Research on Novel Single Image Super-resolution Algorithm through Regularization Approach and Joint Learning Theory: Theoretical Analysis and Applications

Fengling Yin and Bingquan Huo

Binzhou Polytechnic, Shandong, China
yinfengling666@126.com

Abstract

In this research paper, theoretical analysis and applications of a novel single image super-resolution algorithm through regularization approach and joint learning is introduced. Digital image during the process of obtaining the optical fuzzy, movement deformation and degradation factors such as random noise, the influence of the resulting often degradation image, sometimes its resolution is difficult to meet the actual demand of engineering or military applications. In this paper, we combine the joint learning theory together with the regularization standard, through parameter selection, error estimation with omission and solution analysis steps. The proposed framework is based on modified super-resolution model and novel error estimation metrics. In the experiment section, we compare our proposed algorithm with other state-of-the-art and popularly adopted methodologies and use the well-known test image databases to conduct the experiment. The experimental result shows the feasibility and effectiveness of the algorithm. In the future, we plan to do more in-depth research on the parameter selection part to modify our method.

Keywords: Single Image Super-resolution, Joint Learning Theory, Mathematical Regularization, Image Sequences

1. Introduction

Single image super-resolution as one of the most important research areas in the community of image processing, computer vision and pattern recognition is becoming more than popular in the scientific world [1]. Digital image during the process of obtaining the optical fuzzy, movement deformation and degradation factors such as random noise, the influence of the resulting often degradation image, sometimes its resolution is difficult to meet the actual demand of engineering or military applications. Improve imaging system hardware is the most direct method to improve image resolution [2], but should be restricted, technical level, and many other development costs, so the research on image resolution enhancement technique can make up for the inadequacy of existing equipment to some extent. Super-resolution reconstruction technology is not able to totally change the existing imaging system, with complementary information of low resolution image sequence processing, reconstruction of one or more of the high resolution image [3]. Image resolution is the simplest method is to improve image interpolation, such as bilinear interpolation and double three times, but these methods can't generate high frequency information, easy to generate the fuzzy image [4]. Due to its efficiency, feasibility and flexibility in the field of remote sensing, medical and military detection, this crucial technique is widely used world-wide.

Super resolution reconstruction method can be roughly divided into frequency domain method, the airspace method and the method based on learning in recent years. Frequency domain method contains a priori information is limited, used in the calculation of motion model, mass model of a single, limiting its use scope; And based on the method of study,
the design of learning model and the establishment of the training sample set is the
difficulty of this method, this method cannot be widely used. There are some popular
existing algorithms such as: method to estimate the maximum a posteriori probability
(MAP), convex sets (POCS) projection method, maximum likelihood (ML) estimation
method, iterative back projection (IBP) [5-11]. In addition to consider the nature of the
image itself, there are more and more focuses on using composed of pairs of high and low
resolution image block sample collection, trying to get some information from the sample
collection to reconstruct a high resolution image, this method is called image super-
resolution party based on the sample. Traditional regularization method using smooth
constraint and regularization parameters for the overall situation, there is inhibition of the
contradiction between images’ noise and protect the image detail information. In order to
improve the quality of reconstruction image, the inhibition of false information should
reduce the smoothing of image details, accord with the characteristics of the image needs
to be considered a priori model with adaptive regularization parameter calculation
method.

This article first match in the process of super resolution reconstruction error, noise and
other factors are analyzed, to curb the error produced by the false information and
improve the quality of reconstruction image, on the basis of traditional regularization
method, is put forward for different areas of the image using a prior model and suitable to
the characteristics of adaptive regularization parameter, to suppress noise and protect the
balance of the details.

2. The Image Super-resolution Model and Error Estimation

2.1. The Image Super-resolution Model

For super resolution reconstruction of error factors, through transformation model
between original image and image quality were analyzed, and the typical model is shown
in figure 1. Suppose the low resolution image sequence denoted as \( \{ y_k, k \in [1,N] \} \) contains
N sub-size images \( L_1 \times L_2 \). The \( x \rightarrow y_k \) reduction process can be expressed as:
\[
y_k = DB_k M_k x + n_k \quad k \in [1,N]
\]
(1)

Where, \( D \) represents the down sampling matrix, \( B_k \) denotes the caused by the point
spread function (PSF) or diffraction limit fuzzy matrix, \( n_k \) is the outside noise. Moreover,
\( M_k \) is the deformation matrix caused by the movement. Super-resolution reconstruction is
a low resolution image sequence is obtained by the actual process of
\( \{ y_k, k \in [1,N] \} \) reconstruction of high resolution images.

![Figure 1. The Overview of Super-resolution Model](image-url)
2.2. The Error Estimation Technique

Super resolution reconstruction of convex set is commonly used in projection method, maximum a posteriori probability, tend to be based on accurate image registration and accurate estimate of the fuzzy function, is hard to do in the practical application, and image in the process of drop quality tended to be mixed with noise, the error factors which reduces the quality of reconstruction image. We could analyze the feature through the formula 1. The estimate of fuzzy matrix and deformation matrix are shown in the formula 2-3.

\[ M'_k = M_k + \Delta M_k \]  \hspace{1cm} (2)

\[ B'_k = B_k + \Delta B_k \]  \hspace{1cm} (3)

Combine the characteristics of the high and low resolution of vector, using this feature in combination with the whole sample set is divided into several sub collection. As mentioned above, the super-resolution reconstruction problem in high and low resolution one-to-many mapping relationship between, and this method not only requires low resolution piece of similar, also calls for high resolution piece of similar, this limits the variety of high-resolution block. The revised version of observation result is expressed as:

\[ x' = x + (DB_kM_k)^{-1}n_k \]  \hspace{1cm} (4)

Through the combination of 1~4, we could derive the formula 5.

\[ x' = x + (DB_kM_k)^{-1}n_k = x + (DB_kM_k)^{-1}n_k + H_k^{-1}(\Delta B_k, \Delta M_k)n_k \]  \hspace{1cm} (5)

To incorporate the information in the low resolution images to produce a higher resolution image, we will have to make notation of the errors. This method can generate at two magnification of good quality high resolution images, but when the magnification will increase when the reconstruction quality.

3. Tradition Regularization and Joint Learning Algorithm

3.1. Weakness of Tradition Regularization Method

In traditional regularization equation we will always using Laplacian priori model. Through to the solution of the high frequency component do smoothness constraint to stability of the constraints and does not take into account local characteristics of the space image, and the regularization parameters is global. Image details such as edge and texture information and image noise such as false information all belong to the high frequency components of image, traditional regularization equation does not take into account the image differences in different regions of the high frequency components, will cause the image detail fuzzy residual or false information. Using markov random fields (MRFs) model by using the compatible image local area to choose suitable high resolution image block we could generate high resolution images. This method requires a large training sets and large amount of calculation. In super resolution reconstruction of the solution, often using traditional interpolation method (bilinear interpolation and bicubic interpolation) for the reconstruction of the initial image, traditional interpolation method for global method, the kernel function for isotropic template, not considering the local spatial structure of image information, also causes the introduction of the image edge blur and false information.

According to the observation of super-resolution reconstruction model, given the low resolution image sequence under the condition of \( \{ y_k, k \in [1, N] \} \). The MAP estimation model could be expressed as the formula 6. The regularized version is the formula 7.

\[ x^* = \arg \max_{x} \sum_{k=1}^{N} \left\{ p(x|y_k) \right\} \]  \hspace{1cm} (6)
\[ x' = \arg\min_{x \in \mathcal{X}} \{ \rho \left( y_k - DB_k M_x x \right) + \lambda \psi \left( x \right) \} \] (7)

Where the \( \rho(\cdot) \) is image super-resolution reconstruction of observation data fidelity term, \( \psi(x) \) denotes the image super-resolution reconstruction of regularization. \( \lambda \) is the regularization parameters, by adjusting the data fidelity term and the proportion of regularization to control image super resolution reconstruction results.

3.2. Joint Learning Algorithm

Having the characteristics of high and low resolution gradient low rank after component, respectively using two projection matrices map them into a unified feature space. In this unified feature space used in the resolution of the reconstructed neighborhood embedded assumptions can be better. The characteristics of the desired goal are in the public space, the corresponding characteristics after the high and low resolution projection vector of low rank component can close distance as far as possible, so we can build a following formula:

\[
\arg\min_{\{ P_i, P_h \}} J(P_i, P_h) = \arg\min_{\{ P_i, P_h \}} \sum_{i \in G} \left\| P_i^T \left( x'_i \right) \right\| - P_h^T \left( y'_i \right) \right\|^2_2
\] (8)

To use average fusion adjacent blocks overlap each other. The neighborhood embedding method to generate high resolution image is the image into blocks for each block after processed, usually this kind of way to get high resolution images are not satisfied about global reconstruction.

4. Our Proposed Model

4.1. Overview of Our Model

Through the analysis on traditional regularization method has many shortcomings, in order to improve the quality of reconstruction image, this paper puts forward the need to rebuild image division, through the local variance to measure the spatial structure of regional information, by setting threshold, the image is divided into the smooth area, edge/texture area and the smooth area [12-14]. Locally linear embedding in the manifold learning ideas, put forward the high and low resolution of block manifolds with similar local structure assumption, each test low resolution block using a linear combination of the sample collection of a number of nearest neighbor, through calculate the reconstruction weights and pass a weight to a linear combination of the high resolution sample piece to generate for reconstruction of high resolution image block. This method avoids using large training sets, can generate satisfactory reconstruction results. But in the super-resolution reconstruction problem between high and low resolution of one-to-many mapping relation, so the assumption is not necessarily to the manifold. For edge and texture area, choose to better protect the image detail information prior model; for the smooth, choose better can inhibit the prior model of false information [15-17]. The flowchart of the system is shown in the figure 2.

Figure 2. The Overview and Flowchart of Proposed Algorithm
First of all, using triangular interpolation method (DDT) in low resolution image $y$, amplification to get the image $x'_0$ whose size is the same as the original image to avoid the error of the traditional interpolation methods in the reconstruction of expansion. Then, calculate the variance expressed in the formula 9 ad 10.

$$\sigma^2_{x_i}(i,j) = \frac{1}{4ab} \left[ \sum_{m} \sum_{n} x'_i(m,n) - h_{x_i}(i,j) \right]^2$$  \hspace{1cm} (9)

$$h_{x_i}(i,j) = \frac{1}{4ab} \left[ \sum_{m} \sum_{n} x'_i(m,n) - x'(i,j) \right]$$  \hspace{1cm} (10)

### 4.2. The Adaptive of the Prior Model

Choosing the de-noising effect is good for the smooth area, gauss-markov prior model (GMRF), and its regularization can be expressed as [18-22]:

$$\varphi(x) = \sum_{c \in \mathbb{C}} V_c(x)$$  \hspace{1cm} (11)

In this formula, the $c$ denotes the clusters, which could also be named as the image pixel's neighborhood system. $\mathbb{C}$ is the collection of all the side clusters. $V_c(x)$ is the corresponding potential function, the definition is shown in the formula 12. $d_i^v(x)$ is corresponding to the image in a horizontal, vertical, diagonal and against the direction of the angle change expressed in the equation 13.

$$V_c(x) = \sum_{i=1}^{2} \beta_i(d_i^v(x))$$  \hspace{1cm} (12)

$$\begin{cases}
    d_1^h(x(m,n)) & = \ x_{m-1,n} - 2x_{m,n} + x_{m+1,n} \\
    d_1^v(x(m,n)) & = \ x_{m,n-1} - 2x_{m,n} + x_{m,n+1} \\
    d_1^d(x(m,n)) & = \ (\sqrt{2}/2)(x_{m-1,n-1} - 2x_{m,n} + x_{m+1,n+1}) \\
    d_1^d(x(m,n)) & = \ (\sqrt{2}/2)(x_{m-1,n+1} - 2x_{m,n} + x_{m+1,n-1})
\end{cases}$$  \hspace{1cm} (13)

For the smooth area, the selection of image details to protect good Huber markov prior model (HMRF) [23-25], the a priori model will smooth measurement function instead of section function, in order to avoid the transition of image smoothing, do to protect the image detail better to be expressed as:

$$\beta_i(i) = \begin{cases}
    i^2, & i \leq a \\
    2\alpha |i| - \alpha^2, & i > a
\end{cases}$$  \hspace{1cm} (14)

First according to the similarity will be divided into training set is a collection each child in the collection on block is located in the same low dimensional subspace. Then use the low rank matrix recovery method to study the structure of this subspace, solving the characteristic of low rank component although there is no change in length, but they are much closer links between the two countries. High and low resolution of the original characteristics of the low rank component respectively mapped to a unified space. Finally completed in the unified space based on neighborhood embedded super-resolution reconstruction, get the initial high resolution image. The within cluster $c$ of horizontal, vertical and diagonal direction Angle with the smooth measure for the equation is:
\[
\begin{align*}
    d^1_i(x(m,n)) &= x_{m-1,n} - 2x_{m,n} + x_{m,n+1} \\
    d^2_i(x(m,n)) &= x_{m-1,n} - 2x_{m,n} + x_{m,n+1} \\
    d^3_i(x(m,n)) &= 0.5\left(x_{m-1,n-1} - 2x_{m,n} + x_{m,n+1} \right) \\
    d^4_i(x(m,n)) &= 0.5\left(x_{m-1,n-1} - 2x_{m,n} + x_{m,n+1} \right)
\end{align*}
\] (15)

4.3. The Parameter Selection for the Regularization Model

In the smooth areas of the image, with the decrease of \(\sigma^2_{n_0}(i, j)\), human’s subjective visual for smooth area of false information is also the most sensitive. So regularization parameter should select larger values in order to achieve inhibition of false information; Area is often larger, on the other hand, in the edge and texture to the human eye vision of false information is far less sensitive than in the smooth area. Development coupling constraints combine the characteristics of the high and low resolution of vector, using the method combined with characteristics of the whole sample set is divided into a number of child sets. As mentioned above, the super-resolution reconstruction problem in a one-to-many mapping between high and low resolution block off. In order to protect the image detail information, \(\lambda^2_{\sigma^2_{n_0}}(i, j)\) should be smaller. In this paper, the regularization parameter calculation combined with image of local spatial information expressed as the following:

\[
\lambda^2_{\sigma^2_{n_0}}(i, j) = \frac{\tau}{\theta\sigma^2_{n_0}(i, j) + 1}
\] (16)

4.4. The Solution for the Regularization Model

Image super resolution reconstruction of regularization framework can be expressed as:

\[
x^i = \text{arg min}_{x} \left\{ \sum_{l=1}^{N} \left\| y_l - DB_{l} M_x \right\|^2 + \lambda^2_{\sigma^2_{n_0}}(i, j) \sum_{c \in C} \sum_{l=1}^{4} \beta_i\left(d^i_l(x)\right) \right\}
\] (17)

The basic idea is: suppose a matrix column is some belong to the same pattern vector. Because these vectors are from the same pattern, so in many cases they are linear correlation, the properties of matrix have low rank. Low rank matrix recovery method to such a matrix is decomposed into a low rank matrix and a sparse matrix, the matrix has low rank, and therefore has a sparse matrix. The matrix-vector operation into image filter operator of image processing, to improve the efficiency of cost function gradient calculation. After using the steepest descent method (SD) optimization algorithm, iterative algorithm can be expressed as:

\[
x^{m+1} = x^{m} - \text{arg min}_{x} \left\{ \sum_{l=1}^{N} \left\| y_l - DB_{l} M_x \right\|^2 + \lambda^2_{\sigma^2_{n_0}}(i, j) \sum_{c \in C} \sum_{l=1}^{4} \beta_i\left(d^i_l(x)\right) \right\}
\] (18)

After the iteration, we could find out the optimal solution.

5. Experiment and Simulation

5.1. The Experiment Set-up and Initiation

In order to verify the validity of the algorithm in this paper, on the Windows platform, select the Laplacian priori Matlab2011b environment model, TV prior model and GMRF prior model, comparing with the algorithm. In the experiment to select size 256 x 256 ”cameraman” as a reference to the original image, carry on the translation, fuzzy and sampling process after adding noise, specific operation steps as follows: in the translation, in the image of horizontal and vertical direction moving randomly 0 ~ 2 pixels. Through using Gaussian blur image fuzzy process, the point spread function is:
\[ h(i, j) = \begin{cases} 
\frac{1}{e^{-\tau(i^2 + j^2)}}, & (i, j) \in E \\
0, & (i, j) \notin E 
\end{cases} \]  \quad (19)

Is a reference to the original image evaluation method commonly used in the peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) index two indicators to assess for super resolution reconstruction image’ quality. Moreover, we introduce a novel standard denoted as the CC. CC is a quantity that gives the quality of least squares fitting to the original data. It computes how close the reconstructed image is relative to the reference image. The three standards are defined in the following formulas:

\[ R_{PSN} = 10\log_{10} \frac{255^2 MN}{\sum_{m=1}^{M} \sum_{n=1}^{N} x'(m,n) - x(m,n)^2} \]  \quad (20)

\[ SSIM = \frac{(2\mu_B\mu_A + C_1)(2\sigma_{BA} + C_2)}{\mu_B^2 + \mu_A^2 + C_1(\sigma_B^2 + \sigma_A^2) + C_2} \]  \quad (21)

\[ CC = \frac{\sum_{x,y} B'(x,y) A(x,y) - MN \mu_B \mu_A}{\left(\sum_{x,y} B'^2(x,y) - MN \mu_B^2\right) \times \left(\sum_{x,y} A'^2(x,y) - MN \mu_A^2\right)} \]  \quad (22)

### 5.2. The Experimental Result

The above image reconstruction are based on the regularization method, by contrast, you can see that in Figure 3 (d) ~ (f) adopted by the prior model because not only emphasize on limits to the smoothness of image reconstruction, considering the correlation of image region, the rebuilt image edge and texture than the traditional method to get the Figure 3 (c) improved substantially. But these methods have the same prior to the whole image model, using the regularization parameters for the overall situation, the reconstruction effect is better than the Laplacian priori model, but there is still the contradiction between noise and protect the image detail; The method by dividing the image area, combining with the regional information for the a priori model selection and regularization parameter setting, improves the deficiency of existing method, can do to protect the reconstruction image at the same time the unification of the detail and suppress noise. You can see from the image on the premise of protection of details, the residual noise in the Figure 3 (g) than the Figure 3 (d) ~ (f). The second figure shows the reconstruction of the objective evaluation index, and also illustrates the method is superior to the contrast method, this paper proves the better result on the vision. In practical application, have been the norm of observations are often without reference to the original image, and this can be used without reference to the original image evaluation methods such as average gradient, correlation coefficient, and the information entropy index to evaluate the quality of reconstruction image.
6. Conclusion and Summary

In areas such as computer vision, video surveillance, medical image diagnosis and treatment, satellite imagery and many other areas you need to use high resolution image. But in some cases, as a result of the limitation of imaging system and the imaging environment, it is difficult to obtain an ideal resolution image. The limited image resolution will affect the performance of the system, such as low resolution image will cause the system identification performance. Therefore, we need to generate high resolution figure from the low resolution image. In this paper, we conduct research on novel single image super-resolution algorithm through regularization approach and joint learning theory. Image resolution is the simplest method is to improve image interpolation, such as bilinear interpolation and double three times, but these methods can't generate high frequency information, easy to generate the fuzzy image. We combine the joint learning theory together with the regularization standard, through parameter
selection and solution analysis steps, we finalize the proposed methodology. The experimental result shows the feasibility and effectiveness of our proposed methodology compared with other state-of-the-art approaches. In the future, we plan to modify the parameter selection step to combine with more technique to level-up the correctness of proposed method.

References

