Newsvendor Game with Supply Uncertainty

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Abstract

In this paper, we study the newsvendor game with supply uncertainty, where multiple retailers form a coalition to cooperate to place orders to minimize the cost and share risk facing uncertain supplies and demands. After the orders are delivered and the demands are realized, the actual arrival quantities are allocated among the retailers to minimize the total cost. We compute the cost functions by using two-stage stochastic programming model and show that the cooperative game has a nonempty core. Also we propose a method to calculate an allocation in the core.

Keywords: Newsvendor game; supply uncertainty; core; cooperation game

1. Introduction

With the globalization of economics and business, supply uncertainty has become more and more important for a company, and brought great challenge to supply chain management. Factors such as the capacity of suppliers, the quality of products, the variation of lead-time and man-made or natural disasters may make our decisions more complicated than ever [1-4]. Therefore, Supply uncertainty is one of the most important issues in supply chain risk management [5]. In order to reduce the risk brought by supply uncertainty, many mitigation policies have been adopted by firms and analyzed theoretically, e.g., dual sourcing or multi-sourcing and supplier diversification [1] and [6], coordinating to sharing the risk between upstream and downstream firms [6], cooperating to improve supply reliability [7]. [1] pointed out that an interesting direction for future research is to analyze cooperative decision-making in the presence of supply risk, e.g., a group of buyers cooperate to make purchases, which would then lead to question about how they should allocate the supply risk among themselves.

In practice, many retailers which sell the same products often cooperate horizontally to manage supply and demand uncertainty, especially for small or medium firms. That is, a set of retailers form a coalition to cooperate to place orders facing uncertain demands and supplies. After the orders are delivered and the demands are realized, someone find that they have excess inventories and others find that their demands can’t be satisfied. The ones with excess inventory will give inventories to the ones with unsatisfied demands. This phenomenon is called as warming themselves by huddling. In this situation, there are some problems need to be answered: Whether is the coalition stable? How to allocate their costs? Is it difficult to get the allocation mechanism? We adopt the notion of core in cooperative game theory to study the problem.

There are two steams of literature related to this study.
The first stream of literature related to this study is on the management of supply uncertainty. Since 1990s, this topic attracts extensive interest from academia and numerous literature has been appeared. [8-14] gave extensive review. Here we only briefly review some literature closely related to this study.

[15] and [16] studied optimal order policy based on EOQ facing unreliable suppliers. [17] studied optimal order policy for the newsvendor problem that is served by multiple suppliers and some of them are unreliable. [18] considered a supplier selection problem, where a buyer, while facing random demand, is to decide ordering quantities from a set of suppliers with different yields and prices. [19] studied the newsvendor problem that the vendor can place two sequential orders from different suppliers considering demand forecast update. The above literature consider one retailer’s optimal order policy facing multiple unreliable suppliers. [20] considered supply chain coordination with uncertain just-in-time delivery due to an uncertain availability of the supply capacity. [21] studied a decentralized assembly system in which the demand and the suppliers’ yields are random. [22] studied the similar problem and proposed four contracts to coordinate the system under forced compliance. [23] considered a decentralized assembly system in which the unreliable suppliers sell complementary components to an assembler, who faces a random demand. They developed a mechanism to coordinate the supply chain. [24] studied the dynamic pricing and supply chain coordination in which both the demand and yield are stochastic. The above papers the vertical coordination between suppliers and retailers mitigating the supply risk.

The second stream of literature is on inventory centralization game. [25-27] gave reviews. Here we briefly review the literature closely related to this study.

[28-30] studied the inventory centralization game based on the EOQ model. [31-34] studied the inventory centralization game based on the ELS model. These studies assume deterministic demand.

There are some papers studying inventory centralization game under uncertain demand. [35] first used cooperative game theory to study newsvendor game. [36] showed that the newsvendor game is balanced and has a non-empty core for symmetric demands distributions or joint multivariate normal demand distributions. [37-38] showed that newsvendor game has non-empty core for general demand distribution. [39] generalized the results to the case with infinite number of retailers. [40] showed that the core in nonempty by using strong dual theorem of stochastic linear programming and proposed an algorithm to compute an allocation in the core. [41] showed that the newsvendor game with independent symmetric unimodal demand distributions is concave. [42-43] studied the game with general cost function. [44] studied the continuous-time inventory centralization game where demands follow a Poisson process with a constant demand rate. [45] presented a method for computing the nucleoli of large newsvendor games. [46-47] considered a two-stage model with uncertain demand. The retailers place independently their order to their suppliers respectively in the first stage and decide to cooperate by reallocating their inventories after observing the demands in the second stage. The above researches assume the supplies are reliable.

In this paper, we study the cooperating horizontally to manage supply uncertainty under uncertain demands. A set of retailers, each of which faces uncertain supply and demand, decide to form a coalition to manage the supply and demand uncertainties. They first place their orders to their suppliers, which are unreliable, respectively. Then their orders are delivered to the retailers’ warehouses. The quantity received by each retailer is random and no larger than his order quantity. After the retailers receive the orders and their demands are realized, transshipments may occur between the retailers with excess inventories and the ones with
unsatisfied demands. Transshipments incur costs. We show that the core is nonempty and propose a method to compute an allocation in the core.

Our study is related to [40]. But there are some essential differences between Chen and Zhang’ model and ours. First, they assume that all the suppliers are reliable while we assume that the suppliers are uncertain. Second, they assume that the retailers place a joint order to the same supplier. The order is delivered to a central warehouse first and then is allocated to the retailers after the demands are realized. We assume that the retailers place their orders to their suppliers respectively and the orders are delivered to the retailers’ warehouses. Transshipments occur between the retailers when some have excess inventories and others’ demands can’t be satisfied. In addition, transshipments will incur costs.

The rest of paper is organized as follows: In Section 2, we present newsvendor game with supply uncertainty. In Section 3 we study the computation of payoff functions. In Section 4 we show that the game has a nonempty core and propose an allocation in the core. Some conclusions are given in Section 5.

2. Notation and Model

In this section we state the notation used in this paper and the model. Consider a set of \( n \) retailers, which sell the same products. Let \( N = \{1, 2, \ldots, n\} \) be the retailer set. The supplier of each retailer is uncertain, so when retailer \( i \in N \) places an order \( y_i \), he can only get \( \theta_i y_i \), where \( \theta_i \) is a random variable with support \((0,1)\). Let \( c_i \) be the unit purchase cost. Then retailer \( i \in N \) will pay his supplier \( c_i \theta_i y_i \) [48]-[50]. When retailers make the ordering decisions, retailer \( i \in N \) faces a random demand \( d_i(\omega) \) with 

\[
E\left[d_i(\omega)\right] < \infty , \quad \text{where } \omega \text{ is random variable in } \Omega.
\]

Assume that the random variables \( \theta_i \) (\( i \in N \)) and \( d_i(\omega) \) (\( i \in N \)) are independent of each other.

A subset \( S \subset N \) of retailers forms a coalition. For any coalition \( S \), let \( d^S(\omega) = (d_i(\omega))_{i \in S} \) and \( d^S = (d_i)_{i \in S} \), where \( d_i \) is the realized demand of retailer \( i \).

Thus, \( d^S(\omega) \) is random demand of coalition \( S \) and \( d^S \) is the realized demand of coalition \( S \). Coalition \( N \) is the grand coalition.

After the orders are delivered and the demands are realized, the retailers know their realized demand and actual delivery quantity. There must be some retailers who have larger inventory than their demands and other retailers who have larger demands than their inventory. Transshipment may occur among the retailers. Let the quantity of the transshipment from retailer \( i \) to retailer \( j \) be \( x_{ij} \) and the unit cost from retailer \( i \) to retailer \( j \) be \( s_{ij} \). After the transshipments, each retailer’s demand is satisfied his own available inventory. Retailer \( i \)’s excess inventory \( l_i \) will incur a unit holding cost \( h_i \) and unsatisfied demand \( z_i \) lost and incurs a unit penalty cost \( b_i \). Assume that the unit transshipment costs satisfy \( s_{ij} + z_i \geq s_{ij} \) to rule out the trivial case.

The notation used in this paper is listed as follows:

- \( d_i(\omega) \): random demand of retailer \( i \);
- \( d_i \): the realized demand of retailer \( i \);
- \( c_i \): unit purchase cost of retailer \( i \);
- \( y_i \): order quantity of retailer \( i \) (a decision variable);
- \( y^S = (y_i)_{i \in S} \): order quantity vector of coalition \( S \).
$x_{ij}$: quantity of transshipment from retailer $i$ to retailer $j$ (a decision variable);

$s_{ij}$: unit cost transshipment from retailer $i$ to retailer $j$;

$h_i$: unit holding cost of retailer $i$;

$b_i$: unit penalty cost of retailer $i$;

$l_i$: excess inventory of retailer $i$;

$z_i$: unsatisfied demand of retailer $i$;

$\theta_i$: supply uncertainty retailer $i$ faces;

$\theta_i = (\theta_i)_{i\in S}$: supply uncertainty vector of coalition $S$.

For a coalition $S$, it is easy to see that the characteristic function $C(S)$ is the optimal value of the following two-stage stochastic linear program:

$$C(S) = \min_{S} \left[ \sum_{i \in S} c_i \theta_i + \mathbb{E}_\theta \left[ f(y^S, \theta^S, d^S(\omega)) \right] \right]$$

$$= \min_{S} \sum_{i \in S} c_i \mathbb{E}_\theta \left[ f(y^S, \theta^S, d^S(\omega)) \right]$$

$$\text{s.t.} \quad y_i \geq 0, \quad i \in S$$

(1)

where $f(y^S, \theta^S, d^S)$ is defined by

$$f(y^S, \theta^S, d^S) = \min \sum_{i \in S} b_i z_i + \sum_{i \in S} h_i l_i + \sum_{i \in S} \sum_{j \in S, j \neq i} s_{ij} x_{ij}$$

$$\text{s.t.} \quad z_i + \theta_i y_i + \sum_{j \in S, j \neq i} x_{ij} \geq d_i, \quad i \in S$$

$$l_i - \theta_i y_i - \sum_{j \in S, j \neq i} x_{ij} \geq -d_i, \quad i \in S$$

$$\theta_i y_i - \sum_{j \in S, j \neq i} x_{ij} \geq 0, \quad i \in S$$

$$z_i \geq 0, \quad l_i \geq 0, \quad y_i \geq 0, \quad s_{ij} \geq 0, \quad i \in S, \quad j \in S$$

(2)

For a coalition $S \subseteq N$, the computation of the characteristic function is divided into two stages. In the first-stage, retailer $i (i \in S)$ decides the order quantities $y_i (i \in S)$ to minimize the expected total cost for the coalition, which includes the purchase cost, freight cost, inventory holding cost, and penalty cost. In the second-stage, retailer $i (i \in S)$ decides the transshipment quantities $x_{ij} (i, j \in S)$ to minimize the total cost for the coalition, which includes inventory holding cost, penalty cost and transshipment costs after the demands are realized. The first and the second constraints in (2) are inventory-demand balance constraints. The third constraint in (2) means that retailer $i$ cannot transship more to others than he receives. The fourth constraint in (2) is nonnegative constraint.

[40] proposed the newsvendor game where the supplier of each retailer is reliable, i.e., $\theta_i = 1$ for any $i \in S$. We generalize their model to the case with supply uncertainty, i.e., the supplier of each retailer is unreliable ($\theta_i$ is a random variable with support $(0,1]$ for any $i \in S$).
3. Computation of the Characteristic Functions and Allocation

In this section, we discuss the computation of the characteristic functions and allocation by using duality approach.

**Definition 1.** A vector \( i = (l_1, l_2, \ldots, l_n) \) is called an allocation for the game \((N, C)\) if \( \sum_{j \in N} l_j = C(N) \).

3.1. The Case that the Supply Uncertainty and Demands are Given

In this subsection, we consider the case that supply uncertainties and random demands are given. Since the order decision \( y^S = (y_i)_{i \in S} \) is made before supply uncertainties and random demands realize, the retailers expect that they can only get \( E[\theta_i]y_i \) when they make order decision. For a given supply uncertainty vector \( \theta^S = (\theta_i)_{i \in S} \) and a given demand vector \( d^S = (d_j)_{j \in N} \) of coalition \( S \), problem (1)-(2) can be rewritten as the following linear programming

\[
C(\theta^S, d^S) = \min \sum_{i \in S} c_i y_i E[\theta_i] + \sum_{i \in S} h_i z_i + \sum_{i \in S} h_i l_i + \sum_{j \in S \setminus \{i\}} s_{ij} x_{ij},
\]

s.t.

\[
\begin{align*}
& z_i + \theta_i y_i + \sum_{j \in S \setminus \{i\}} x_{ij} \geq d_i, & i & \in S \\
& l_i - \theta_i y_i - \sum_{j \in S \setminus \{i\}} x_{ij} \geq -d_i, & i & \in S \\
& \theta_i y_i - \sum_{j \in S \setminus \{i\}} x_{ij} \geq 0, & i & \in S
\end{align*}
\]

(3)

The dual problem of problem (3) is

\[
\max \sum_{i \in S} \left( \alpha_i - \beta_i \right) d_i,
\]

s.t.

\[
\begin{align*}
& \alpha_i \leq h_i, & i & \in S \\
& \beta_i \leq h_i, & i & \in S \\
& \theta_i \left( \alpha_i - \beta_i + \gamma_i \right) \leq c_i E[\theta_i], & i & \in S \\
& \alpha_i - \beta_i - \gamma_i \leq s_{ij}, & i & \in S, j \in S, j \neq i \\
& \alpha_i \geq 0, \beta_i \geq 0, & i & \in S \setminus \{i\}, \gamma_i \geq 0, & i & \in S
\end{align*}
\]

(4)

The strong duality is applied since problems (3) and (4) are linear programming. Then the optimal objective value of problem (4) is also equal to \( C(\theta^N, d^N) \).

Consider the grand coalition \( N \) and its characteristic function \( C(\theta^N, d^N) \). Let \( l_i = (\alpha_i - \beta_i) d_i \) correspond to the computation of \( C(\theta^N, d^N) \). Then \( l_i = (l_1, l_2, \ldots, l_n) \) is a cost allocation and \( (\alpha_i - \beta_i) \) is the shadow price of retailer \( i \) for a given supply uncertainty vector \( \theta^N = (\theta_i)_{i \in N} \) and a given demand vector \( d^N = (d_j)_{j \in N} \).

3.2. The Case with Supply Uncertainties and Random Demands

In this subsection, we discuss the case that retailers face supply uncertainties and random demands.

For the stochastic programming problem (1)-(2), the dual is
\[ \begin{align*}
\max_{\omega} & \sum_{i \in S} \left( \alpha_i(\theta^i, \omega) - \beta_i(\theta^i, \omega) \right) d_i(\omega) \\
\text{s.t.} & \quad \alpha_i(\theta^i, \omega) \leq h_i, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \beta_i(\theta^i, \omega) \leq h_i, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \theta_i \left( \alpha_i(\theta^i, \omega) - \beta_i(\theta^i, \omega) + \gamma_i(\theta^i, \omega) \right) \leq \varepsilon E[\theta_i], \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \alpha_i(\theta^i, \omega) - \beta_i(\theta^i, \omega) - \gamma_i(\theta^i, \omega) \leq s_{ji}, \quad i \in S, j \in S, j \neq i, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \alpha_i(\theta^i, \omega) \geq 0, \beta_i(\theta^i, \omega) \geq 0, \gamma_i(\theta^i, \omega) \geq 0, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega
\end{align*} \]

Let \( \pi_i(\theta^i, \omega) = \alpha_i(\theta^i, \omega) - \beta_i(\theta^i, \omega) \), problem (5) can be rewritten as

\[ \begin{align*}
\max_{\omega} & \sum_{i \in S} \pi_i(\theta^i, \omega) d_i(\omega) \\
\text{s.t.} & \quad \pi_i(\theta^i, \omega) \leq h_i, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \pi_i(\theta^i, \omega) \geq -h_i, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \theta_i \pi_i(\theta^i, \omega) + \theta_j \gamma_j(\theta^i, \omega) \leq \varepsilon_i E[\theta_i], \quad i \in S, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \pi_i(\theta^i, \omega) - \gamma_j(\theta^i, \omega) \leq s_{ji}, \quad i \in S, j \in S, j \neq i, \theta_i \in (0,1], \omega \in \Omega \\
& \quad \gamma_j(\theta^i, \omega) \geq 0, \quad i \in S, \theta_i \in (0,1], \omega \in \Omega
\end{align*} \]

**Theorem 1.** For any coalition of retailers \( S \subset N \), the optimal objective value of the dual problem (6) is equal to \( C(S) \).

**Proof.** For any supply uncertainty vector \( \theta^i \), let

\[ g(\theta^i) = E_{\omega} \left[ f(y^\omega, \theta^i, d^\omega(\omega)) \right]. \]

Since \( E[d_i(\omega)] < \infty \), we can get that for any feasible \( y^\omega = (y_i)_{i \in S} \)

\[ g(\theta^i) = E_{\omega} \left[ f(y^\omega, \theta^i, d^\omega(\omega)) \right] < \infty. \]

Note that retailer \( i \)'s supply uncertainty \( \theta_i \) is a random variable with support \((0,1]\), we have

\[ E_{\theta_i} \left[ f(y^\omega, \theta^i, d^\omega(\omega)) \right] = E_{\omega} \left[ g(\theta^i) \right] < \infty. \]

Therefore, from Theorem 1 in [40], we know that \( C(S) \) is equal to the optimal value of problem (6), for any collection of retailers \( S \subset N \).

[40] showed the strong duality for the inventory centralization games with reliable suppliers. We generalize their results to the newsvendor game with supply uncertain.

It is intuitive to understand the relationship between the original problem (1)-(2) and the dual problem (6). There are two selectable approaches to satisfy demands. In the first approach, retailers cooperate to make order and allocate product among them to minimize the expected total cost for the coalition. This is exactly problem (1)-(2). In the second approach, the retailers outsource the ordering and allocation to another company, and pay a charge depending on the realization of supply uncertainty and random demand to the company. The company maximizes the total charge. The charge includes purchase cost, freight cost, inventory holding cost, and penalty cost. This is exactly problem (6). Theorem 1 shows that the costs of these two approaches are the same.
4. Nonemptiness of the Core

In this section we will show that the newsvendor game with supply uncertainty has nonempty core. We first give the notion of core in cooperative game theory.

**Definition 2.** An allocation $l$ is in the core of the game $(N, C)$, if $\sum_{i \in N} l_i = C(N)$ and $\sum_{i \in S} l_i \leq C(S)$ for any subset $S \subseteq N$.

Let $(\pi_i(\theta^N, \omega), \gamma_i(\theta^N, \omega))$ be an optimal solution of the dual problem (6) with $S = N$.

Define

$$l_i = E_{\theta^N, \omega} \left( \sum_{i \in N} \pi_i(\theta^N, \omega) d_i(\omega) \right)$$

(9)

**Theorem 2.** The vector $l = (l_1, l_2, \ldots, l_n)$ defined by (9) is an allocation in the core of the newsvendor game with supply uncertainty.

**Proof.** From Theorem 1 we know that $C(S)$ is equal to the optimum of problem (6) for all coalition $S \subseteq N$. Because $(\pi_i(\theta^N, \omega), \gamma_i(\theta^N, \omega))$ is an optimal solution of problem (6) with $S = N$, then we have

$$\sum_{i \in S} l_i = \sum_{i \in S} E \left[ \pi_i(\theta^N, \omega) d_i(\omega) \right] = C(N).$$

$l = (l_1, l_2, \ldots, l_n)$ is an allocation of the game.

On the other hand, since $(\pi_i(\theta^N, \omega), \gamma_i(\theta^N, \omega))$ is an optimal solution of the dual problem (6) with $S = N$, $(\pi_i(\theta^N, \omega), \gamma_i(\theta^N, \omega))_{i \in S}$ (actually by restricting it to the coalition $S$) is also a feasible solution of problem (6) with $S \neq N$. Then

$$C(S) \geq E \left[ \sum_{i \in S} \pi_i(\theta^N, \omega) d_i(\omega) \right] = \sum_{i \in S} E \left[ \pi_i(\theta^N, \omega) d_i(\omega) \right] = \sum_{i \in S} l_i.$$

This shows that the allocation $l = (l_1, l_2, \ldots, l_n)$ is in the core of the game.

[40] studied the newsvendor game with reliable supplier, and show that one retailer’s cost allocation has nothing with other retailers. Here we consider the newsvendor games with unreliable suppliers. The cost allocation one retailer’s cost depends on not only himself but also all the other retailers’ supply uncertainties.

From Theorem 2, we can get Corollary 1.

**Corollary 1.** Newsvendor game with supply uncertainty has a nonempty core.

5. Conclusion

We analyze the newsvendor game with supply uncertainty, where multiple retailers form a coalition to cooperate to share risk facing uncertain supplies and demands. We formulate a two-stage stochastic programming model to compute the cost functions of each coalition. By using dual method, we show that newsvendor game with supply uncertainty has a nonempty core and propose a way to find a cost allocation in the core.

Based on our study, there are some topics need to be studied further: (1) newsvendor game with price-dependent demand and supply uncertainty. (2) inventory game with supply uncertainty in multiple periods.
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References


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