A Supervised Patch-adaptive Super Resolution Algorithm Based on Compressive Sensing

Haitian Zhai*, Hui Li and Weiting Gao

Department of Electronics and Information, Northwestern Polytechnical University, Xi’an, Shaanxi 710072, China
Haitian_1988@mail.nwpu.edu.cn

Abstract

This paper introduces a novel solution to generate a super-resolution image from a set of low-resolution input based on patch information. Recent research has shown that super-resolved data can be reconstructed from an extremely small set of measurements compared to that currently required. This paper incorporates the compressive sensing framework to the reconstruction model. Moreover, in order to remove outliers introduced by image parallax, the supervised patch-adaptive matching method which uses photometrical similarity and geometrical distance to determine the matching patch is proposed to reconstruct the high resolution image. The performance of the proposed algorithm on both synthetic and real images is evaluated with several grayscale and color image sequences and found successful when compared to other algorithms.

Keywords: image reconstruction, super resolution, compressive sensing, sub-pixel registration

1. Introduction

A great number of applications frequently require high-resolution images such as biometrics identification, satellite imaging, and so on. However, in many imaging applications, acquiring an image of a scene with high spatial resolution is not possible due to a number of theoretical and practical limitations. To increase the image resolution, either increase the chip size of sensors or reduce the pixel size by sensor manufacturing techniques are severely constrained by the physical limitation of imaging systems [1]. Therefore, it is of great interest to reconstruct a high resolution (HR) image using only digital image processing techniques. Most of the image magnification processes are usually achieved by pixel interpolation using a linear filter which cannot recover the real image detail even if the pixel number of the display is increased.

The problem of the reconstruction of a high-resolution image from a set of low-resolution observations of the same scene, known as super resolution (SR), has been an active research topic in the areas of image processing and computer vision. There are many methods for super-resolution image reconstruction [2-6], [11-14]. Among them, the frequency domain method is one of the most promising solutions [2]. They use a discrete Fourier transform (DFT)-based algorithm to model global translational scene motion. How to recover the high-frequency information that was lost in the process of generating the low-resolution inputs is the main challenge. In terms of Nyquist-Shannon sampling theorem, the high-frequency information was eliminated by the band-limiting filter of the photographic process was due to imperfections in the optics and integration over the pixels of the sensor if the low resolution (LR) images were directly captured by a camera. More recently, Fourier-wavelet deconvolution and denoising algorithms are combined and extended to the multiframe SR application [3]. They use a fast Fourier-based-

* Corresponding Author
multiframe image restoration method to produce a sharp, but noisy estimate of the HR image and then apply a space-variant non-linear wavelet thresholding technique that addresses the non-stationarity inherent in resolution-enhanced fused images.

Another notable super-resolution method which only uses a single LR image for estimating the HR image, known as single image super resolution [7-10], can be classified into two major categories. The first class of methods are reconstruction based super resolution algorithms which define constraints for the desired HR image to improve the quality of the reconstruction. Algorithms of this type reconstruct image details by interpolating the LR input while making edges sharper. The most well-known method is the back projection algorithm which gradually sharpens the edges in each iteration. Algorithms of the second type are learning based super resolution algorithms which learning correspondences between LR and HR image patches from the training database to invert the process of down-sampling in a reasonable manner. Chakrabarti et al., [8] proposed a kernel principal component analysis based on the prior model to produce SR face images. They applied the prior probability based on the energy outside of the principal components to regularize the solution. Freeman et al., [9] proposed an example-based SR resolution technique which applied the Markov Random Field to framework and solved through belief propagation. This technique requires the nearest neighbor-based estimation of high-frequency patches based on the corresponding patches of a recorded low frequency image. T. Goto et al., [10] proposed a learning-based superresolution method that utilizes total variation (TV) regularization. Zhou et al., [7] proposed a learning-based approach through sparse representation. They jointly trained the HR and LR dictionaries for signal sparse representation.

For the growing requirements for super-resolution (SR) images, compressive sensing (CS) theory [15], which rapid progress in the past few years, provides a new solution to such an issue as opposed to traditional thinking of continually enlarging the sensor array. The sparsity of the representation is the focus of the present study, specifically its effects on reconstruction speed. Recently sparse representation has been successfully applied to many other related inverse problems in image processing, such as denoising and restoration, often improving on the state-of-the-art. Therefore, SR through compressed sensing was attempted.

In this paper, a novel compressive sensing based super resolution is proposed. Through the utilization of a systematic modeling of the desired HR image, the proposed framework incorporates the patch-adaptive reconstruction method and provides an estimated HR image with high quality and compares favorably to existing super resolution algorithms.

The rest of the paper is organized as follows. Section 2 introduces the basic concept of compressive sensing model and the greedy algorithm. Patch adaptive SR based on compressive sensing framework is proposed in Section 3 and the experimental results are compared with a number of methods in Section 4. Finally, conclusions are drawn in Section 5.

2. **Compressive Sensing Model**

Compressive sensing is a novel and popular approach for sparse signal reconstruction. In image processing the main assumption is that the image has a sparse representation in an orthonormal basis or a tight frame. Assuming \( x \in \mathbb{R}^n \) is \( n \) dimensional signal of \( K \)-sparse representation in the orthogonal basis \( D \), written as,

\[
x = D\alpha
\]  

where coefficient \( \alpha \) contains only \( K \) non-zero or significant elements. In CS theory, \( x \) represents any one dimensional signal. However, this signal is assumed to be an \( m \)-pixel image that has been reconstructed into an \( n \)-pixel vector. Clearly, \( x \) and \( \alpha \) are
equivalent representations of the same signal, with $x$ in the time domain and $\alpha$ in the $D$ domain. One of the outstanding results in CS theory is that the signal $x$ can be reconstructed using optimization strategies aimed at finding the sparsest signal that matches with the $m$ projections.

$$y = \varphi x$$

(2)

where $\varphi$ is an observation matrix that optimized by the gradient decent method. The measurement process can be rewritten as

$$y = \varphi x = \varphi D \alpha = \psi \alpha$$

(3)

Within the framework of CS, the desired sparsest $\alpha$ can be determined by solving the following $l_0$-norm problem:

$$\min \| \alpha \|_0 \quad \text{s.t.} \quad y = \psi \alpha$$

(4)

The problem is known to be an nondeterministic polynomial time complete problem. However, recent studies indicate that $l_1$-norm minimization can yield a result similar to that of $l_0$-norm minimization as long as the number of samples $N = O \left( K \log \left( \frac{L}{K} \right) \right)$

$$\min \| \alpha \|_1 \quad \text{s.t.} \quad y = \psi \alpha.$$  

(5)

To apply CS in SR image reconstruction, the test image is considered the sub-sampled image of the HR image. However, the sparsity of a signal and the incoherence between measurement and representation matrices must be maintained to determine the unique sparse solution.

2.1. Sparse Reconstruction Condition

The measurement matrix is designed to ensure that the compressible signal is not damaged by dimensionality reduction. Many well-known pairs of incoherent bases exist, including the random Fourier sample measurement matrix with the identity matrix and the Gaussian measurement matrix with any other basis. Random observation is the original method for designing the measurement matrix, after which deterministic observation was proposed. $y = \psi \alpha$ cannot be solved for $\alpha$ with any arbitrary $\psi$ if $m \geq n$, even if $m \geq 2K$. However, compressed sensing framework can be applied only if the measurement matrix $\psi$ meets the Restricted Isometry Condition (RIC) [16]:

$$(1 - \zeta) \| \psi \|_2 \leq \| \psi \|_2 \leq (1 + \zeta) \| \psi \|_2$$

(6)

with parameters $(\zeta, \zeta)$, where $\zeta \in (0,1)$ for all $z$-sparse vectors $\nu$. Essentially, the RIC states that a measurement matrix will be valid if every possible set of $z$ columns of $\psi$ forms an approximate orthogonal set. In effect, sampling matrix $S$ need to be as incoherent to the compression basis $D$ as possible. Examples of matrices that have been proven to meet RIC include Gaussian matrices (where the entries are independently sampled from a normal distribution), Bernoulli matrices (binary matrices drawn from a Bernoulli distribution), and partial Fourier matrices (randomly selected Fourier basis functions).

In this work, wavelets are used as the compression basis $D$ because they are much better at sparsely representing images than non-localized bases such as Fourier. However, in super-resolution the downsampling matrix $\varphi$ involves point-sampled measurements, which could result in a measurement matrix $\varphi$ that does not meet the Restricted Isometry Condition because point-sampling measurements are not incoherent with the
wavelet compression basis. Intuitively, it can be seen that the better a basis is at representing localized features (such as wavelet), the more coherent it will be to point sampling because it can represent small spatial features (e.g., point samples) with only a few coefficients, by definition. Therefore, in order to successfully apply a wavelet basis to this problem, we must find a way to increase the incoherence between the bases.

2.2. Greedy Reconstruction Algorithm

Due to the simple geometric interpretation and low complexity, the iterative greedy algorithms, which include the Orthogonal Matching Pursuit (OMP) [17], the Regularized OMP (ROMP) [18], and the Stage-wise OMP (StOMP) algorithms [19], received significant attention. The idea behind these greedy algorithms is to find the support of the unknown signal sequentially. The main idea of greedy algorithm is to find the coefficient of \( \alpha \) with the largest magnitude by projecting \( y \) onto each column of \( \psi \) and selecting the largest \( \langle y, \phi_i \rangle \), where \( \phi_i \) is the \( i \text{th} \) column of \( \psi \). Once the largest coefficient of \( \alpha \) is identified, a least-square problem is resolved assuming it is the only non-zero coefficient. The new estimate for \( \alpha \) is utilized to compute the estimated original HR signal \( x \) and subtract it from \( D\alpha \). This process is iterated using the residual signal to find the next largest coefficient of \( \alpha \). By iterating \( K \) times, a \( K \)-sparse approximated representation is acquired.

The computational complexity of the OMP algorithm depends on the sparsity of the original signal: reconstruction complexity of standard OMP is roughly \( O(KmN) \), since it is a \( K \)-sparse signal. This complexity is significantly smaller than that of linear programming methods, especially when the signal sparsity level \( K \) is small. However, for OMP techniques to operate successfully, the correlation between all pairs of columns of \( \psi \) should be at most \( 1/2K \), which by the Gershgorin Circle Theorem, represents a more restrictive constraint than the RIP. Although ROMP is faster and more robust on restricted isometry condition, its search strategy is overly restrictive. A modified version of it called subspace pursuit (SP) algorithm [20] was proposed that has superiority over both OMP and ROMP. The SP algorithm generates a list of candidates sequentially with back-tracing, which incorporates a method for re-evaluating the reliability of all candidates at each iteration of the process.

3. Proposed Approach

3.1. Patch Adaptive Super Resolution

In this section, the patch based super resolution algorithm is proposed to produce improved SR images. The basic idea behind the proposed super resolution algorithm is borrowed from sequential coding theory with backtracking. The prevailing image registration methods assume all the feature points are coplanar and build a homography transform matrix to do registration. The advantage is that they have low computational cost and can handle planar scenes conveniently; however, with the assumption that the scenes are approximately planar, they are inappropriate in the registration applications when the images have large depth variation due to the high-rise objects, known as the parallax problem. Parallax is an apparent displacement of difference of orientation of an object viewed along two different lines of sight, and is measured by the angle or semi-angle of inclination between those two lines. Nearby objects have a larger parallax than further objects when observed from different positions. Therefore, the objects close to camera appear to move faster than the objects in the distance as the viewpoint moves side to side. So these two images cannot be merged directly.

From the first step, reference image is subdivided into patches (of size \( n \)) that are traversed in raster-scan order. For each patch, the matching patch located in the input image, needs to be found. The matching patch should satisfy two conditions: first, the
patch should come from a location in the input image that has a shortest distance; second, the patch should has the largest similarity with the reference patch. Define $P_k$ as the probability of the matching patch,

$$P_k = \exp \left( \sqrt{2S - D(b_k,t_j)} \right), \quad k = 1, \ldots, M_1, \quad j = 1, \ldots, M_2$$

(7)

where $\{(b_k), \quad k = 1, \ldots, M_1\}$ are the vectors of the blocks in the reference LR image, $\{(t_j), \quad j = 1, \ldots, M_2\}$ are the vectors of the blocks in the input LR image, $M_1$ and $M_2$ are the numbers of the patches in the reference image and input image, respectively, and $D(b_k,t_j)$ is the geometric distance between $b_k$ and $t_j$,

$$D(b_k,t_j) = \left\| b_k - t_j \right\|_n$$

(8)

where $\left\| \cdot \right\|$ is Euclidean distance. The prior probability, which is derived from the cross correlation between $b_k$ and $t_j$, is formulated below

$$r_k = \frac{\sum_i \left[ (b_k(i) - \bar{b}_k) \times (t_j(i) - \bar{t}_j) \right]}{\sqrt{\sum_i \left[ (b_k(i) - \bar{b}_k) \right]^2 \times \sum_i \left[ (t_j(i) - \bar{t}_j) \right]^2}}$$

(9)

where $\bar{b}_k$ and $\bar{t}_j$ are the means of $b_k$ and $t_j$, respectively. Then the matching patch $\hat{b}_k$ is given by

$$\arg \max_{i} \left( c_1P_{i} + c_2r_{k} \right) / (c_1 + c_2), \quad k = 1, \ldots, M_1$$

(10)

where $c_1$ and $c_2$ are the regularization parameters that determine how strongly the geometric distance and photometrical similarity are constrained to the total probability. Then, the input LR image can be formulated as below

$$\tilde{y} = (\hat{b}_{1}, \hat{b}_{2}, \cdots, \hat{b}_{n})$$

(11)

The constrained optimization (3) can be similarly reformulated

$$\tilde{y} = \Psi \alpha$$

(12)
As discussed in previous section, even wavelets is good at representing sparse image data, we cannot use it directly in image reconstruction procedure since it does not meet the RIC when the image is downsampled. To fulfill RIC requirement, a blurring matrix $\Phi$ is used to filter the high resolution image $y$. The desired high resolution image can be written as $\hat{s}$, which is then filtered by the blurring matrix $\Phi$.

$$\begin{align*}
s &= \hat{s}
y &= \Phi s
\end{align*}$$

(13)

The Gaussian filter $\Phi$ is chosen as the blurring filter. Since this filter can be considered as a multiplication by a Gaussian in the frequency domain, it can be defined that $\Phi = \mathcal{F}^* G \mathcal{F}$. Then (13) can be written as

$$\begin{align*}
y &= \psi \mathcal{F}^* G \mathcal{F} \alpha
\end{align*}$$

(14)

where $G$ is a Gaussian matrix with values of the Gaussian function along its diagonal and zeros elsewhere and $\mathcal{F}$ is the Fourier transform matrix. Also, by assuming that the transform of $\alpha$, denoted by $\hat{\alpha}$, is sparse in its wavelet domain, (14) can be modified as

$$\begin{align*}
y &= \psi \mathcal{F}^* G \mathcal{F} D \hat{\alpha}
\end{align*}$$

(15)

There are many solutions to the underdetermined (15). In order to make the result unique, the desired $\hat{\alpha}$ is considered to have minimum number of nonzero coefficients. In

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**Algorithm 1: Subspace Pursuit Algorithm**

**Input:** $K$, $\phi$, $y$

**Initialization:**

- $\tilde{T} = \{K\}$ indices corresponding to the largest absolute values of $\phi^* y$
- $y_r = \text{residue}(y, \phi_{\tilde{T}})$

**Iteration:**

- If $y_r = 0$, quit the iteration; otherwise continue
- $T' = \tilde{T} \cup \{K\}$ indices corresponding to the largest magnitudes of $\phi^* y_r$
- Let $x'_r = \phi_{T'}^* y$ where $\phi_{T'}^* := (\phi_{T'} \phi_{T'})^{-1} \phi_{T'}$
- $\tilde{T} = \{K\}$ indices corresponding to the largest elements of $x'_r$
- $\tilde{y}_r = \text{residue}(y_r, \phi_{\tilde{T}})$
- If $\|\tilde{y}_r\| > \|y_r\|$, quit the iteration; otherwise, let $\tilde{T} = \tilde{T}$ and $y_r = \tilde{y}_r$, and continue with a new iteration

**Output:**

The estimated signal $\hat{a}_r$ satisfies $a_r^{(n=x)} = 0$ and $\hat{a}_r^T = \phi_{T}^* y$
other word between all possible answers for $\hat{a}_s$, the one which gives minimum number of coefficients is chosen. Therefore $\hat{a}_s$ can be found by solving an $l_1$-norm optimization problem. Greedy algorithms introduced in section III-B, provide a solution for $l_1$-norm optimization. Therefore, the $l_1$-optimization problem is expressed as follow:

$$\begin{aligned}
\min_{\hat{a}_s} & \|\hat{a}_s\|_1 \\
\text{s.t.} & \quad y = \psi F^T G F D \hat{a}_s
\end{aligned}$$

(16)

By having $\phi = \psi F^T G F D$, (16) would be modified to

$$\begin{aligned}
\min_{\hat{a}_s} & \|\hat{a}_s\|_1 \\
\text{s.t.} & \quad y = \phi \hat{a}_s
\end{aligned}$$

(17)

Therefore, $\hat{a}_s$ can be found by solving (17). In this work, subspace pursuit algorithm has been used for solving (17). Compared with ROMP which is also a greedy algorithm, Subspace Pursuit algorithm has lower reconstruction complexity of matching pursuit techniques as long as better reconstruction capability. In SP algorithm, both the forward matrix $\phi$ and backword $\phi^*$ are needed

$$\phi^* = D^T \Phi^{-1} \psi^T = D^T F^T G^{-1} F \psi^T$$

(18)

The output of the subspace pursuit algorithm ($\hat{a}_s$) is in wavelet domain, by taking inverse of wavelet transform the desired high resolution image ($a_s$) would be recovered.
4. Experimental Results

In this section, the performance of the proposed algorithm on both synthetic and real images is analyzed by comparing with different algorithms such as robust SR [5], total-variation regularization SR [13], and normalized convolution SR [14]. Shown on Figure 1(a) is one of the 8 synthetic LR image generated from the HR image through translating, rotating, blurring, and downsampling by a factor of 4. (e) shows the result of an SR image reconstructed by the proposed method. The variance of $G$ is set to $\sigma^2 = 1666.7$, and the coefficients are set to $c_1 = 0.7$, $c_2 = 0.3$. For the purpose of comparison, three different algorithms are also implemented on the same set of LR images and the results are shown in (b-d). It can be seen from the zooming area of Figure 1 that in proposed method the LR effect is significantly reduced and the resolution is highly enhanced. Due to the correction of the robust patching searching and matching, the details of the lion’s mane looks clearer for the proposed algorithm.

In order to measure performance analysis, the HR image is first downsampled into LR images and then reconstructed using the proposed algorithm to HR image. Since the original HR image is available, the restoration quality is measured by peak signal-to-noise ratio (PSNR) of the image as

$$PSNR = 10 \log_{10} \left( \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} 255^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - \hat{I}(i,j))^2} \right)$$  \hspace{1cm} (19)$$

where $I$ is the original HR image and $\hat{I}$ is the reconstructed image. Figure 2 shows a plot of PSNR with the number of frames taken for super-resolution. From the figure, it is evident that increasing the number of frames does not have significant influence on PSNR but causes additional computational cost and the proposed algorithm has the largest PSNR compared with other algorithms.

![Figure 2. Effects of the Number of Frames on PSNR](image-url)
The proposed method was also tested on a real video image sequence as shown in Figure 3. The variance of $G$ is set to $\sigma^2 = 1666.7$, and the coefficients are set to $c_1 = 0.7$, $c_2 = 0.3$. (a) shows the LR image which is of size $128 \times 96$. The image sequence is magnified three times by the proposed algorithm and other methods as shown in (b-e). Another test was conducted as is shown in Figure 4. The LR image in (a) is of size $48 \times 74$. As can be seen from (b-e), the proposed algorithm smooths the image background while preserving sharp edges. Therefore, in practical applications where quality of image is the priority, proposed algorithm could be a better choice than the other methods.

5. Conclusion

In this paper, a novel algorithm for multi-frame super-resolution using adaptive patch matching technique which uses photometrical similarity and geometrical distance to determine the matching patch is proposed to reconstruct the high resolution image. Also, by incorporating the ideas of compressive sensing theory and the greedy reconstruction algorithm, the proposed algorithm has advantages over other methods since it can reserve the edges in the image and yields better results. Experimental results show that the proposed method leads to a better preservation in both flat regions and edges of the HR image.
Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No. 61171155, Natural Science Foundation of Shaan Xi Province under Grant No. 2012JM8010, and Doctorate Foundation of Northwestern Polytechnical University under Grant No. CX201316.

References


Figure 4. An example of HR Reconstruction from Different Super Resolution Methods. Results (3x Resolution Increase) by (a) Original Low Resolution Image Interpolated by Nearest Neighbor Interpolation, (b) Robust Method, (c) Total Variation Reconstruction, (d) Normalized Convolution SR, (e) Proposed Method

Authors

Haitian Zhai received his B.S. degree in Electrical Information and M.S. degree in Circuits and Systems from Northwestern Polytechnical University, China in 2009 and 2011, respectively. He is currently pursuing the Ph.D. degree from the Department of Electronics and Information, Northwestern Polytechnical University, China. His current research interests include image super resolution, compressive sensing, and information fusion.

Hui Li received his B.S. degree in Electrification Professional, M.S. degree in Circuits and Systems, and Ph.D. degree in Systems Engineering from Northwestern Polytechnical University, China in 1991, 1996, and 2000, respectively. He is currently a Professor in Northwestern Polytechnical University, China. His current research interests include image super resolution, information fusion, and signal processing.

Weiting Gao received his B.S. of Electrical Information degree and M.S. degree of Circuits and Systems from Northwestern Polytechnical University, China in 2007 and 2011, respectively. He is currently pursuing the Ph.D. from the Department of Electronics and Information, Northwestern Polytechnical University, China. His current research interests include signal and image processing, information fusion and learning-based image super resolution.