Two Stage Detector Comprising of Weighted-ED and Correlated-GLRT for Cognitive Radio Networks

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Abstract

Cognitive radio network is one of the prime solutions of band width crises problem. To resolve band width crises firstly we have to sense primary user (PU) licensed signal. To sense PU signal there are various sensing techniques which have been proposed by researchers, but each of them are having some limitations. In this paper we present a two stage detector for spectrum sensing (SS) in cognitive radio (CR). It comprises of a weighted-energy detector (W-ED) and a correlated-generalized likelihood ratio test (C-GLRT). The first stage detects the energy and then if required, the C-GLRT makes the final decision in second stage. Performance of the proposed two stage detector is compared with the existing energy detector (ED), generalized likelihood ratio test (GLRT), and adaptive spectrum Sensing (ASS) detectors. The numerical results show that proposed sensing technique has better detection performance and require less sensing time.

Keywords: Cognitive Radio, C-GLRT, GLRT, W-ED

1. Introduction

There is tremendous increase in wireless services, which has increased the demand of the radio spectrum. However, the radio spectrum is a limited resource and its efficient utilization is essential. The licensed spectrum users are referred to as primary users. Most of the spectrum is wasted because it is not being used by the primary user at all the times and also at all geographical locations. Cognitive Radio (CR) is used to detect such situation in order to make use of the spectrum when it is not used by the primary users. Spectrum Sensing (SS) Techniques are used to allocated the spectrum to the secondary unlicensed users for the specific time being and geographical location [1]. The SS here is performed by the two stage detector. The proposed two stage detector makes use of the W-ED as the first stage and C-GLRT as the second stage of the detector [2]. When the noise variance is known the energy detector (ED) is a simple and robust SS technique [3, 4]. However if we do not have exact knowledge of the noise variance it leads to incorrect evaluation of the threshold and hence leads to increase in the false alarm probability [5]. When the noise variance is unknown the eigen value based detector GLRT can be used [6]. These detectors are based on the correlated multiple antennas and hence are suitable for the practical use. Further, in [7], authors presented two-stage detection scheme known as ASS, where out of two stages only one stage detector performs sensing operation at a time. But it does not perform well at low SNR.

The rest of the paper is organized as follows: Section II presents system description. Section III describes proposed system model. Section IV shows the numerical results and analysis. Finally, Section V concludes the paper.
2. System Description

Suppose that the secondary user is deployed with $M$ antennas then the exponential correlation model is generally used to define the correlation among the antennas. The correlation matrix $C$ of antennas is a symmetric toeplitz matrix. The components of correlation matrix $C$ can be written as

$$
C_{ij} = \begin{cases} 
\rho^{j-i}, & i \leq j \\
C_{ji}, & i > j 
\end{cases}
$$

Where, $i, j = 1, 2 \ldots M$, and $0 \leq \rho \leq 1$, $\rho$ is the antenna correlation coefficient between two adjacent antennas, and defined as $\rho = \exp \left( -\frac{23\lambda^2 d^2}{4} \right)$, It relies on the angular spread $\Lambda$, wavelength $\lambda_c$, and the distance $d$ between two adjacent antennas. Angular spread $\Lambda$ ranges from 0 to 1, with 0 denoting the case of a single multipath component from a single direction, and 1 denoting no clear bias in the angular distribution of received power.

3. Proposed System Model

We consider the scenario where multiple antennas are deployed at the single secondary node in order to detect the primary signal. The number of antennas which are used at secondary node is being represented by $M$. Then in order to detect the signal with a very low signal to noise ratio (SNR) In the proposed method various time instants have been considered, which is represented by $N$. Firstly considering the noise signal represented by $w(n)$ having Gaussian distribution with mean zero and co-variance matrix $\sigma_w^2 I$. The signal being transmitted by the primary user is represented by $s(n)$ and is complex phase shift keying modulated having an average power equal to $p$ which we have assumed to be 1. The signal is transmitted over a channel which can be white Gaussian channel or Rayleigh fading channel. The channel gain is represented by $h(n)$ and have Gaussian distribution with mean as zero and variance as $\sigma_h^2$. The received signal is represented by $x(n)$. Since $M$ antennas are considered at the secondary user in this case, the received signal vector $x(n)$ is a $M \times 1$ vector. The entire received signal matrix being represented by $X(n)$ which is formed by considering the received signals at each antenna at different instants of time. Therefore $X(n)$ is given as:

$$
X(n) = [x(1) \ x(2) \ \ldots \ \ldots \ x(N)]
$$

The SS problem can be formulated on the basis of two hypothesis test: $H_0$ and $H_1$. In the hypothesis $H_0$ we assume that we are receiving only the noise signal. Hence the received signal $x(n)$ is equal to the noise signal i.e.,

$$
H_0 : x_i(n) = w_i(n)
$$

In the hypothesis $H_1$ it is assumed that signal and noise both are received. Hence the received signal here is given as

$$
H_1 : x_i(n) = h_i(n) \times s(n) + w_i(n)
$$

Therefore, the distribution of $X(n)$ is given as

$$
X \approx \begin{cases} 
C_N(0, \sigma_w^2 I) & \text{under } H_0 \\
C_N(0, P\sigma_h^2C + \sigma_w^2 I) & \text{under } H_1 
\end{cases}
$$

Then, likelihood functions under $H_0$ and $H_1$ are:
\[
p(X|H_0, \sigma^2_w) = \prod_{n=1}^{N} \frac{1}{\pi^M \sigma^2_w} \times \exp \left( -\frac{x(n)H_x(n)}{\sigma^2_w} \right)
\] (6)

\[
p(X|H_1, \sigma^2_w) = \prod_{n=1}^{N} \frac{1}{\pi^M \text{det}(P \sigma^2_h C + \sigma^2_w I)} \times \exp \left( -x(n)^H(P \sigma^2_h C + \sigma^2_w I)^{-1} x(n) \right)
\] (7)

The novelty of this paper is that it utilizes the property of both W-ED and C-GLRT detectors. Due to W-ED it assigns the higher weight coefficients to the signal component, which corresponds to larger eigen values, while according to C-GLRT it considers the correlation effect of the multiple antennas being used at the secondary user, hence this arrangement significantly improves the detection performance.

Figure 1, shows the flow chart of proposed two stage detector, the ED being used at the first stage; performs the coarse detection. If it declares that the channel is occupied the result is simply accepted however if the ED declares the channel to be unoccupied the result is given to the second stage of the detector. The C-GLRT forms the second stage of the detector which performs the final detection. The first stage W-ED is based on the Neyman Pearson theorem when the noise variance is known, the W-ED is given as:

\[
L(X) = \frac{p(X|H_1, \sigma^2_w)}{p(X|H_0, \sigma^2_w)}
\] (8)

The test statistic is then given as:

\[
T_{W-ED}(X) = \ln L(X) \overset{H_1}{\overset{H_0}{\approx}} \varepsilon
\] (9)

Where, \( \varepsilon \) is the decision threshold. It is seen that for a W-ED, the observed data \( x(n) \) is first linearly transformed to \( y(n) \), then the transformed signal component corresponding to the larger eigen value is used with a higher weight coefficients. When the antennas are independent, \( C \) is a diagonal matrix, all the eigen values are the same, and the W-ED becomes the ED. When the constraint on probability of detection is given, the threshold \( \varepsilon \) is calculated on the basis of the false alarm probability.

The probability of false alarm is given as

\[
P_{F}^{ED} = P_{r}(T_{ED} > \tau|H_0) = 1 - F_{T_{ED}|H_0}(\tau)
\] (10)

The probability of detection is given as

\[
P_{D}^{ED} = P_{r}(T_{ED} > \tau|H_1)
\] (11)
Figure 1. Flow Chart of the Proposed Two Stage Detector

The Second stage C-GLRT based method first obtains the maximum likelihood estimate (MLE) of the unknown parameter \( \theta \) under \( H_0 \) and \( H_1 \); then it forms the GLRT test statistic as:

\[
L_G(X) = \frac{p(X|H_0, \theta_1)}{p(X|H_0, \theta_1)}
\]  

(12)

Based on the likelihood functions under \( H_0 \) and \( H_1 \); The MLE of \( \sigma_w^2 \) is obtained as:

\[
\sigma_w^2 = \frac{1}{MN} \sum_{n=1}^{N} \frac{x(n)Hx(n)}{\sigma_w^2} = \frac{1}{M} \sum_{i=1}^{M} \lambda_R^i
\]  

(13)

Where \( \lambda_R^i \) is the eigen value of the sample covariance matrix \( R_x \) defined as \( R_x = 1/NXX^H \). The derivative is set to zero and the MLE of \( \sigma_w^2 \) is derived under \( H_1 \) as:

\[
\sigma_w^1 = \max \left( 0, \frac{1}{M} \sum_{i=1}^{M} \lambda_R^i - \frac{p\sigma_w^2}{M} \sum_{i=1}^{M} \lambda_l \right)
\]  

(14)

Using the MLE of \( \sigma_w^2 \) and \( \sigma_w^1 \) in the likelihood functions of \( H_0 \) and \( H_1 \) the test statistic of C-GLRT is derived as:

\[
T_{C-GLRT}(X) = \ln L_G(X) \geq_{H_0}^H \epsilon
\]  

(15)

The probability of false alarm is given as

\[
P_F^{C-GLRT} = p\{T_{C-GLRT} > \tau|H_0\}
\]  

(16)

\[
P_F^{C-GLRT} = 1 - F_{T_{C-GLRT}|H_0}(\tau)
\]  

(17)
The probability of detection is given as
\[ P_{D_{C-GLRT}} = P_r \{ T_{C-GLRT} > \tau | H_1 \} \]  

(18)

4. Numerical Results And Analysis

In the proposed two stage detection scheme, the overall false alarm probability is calculated by using the false alarm probabilities of both the W-ED and C-GLRT detectors. False decision is made when the decision statistic is greater than the threshold under \( H_0 \) and when the decision statistic is smaller than the threshold under \( H_1 \). When the W-ED makes a false decision the result is given to the C-GLRT, based on this the decision of the second stage becomes the final result. The transmitted power is taken as \( P \) which is assumed to be 1 and the channel parameter is taken as 1. The numbers of samples considered are 20 and the numbers of antennas used are 6. The constraint on the false alarm probability is 0.01. The detection performance of the proposed two stage detector is compared with existing ED, GLRT, and ASS detectors. Figure 2 shows the graph between the probability of detection and SNR. Numerical results show that the proposed two stage detector scheme optimizes detection performance and outperforms the GLRT, ASS, and ED sensing techniques by 6.0\%, 23.0\%, and 63.0\% at \(-5 \) dB SNR respectively.

![Figure 2. Detection Performance Comparison between ED, GLRT, ASS, and Proposed Two Stage Detector](image.png)
Sensing time is the total time taken by cognitive radios to detect PU signal. If sensing time is increased then PU can utilize its frequency band in a better manner and the limit is decided that CR can’t interfere throughout that much of time. More PUs will be detected if more the SS, due to this the level of interference will be less. The sensing time depends on the number of samples received by the CR. Figure 3 shows the graph of spectrum sensing time versus SNR. The proposed scheme requires lesser sensing time than the existing ED, GLRT, and ASS detectors. It is observed that there is an inverse relation between SS time and SNR. As SNR increases, sensing time decreases. At -20 dB SNR, proposed scheme requires approximately 66.5 ms while existing schemes (ED, GLRT, and ASS detectors) requires around 69.0 ms, 68.2 ms and 67.0 ms sensing time.

\[ T = T_F + T_S \]  \hspace{1cm} (19)

In equation (19), \( T \) shows total SS time of CR user. \( T_F \) and \( T_S \) are the first stage SS time and second stage SS time of individual CR user’s respectively.

5. Conclusion

In this paper, we have proposed a two stage detector comprising of weighted-ED and correlated-GLRT for CRN. This scheme enhances detection performances as well as improves sensing time. Numerical results show that proposed two stage detector scheme outperforms other existing schemes (ASS, GLRT, and ED) by 5.0 %, 22.0 %, and 63.0 % at - 5 dB SNR. It is also shown that the proposed scheme yields lesser sensing time than existing schemes (ASS, GLRT, and ED), by 0.5 ms, 1.7 ms, and 2.5 ms at - 20 dB SNR respectively. All results conclude that the proposed detector performance better than other existing detectors. Hence it is suitable to be used in present conditions.

Acknowledgments

The authors would like to thank their parents for their support and motivation, for without their blessings and God’s grace this paper would not be possible.
References


