (\lambda - 1)(1 - F(r_1)) + \frac{(a - b + p)(a - b)^2 f(r_1)}{p^2} + (\lambda - 1)(a - b + p) - (b - c)(a - c) > 0 \]

So optimal order quantity \( \hat{\theta} \) increasing with the price \( p \).

Especially, when \( \lambda = 1 \), the risk neutral newsvendor optimum order quantity \( \theta_i \) meets to

\[ (a - b + p)(1 - F(\theta_i)) - (b - c)F(\theta_i) = 0 \]
(9)

Theorem 3:

If \((a - b + p)(1 - F(r_1)) > (b - c)F(r_1)\), then \( \hat{\theta} \|_{\partial \lambda} > 0 \) and \( \hat{\theta} > \theta_i \);

If \((a - b + p)(1 - F(r_1)) = (b - c)F(r_1)\), then \( \hat{\theta} \|_{\partial \lambda} = 0 \) and \( \hat{\theta} = \theta_i \);

If \((a - b + p)(1 - F(r_1)) < (b - c)F(r_1)\), then \( \hat{\theta} \|_{\partial \lambda} < 0 \) and \( \hat{\theta} < \theta_i \).

Proof: The function (9) derivative on both sides with \( \theta_i \), we can get

\[ \frac{\partial^2 \theta}{\partial \theta^2} = (a - b + p)(1 - F(\hat{\theta})) - (b - c)F(\hat{\theta})[f(\hat{\theta})(a + p - c) + (\lambda - 1)[f(r_1)(a - b + p)^2 + f(r_1)(b - c)^2]}{p^2} ] \]
(10)

For the function (10), we define \((a - b + p)(1 - F(\hat{\theta}))\) as unit opportunity shortage penalty, it refers to opportunity cost of unit shortage plus with the probability of negative returns caused by the shortage of production. We define \((b - c)F(\hat{\theta})\) as unit opportunity expected excess loss, it means that when the order is greater than the demand, the opportunity cost plus with the probability of negative returns caused by the order exceed. Using this concept can be better explained newsvendor model. From function (8), we get

(1) When \((a - b + p)(1 - F(r_1)) > (b - c)F(r_1)\), \( \hat{\theta} \|_{\partial \lambda} > 0 \). From function (8) and function (9), we obtained \( F(\hat{\theta}) > F(\theta_i) \).

As the \( F \) monotonically increasing with \( \theta \), so we have \( \theta_i > \theta_i \). In this case, unit opportunity shortage penalty larger than unit opportunity expected excess loss, the greater
risk aversion coefficient, the more expected utility loss of shortage than excess, so the optimal order quantity should be correspondingly increased.

(2) When \((a - b + p)(1 - F(r_j)) = (b - c)F(r_j), \) \(\partial \theta_j / \partial \lambda = 0, \) \(F(\theta_j) = F(\theta_1),\) then \(\theta_j = \theta_1.\)

In this case, unit opportunity shortage penalty is equal to unit opportunity expected excess loss, the optimal order quantity is independent of and risk factors, and it’s the same as risk-neutral optimal order quantity.

(3) When \((a - b + p)(1 - F(r_j)) < (b - c)F(r_j), \) \(\partial \theta_j / \partial \lambda < 0, F(\theta_j) < F(\theta_1).\)

As the \(F\) monotonically increasing, so we have \(\theta_j < \theta_1.\) In this case, unit opportunity shortage penalty smaller than unit opportunity expected excess loss, the greater risk aversion coefficient, the more expected utility loss of excess than shortage, so the optimal order quantity should be reduced accordingly to avoid ordering more than demand.

5. Newsvendor Optimal Ordering and Advertising Decision Model

In the last section we discussed the optimal ordering quantity with risk averse newsvendor, on the basis of the section, we will discuss the risk neutral newsvendor advertising and order quantity optimal decision problem. Suppose \(m\) is the advertisement costs and \(\lambda = 1.\)

Let \(r\) is total demand products after newsvendor make advertising, which is random variable and relative to advertising costs, the mean and variance are respectively \(\mu\) and \(\sigma^2.\) Let \(r\) is a concave increasing function with \(m\) and there is an upper bound. Suppose \(f(r)\) is the probability density function and \(F(r)\) is the cumulative distribution function.

\(CV = \sigma / \mu\) represents the variation coefficient of demand, according to Jones (1995), suppose

\[
E(r) = \mu + \mu w m^a
\]

which \(\mu\) is the average market demand before advertising, \(w\) and \(\alpha\) are both constant, \(0 \leq a \leq 1.\) When \(w = 0,\) it means advertising does not have any impact on product market demand. Obviously, for arbitrary \(w > 0,\) the bigger \(\alpha,\) the more demand causing by advertising, as shown in Figure 2.

\[E(r) = \mu + \mu w m^a\]

\[E[\prod \{\theta, m\}] = \int_a^{\infty} \int_a^{\infty} \theta f(r) dr - p \int_a^{\infty} \theta f(r) dr - (a - c) \int_a^{\infty} \theta f(r) dr - (b - c) \int_a^{\infty} \theta f(r) dr - m
\]

Due to the effect of advertising on the market, product demand changes, compared without advertising, there are the following two kinds of circumstances:

\[
E[r] = \mu + \mu w m^a
\]
5.1. Advertising Increase the Average of Demand, but does not Change the Variance

According to equation (12) of the first order conditions:

\[(a - b + p)(1 - F(\theta - \mu w m^a)) - (b - c)F(\theta - \mu w m^a) = 0\]  

Obtain the optimal ordering quantity and satisfy \( F(\theta^* - \mu w m^a) = \frac{a - b + p}{a - c + p} \), then we can get

\[\theta^* = \mu w m^a + \theta^*_1\]  

Which \( \theta^*_1 \) is optimal order quantity when newsvendor is risk neutral under shortage penalty.

Theorem 4: If advertising increase the average of demand, but does not change the variance, then the optimal advertising investment \( m^* = [(a - b)\alpha \mu w]^{1/(1-\alpha)} \), optimal order quantity \( \theta^* = \mu w [(a - b)\alpha \mu w]^{\alpha} + \theta^*_1 \).

Proof: According to equation (12) and (14), let \( \partial E(\pi) / \partial m = 0 \), can easy get \( m^* = [(a - b)\alpha \mu w]^{1/(1-\alpha)} \), then the optimal quantity \( \theta^* = \mu w [(a - b)\alpha \mu w]^{\alpha} + \theta^*_1 \).

Due to \( 1 / (1 - \alpha) > 1 \), so the optimal advertising costs increase with the increase of margin profit and the average market demand before advertising.

When the demand follows a uniform distribution, suppose demand distribution range without advertising is \([a, b]\), and with advertising is \([a', b']\), then we can get

\[a' = a_0 + \mu w m^a\]  

\[b' = b_0 + \mu w m^a\]  

from equation (7) which \( \mu = (b_0 - a_0) / 2 \).

According to theorem 2, we obtain \( m^* = \frac{(a - b)(b_0 - a_0)(\alpha w)}{2^{1/(1-\alpha)}} \) and

\[\theta^* = \frac{w(b_0 - a_0)(a - b)\alpha (b_0 - a_0)w^{1/(1-\alpha)}}{2^{1/(1-\alpha)}} + \frac{(a - b + p)(b_0 - a_0)}{a - c + p} + a_0\].

5.2. Advertising Increase the Average of Demand also Change the Variance, but the Coefficient of Variation does not Change

In this case, because the coefficient of variation does not change, so \( \delta^* = \delta(1 + w m^a) \), when the demand follows a uniform distribution, the lower bound of demand distribution interval after advertising is \( a' = a_0 + a_0 w m^a \), the upper bound is \( b' = b_0 + b_0 w m^a \), then we can get:

\[\theta^* = (1 + w m^a)\theta^*_1 = (1 + w m^a)(a_0 + a_0 w (b_0 - a_0) + \frac{a + p - b}{a + p - c})\]

\[m^* = \frac{(a_0 + a_0 w (b_0 - a_0) + \frac{a + p - b}{a + p - c})(\theta^* - \theta^*_1) - p(b_0 - a_0)\theta^*_1}{2(b_0 - a_0)}\].
6. Model Simulation

Suppose $a = 12, b = 8, c = 6, p = 1, a = 0.6, \omega = 0.003$, the demand follows a uniform distribution in the interval $[500, 10000]$ without advertising, the calculation results are shown in Table 1:

It can be seen from table 1, advertising increase the profit of the newsvendor. When advertising efficiency parameter is small, in this case the advertisement has influence only on the average demand, the optimal order quantity is almost not affected by advertising, but the profit is increased. The main reason is that although order quantity is not increased by advertising, but there is a certain effect on the market demand, the expected loss including return loss and shortage penalty is reduced, so that the total profit increase. If the influence of advertising on the mean and variance of demand, the advertisement will not only increase the market demand, but also makes the retailer orders increased, although at the moment the advertising level is lower, but the profit is higher than that of the advertising only change mean.

### Table 1. Results of the Optimal Retailer Advertising Decision under Different Conditions

<table>
<thead>
<tr>
<th></th>
<th>Without advertising</th>
<th>Advertising only change mean</th>
<th>Advertising change mean and variance both</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal order quantity</td>
<td>7285.71</td>
<td>7285.84</td>
<td>10114.4</td>
</tr>
<tr>
<td>optimal advertising costs</td>
<td>———</td>
<td>6840.14</td>
<td>3311.26</td>
</tr>
<tr>
<td>optimal profit</td>
<td>14214.3</td>
<td>15781.8</td>
<td>16421.8</td>
</tr>
</tbody>
</table>

7. Summary

This paper discussed the shortage penalty and loss aversion newsvendor model, according to the concepts of unit opportunity shortage penalty and unit opportunity expected excess loss, analyzed the relationship between optimal order quantity and the loss aversion coefficient. On this basis, from two aspects of the impact of advertising on the market, we studied the retailer optimal advertising and order quantity decision problems, got the optimal level of advertising and the optimal order quantity, at last an example illustrated the influence of advertising to the newsvendor.

This paper get the conclusion base on prices unchanged, in reality, the market price is not fixed, but is determined by market supply, then demand is a function of price, in this case newsvendor optimal order quantity will be considered in the future for further study.

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References


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