Various Techniques to Reduce PAPR in OFDM Systems: A Survey

Mamta Bisht\textsuperscript{1} and Alok Joshi\textsuperscript{2}

Department of Electronics and Communication Engineering
\textsuperscript{1}B. T. Kumaun Institue of Technology, Dwarahat, Almora
\textsuperscript{2}Jaypee Institute of Information Technology, Noida
\textsuperscript{1}bishtmamta29@gmail.com, \textsuperscript{2}20.alok@gmail.com

Abstract

A non-constant envelop with high peaks is a main disadvantage of Orthogonal Frequency Division Multiplexing (OFDM). These high peaks produce signal excursions into non-linear region of operation of the Power Amplifier (PA) at the transmitter, thereby leading to non-linear distortions and spectral spreading. Many Peak to Average Power Ration (PAPR) reductions methods have been proposed in the literature. The objective of this review is to give a clear understanding of different techniques to reduce PAPR of the signal.

Keywords: OFDM, PAPR, PA

1. Introduction

Various works has been done on OFDM. In OFDM, as all the carriers are added using an IFFT operation, this may lead to a signal with large peaks and dynamic range in time domain [1]. For an OFDM signal $x(t)$, the PAPR is given as:

$$
PAPR \{x(t)\} = \frac{\max_{0 \leq t \leq T} \{ |x(t)|^2 \}}{E\{ |x(t)|^2 \}} \quad (1)
$$

Where $\max_{0 \leq t \leq T} |x(t)|^2$ the peak signal power and $E\{ |x(t)|^2 \}$ is the average signal power.

High value of PAPR is serious concern when OFDM signal pass through nonlinear devices such as in 4G systems where OFDM is a downlink method and on-board high power amplifier have non-linear input output characteristics at high power values as shown in Figure 1.

![Figure 1. Characteristics of HPA [2]](image_url)
Reduction of peak-to-average power ratio is always a concern for researchers. Various methods have been implemented to reduce PAPR like clipping and filtering [4-6], companding [10-14], SLM [15-21], PTS [22-29], tone injection [33-36], tone reservation [37-42], active constellation extension [43-45] and coding [46-69].

2. PAPR Reduction Techniques

A large PAPR would drive PAs at the transmitter into saturation, producing interference among the subcarriers that degrades the BER performance and corrupts the spectrum of the signal. To avoid driving the PA into saturation, the average power of the signal may be reduced. However, this solution reduces the signal-to-noise and consequently, the BER performance. Therefore, it is preferable to solve the problem of high PAPR by reducing the peak power of the signal. PAPR reduction techniques can be broadly classified into three main categories [3]:

1. Signal distortion techniques,
2. Multiple signalling and probabilistic techniques and
3. Coding techniques.

2.1. Signal Distortion Techniques

Signal distortion techniques reduce the PAPR by distorting the transmitted OFDM signal before it passes through the PA. The well-known distortion techniques are as:

2.1.1. Clipping and Filtering: This method employs a clipper that limits the signal envelope to a predetermined clipping level (CL) if the signal exceeds that level; otherwise, the clipper passes the signal without change [4], as defined by:

$$ T(x[n]) = \begin{cases} x[n] & \text{if } |x[n]| \leq CL \\ CL & \text{if } |x[n]| > CL \end{cases} $$

(2)

Where $x[n]$ is the OFDM signal, CL is the clipping level and, $\angle x[n]$ is the angle of $x[n]$. Clipping is a non-linear process that leads to both in-band and out-of-band distortions [5]. The out-of-band distortion causes spectral spreading and can be eliminated by filtering the signal after clipping but the in-band distortion can degrade the BER performance and cannot be reduced by filtering [6]. However, oversampling by taking longer IFFT can reduce the in-band distortion effect as portion of the noise is reshaped outside of the signal band that can be removed later by filtering.

2.1.2. Peak Windowing: In this scheme a predetermined threshold level is defined and if the high peak goes beyond this predetermined threshold, it is multiplied by a weighting function known as window function. The most commonly used window functions include Cosine, Hamming, Hanning, Kaiser and Gaussian Windows. In [7] author described a scheme to perform windowing on a clipped and filtered signal repeatedly for PAPR reduction and achieved 7dB PAPR reduction at CCDF value of $10^{-3}$, within 1 dB increase of $E_b/N_0$ at $10^{-4}$ BER. An advanced peak windowing method is discussed in [8], where new weighting coefficients are introduced whenever successive peaks are generated within a half of the window length. The successive peaks can be restrained to a given threshold level after applying the new weighting coefficients. In [9] author introduced sequential asymmetric superposition (SAS) which is a two new peak windowing methods and optimally weighted windowing (OWW) to deal with closely spaced peaks to avoid high PAPR values.

Peak windowing scheme does not employ hard clipping and therefore, gives better result as compared to clipping technique but still distortion can’t be avoided completely.
2.1.3. Companding Transforms: This method basically applied for audio signals. Companding consist compression and expansion. After companding, the lower peak values are increased but higher peaks remain constant and hence, average power of OFDM signal is increased. Hence the peak to average power ratio decreases. Companding transform can be generally classified into four classes: linear symmetrical transform (LST), linear asymmetrical transform (LAST), nonlinear symmetrical transform (NLST), nonlinear asymmetrical transform (NLAST). Many companding transforms which belongs to the above four mentioned classes, are discussed in the literature.

In [10-12], µ-law companding transform is used to reduce PAPR. The authors in [10] examined the effect of companding on the BER performance of the OFDM system in the presence of AWGN and concluded that a reasonable symbol error rate is achieved by properly choosing companding coefficients. A NLAST to reduce PAPR is proposed in reference [13] using the error function transformation given by

\[ x_c[n] = k_1 \text{erf}(k_2 x[n]) \]

where \( k_1 \) and \( k_2 \) are properly chosen coefficients based on statistics of the transmitted OFDM signal. If \( k_1 \) and \( k_2 \) are chosen properly, then it projects the high peaks of the signal envelope into the nonlinear region of the companding function, while the lower magnitudes are projected onto the linear region. This enhanced the low values, while high peaks are relatively attenuated. The error function transforms the Gaussian distributed OFDM signal into a quasi-uniform distributed one. This action increases the mean power and decreases the peak power, and, consequently, reduces PAPR. In [14], a similar NLAST which uses the error function is proposed to transform the Rayleigh distributed envelope or the exponentially distributed power of the original OFDM signal into uniform distribution.

But this method is not in much use because the average power of OFDM signal increased to reduce PAPR, which increases more burdens on transmitter to transmit more power than before.

2.2. Multiple Signalling and Probabilistic Techniques

This method either generate multiple permutation of the OFDM signal and transmit the one with minimum PAPR or to modify the OFDM signal by introducing phase shifts, adding peak reduction carrier or changing constellation points. Major techniques under this category are follows:

2.2.1. Selective Mapping (SLM)

The basic idea in SLM technique is to generate a set of sufficiently different candidate data blocks by the transmitter where all the data blocks represents the same information as the original data block and select the favorable having the least PAPR for transmission. The block diagram of the SLM technique is shown in Figure 2.
Let the input data block \[X = [X_0, X_1, \ldots, X_{N-1}]\], this data block is multiplied with \(M\) different phase factors \[b_m = [b_m^0, b_m^1, \ldots, b_m^{N-1}],\] where \(m = 0,1,2,\ldots, M-1;\) \[b_m^n = e^{j \theta_m^n}\] and \(\theta_m^n \in [0,2\pi]\) for \(n=0,1,2,\ldots,N-1\). After taking IFFT, this multiplication generates \(M\) sequences in time domain given by:

\[x_m[n] = \sum_{n=0}^{N-1} X_n b_m^n e^{j 2\pi n \frac{n}{N}}\]  

for \(m = 0,1,\ldots,M\)

Among all of the generated \(x_m\), we selected the lowest PAPR for transmission and the condition is given by:

\[\tilde{x} = \text{arg} \max_{m=0,1,\ldots,M-1} \left\{ \max_{n=0,1,\ldots,N-1} |x_m[n]| \right\}\]  

(4)

Side information [SI] about the phase factor is needed to be transmitted separately to decode the OFDM symbol at the receiver side. For \(M\) phase sequences \([\log_2 M]\), side information bits are required. In SLM, an \(N\)-point IFFT involves \(\frac{N}{2} \log_2 N\) complex multiplications and \(N \log_2 N\) additions; these are increased by a factor of \(L\) if oversampling is performed. Computational complexity, PAPR reduction capability and avoiding SI are the major issues associated with SLM. Various schemes are available in literature to modify SLM. In [15], author proposes a scheme for removing SI in SLM where phase sequence could be easily decoded at the receiver after a level is inserted with each candidate as an identifier tag and scrambling is used to avoid any direct manipulation of SI at the receiver. Here only a small amount of redundancy increases the cost of hardware implementation. In this scheme removal of SI is not proposed, only representation is changed. A magnitude scaled-SLM method is proposed in [16], where a set of envelope function derived from Walsh sequences are used to scale the power profile of OFDM signal and at the receiver envelope function along with a detection matrix could easily identify the used phase at the transmitter. Author proposed use of m-sequences in [17] for detection of SI at the receiver but at transmitter SI is embedded in the sequences with the help of block partitioning and rotation, at the receiver cyclic shifting of m-sequences which are derived from a Walsh Hadamard matrix is used to choose phase factor.

A lot of computations are required to choose best candidate for large block sizes. To solve this complex problem, a lot of work has been done in this field. A method to reduce computational complexity is proposed in [18] where an intermediate k-stage IFFT block is used to partially IFFT the block and then phase sequences are multiplied to it, remaining n-k stage IFFT is done after it. The computational complexity reduction ratio (CCRR) is tabulated for various values of n-k, M and N. For lower value of n-k up to 72% CCRR is achieved. In [19], a scheme is proposed to generate a candidate signal by combining OFDM signals and its cyclically delayed version of varying delay and phase, same PAPR performance is achieved at reduced complexity of 50% to 76%. In [20], additive sequences are used for generation of new candidates from the existing one, scheme achieves a CCRR up to 88% for the multiplication and78% for addition at M=40. In [21], intermediate IFFT stages are used, however in place of multiplying phase rotation, the proposed scheme generates OFDM candidates by cyclically shifting the connections at the intermediate IFFT stages, achieved CCRR for multiplications and addition is 70% for 1024 subcarriers and M=8.
2.2.2. Partial Transmit Sequence (PTS): In PTS, an input data block of length N is partitioned into a number of disjoint sub-blocks. Then each of these sub-blocks are padded with zeros and weighted by a phase factor. The schematic is shown in Figure 3:

![Figure 4. Block Diagram of PTS Technique](image)

The data block \(X = [X_0, X_1, X_2, \ldots, X_{N-1}]^T\) is divided in \(V\) disjoint sets, \(\{X_v, v=1,2,\ldots,V\}\), using same number of carriers for each group, the alternative frequency domain signal sequence is given by:

\[
x' = \sum_{v=1}^{V} X_v b_v
\]

where \(b_v = e^{j\phi_v}\) are the phase factors and \(\phi_v = \frac{2\pi i}{W}, i = 0,1,\ldots, W - 1\). In time domain \(x_v\), IFFT of \(X_v\) is called partial transmit sequence. The phase factor is chosen such that PAPR of candidate signal is minimum.

For \(V\) sub-blocks and \(W\) phase weights, we have to search \(W^{V-1}\) possible candidates, as for the first block phase factor is always chosen as 1. In calculation of each candidate \(V-1\) additions and multiplication takes place.

Various techniques are suggested in literature to reduce computational complexity of the PTS scheme. In [22], a scheme is proposed that updates the set of phase factors iteratively till PAPR drops below a specified threshold. A simple iterative flipping algorithm is proposed in [23] to reduce the complexity of the PTS method by converging to a sub-optimal choice of the phase factors. Algorithms are described in [24] for combining partial transmit sequences with reduced complexity and very little performance degradation. A gradient descent search for phase factors is proposed in [25] which reduce search complexity at the expense of some performance degradation too. Reference [26] proposed a PTS scheme based on listing the phase factors into multiple subsets table and to reduce computational complexity, utilize the correlation among phase factors in each subset.

In [27], a low computational complexity PTS scheme is proposed where two search steps are employed to find a subset of phase rotating vectors with good PAPR reduction performance. In first step, Kasami sequences [28] with low correlation or quaternary sequences of family A [29] are used as initial phase rotating vectors for PTS scheme. In the second step, to find additional phase rotating vectors, a local search is performed based on the initial phase vectors with good PAPR reduction performance.
2.2.3. **Interleaved OFDM:** This technique is very similar to SLM, the only difference is that interleaver is used instead of phase sequences [30-32]. Interleaver is a device that operates on a block of N symbols and reorder or permuted them in a specific manner. The block diagram of this scheme is shown in Figure 4:

![Interleaved OFDM Techniques](image)

Interleavers and de-interleavers are usually denoted by the symbol $\pi$ and $\pi^{-1}$, e.g., if $X = [X_0, X_1, \ldots, X_{N-1}]^T$ after interleaving it becomes

$$X' = [X_{\pi(0)}, X_{\pi(1)}, \ldots, X_{\pi(N-1)}]^T$$

where \(\{n\} \leftrightarrow \{\pi(n)\}\) is a one to one mapping $\pi(n) \in \{0, 1, \ldots, N-1\}$ and for all n. To make K modified data blocks, interleavers are used to produce permuted data blocks from the same data block. The PAPR of (K-1) permuted data blocks and that of the original data block are computed using K IDFT/IFFT operations and then the data block with the lowest PAPR is chosen for transmission. The receiver need only know which interleaver is used at the transmitter to recover the original data block. Thus the number of required side information bits is $\lceil \log_2 K \rceil$. The permutation indices $\{\pi(n)\}$ is stored by both the transmitter and receiver in memory. Thus, interleaving and de-interleaving can be done simply and the amount of PAPR reduction depends on the number of interleavers (K-1) and the design of the interleavers.

SLM, PTS and interleaved techniques are however distortion-less but they require transmission of side information causing reduced bandwidth efficiency.

2.2.4. **Tone Injection (TI):** This technique increases the constellation size so that each of the point in the original basic constellation can be mapped into several equivalent points in the expanded constellation [33]. Since substituting a point in the basic constellation for a new point in the larger constellation is equivalent to injecting a tone of the appropriate frequency and phase in the multicarrier signal, therefore, this technique is called tone injection. The extra degrees of freedom, which is generated as each symbol in the data block can be mapped into one of the several equivalent constellation points, can be utilized for PAPR reduction. The extended constellation of a constellation point in QPSK/4-QAM is shown in Figure 5:
In Figure 5 distance between the constellation point is $d$, in such case real and imaginary parts of the symbol $X_n$ can be take value $(\sqrt{M - 1}) \frac{d}{2}$, where $M$ is the number levels in M-QAM. Using TI, these points could be extended to new points. The distance between the original and extended points is $D$ which is chosen such that BER at the receiver remain unaltered. Usually the value of $D$ is $\rho d \sqrt{M}$, where $\rho \geq 1$. $D$ is an important parameter, higher value of $D$ increases the average power but BER will be low and lower value of $D$ causes poor BER as constellation points come close to each other. Reference [34] proposes a tone injection technique with hexagonal constellation in place of QAM, as hexagonal geometry allows more number of signal points to be spaced uniformly in same area as compared to QAM constellation and the average magnitude is less than average value of QAM constellation, thus requires less transmission power than corresponding QAM constellation. Cross entropy method is used in [35] to solve the problem of large number of searches to find optimum constellation. A low complexity TI method is proposed in [36] that use the clipping noise to find the optimal equivalent constellation and is based on minimum mean error between the clipping noise and possible constellation points.

2.2.5. **Tone Reservation (TR):** In this technique a subset of tones having low SNR is reserved for PAPR reduction. These tones carry no information data and added to the existing OFDM symbols so that the summation has lower PAPR values. Finding and optimized the set of peak reduction tones or peak reduction carriers (PRT’s/PRC’s) increases the complexity of transmitter and also increases required transmission power. Various works are available in literature mainly focusing on complexity reduction of optimization problem. In [37], a gradient algorithm is proposed where the gradient of clipping noise mean square error is calculated and optimization of signal to clipping noise ratio is done in place of PAPR and order of complexity is $O(N)$. A truncated IDFT algorithm is proposed in [38] where in place of calculating entire IDFT values, it calculates on maximal IDFT element thus reducing complexity of optimization process. The basic idea is to divide the group in two halves of $N/2$ and leave the half with lesser energy and move in similar way till we reach the maximum energy element, however this scheme may not always give correct maximal IDFT element, it also cost in lower PAPR reduction. For designing the peak cancelling signal clipping noise is analysed in [39] and several iterations of clipping and filtering is used till desired peak cancellation signal is generated. Author also proposed an adaptive-scaling algorithm and constant-scaling algorithm for tone reservation. In [40] LSA-TR method with fast convergence is proposed which is based on least square approximation and used to find the peak cancelling signal faster than clipping control TR method. Reference [41] used genetic algorithm to find the optimal PRC set, it also discuss an adaptive amplitude clipping tone reservation algorithm (AAC-TR) to solve the optimization problem in clipping control TR method. A curve fitting based tone reservation method is proposed in [42] using clipped noise introduced.
by clipping to generate a peak-cancellation signal. Since, proposed scheme need just one IFFT operation, therefore, reducing the computational complexity by huge margin.

2.2.6. Active Constellation Extension (ACE): This technique is similar to Tone Injection (TI). The only difference is that in ACE, only the outer constellation points are dynamically extended away from the original constellation. Extending outer point from decision boundary increases the spacing between the constellation point and thus reducing BER and if adjusted properly PAPR could also be reduced. Various literatures are available on ACE and suggested modification. In [43] author focuses on poor BER performance due to clipping used in ACE and gives a generalization of the ACE constraints to limit BER degradation. Reference [44] uses pre-distortion technique in place of clipping based ACE (CB-ACE), the metric pre-distort those frequency domain symbols which have large contribution to output thus the PAPR. An adaptive clipping control algorithm is proposed in [45] to achieve better PAPR as compared to clipping based ACE at reduced number of iterations. In this way ACE offers dual advantage of BER and PAPR reduction. ACE technique does not require transmission of side information and hence there is no data rate loss too. Only the drawback of this scheme is that it increases the requirement of transmission power.

2.3. Coding Techniques

The basic idea behind coding technique is to select those codeword that reduce the PAPR for transmission. A forward error correction (FEC) code is defined by (n,k), where n are the data bits and k represents redundant bits, so the idea is to add redundant bit in a manner that overall PAPR value is minimized. FEC are classified as block codes and run length codes. In block codes, a block of data bits are used together to encode them whereas run length code employs memory and lower values of n. Linear block codes, Golay complementary codes, Reed Mullar, Bose Chaudhari Hochquenghem (BCH), low density parity check (LDPC) are few block codes which have been used for PAPR reduction. Turbo codes which are derived from convolution codes are also discussed in literature for PAPR reduction.

2.3.1. Linear Block Coding: In [46], a simple linear block coding (LBC) was proposed where 3 bits are mapped into 4 bits by adding a parity bit. A simple rate- ¾ cyclic code is used in [47] for any number of subcarriers that is a multiple of 4 to reduce PAPR by more than 3 dB. A combined (8,4) LBC is used in [48] to provide error control capability and reduce PAPR of a multicarrier modulation by 4 dB. Reference [49] proposed an another simple LBC based on the observation that regardless of the number of subcarriers, codewords with equal odd and even bit values have high PAPR. Therefore, eliminating these codewords by adding a simple bit code, PAPR can be reduced easily. In [50], a low complexity complement block coding (CBC) scheme is proposed where few complement bits are inserted in the middle of the information bits to form a codeword with reduced PAPR. A standard arrays of linear block codes are used in [51] for PAPR reduction which may be regarded as a modified version of SLM. In this scheme the coset leaders of a linear code are used for scrambling, hence no side information is required to be transmitted and the received signal can be decoded by syndrome decoding. To control PAPR of OFDM signals the authors proposed the use of fountain codes in [52]. LT codes [53] was the first practical realization of fountain codes and later a further enhancement was proposed by the Raptor codes [54]. The best fountain coded OFDM packets can be generated with a low PAPR which is the motivation behind this scheme.

2.3.2. Golay Complementary Sequences: Golay Complementary Sequences [55] can be used as codewords to modulate the subcarriers of the OFDM systems, resulting a signal of PAPR with an upper bound of 2. In [56], relation between Golay complementary
sequences and second order Reed-Mullar code is exploited to achieve low PAPR of almost 3dB. In [57-62], Golay codes were further investigated for PAPR reduction for various constellation sizes such as 16-QAM and 64-QAM.

2.3.3. Turbo Coding: Turbo codes being a capacity approach codes are very popular and these codes are also being used for PAPR reduction. In [63], three turbo coded OFDM system or PAPR reduction were proposed where first using m-sequences for PAPR reduction and short codes for side information. Second uses interleaving and third is combination of first two schemes. A tail-biting turbo coded OFDM system is proposed in [64] to generate candidates in a selective mapping scheme, without need of side information protection.

2.3.4. Bose Chaudhari Hochquenghem (BCH): A coding schemes derived from dual BCH codes also been employed for PAPR reduction technique [65], as BCH codes don’t require practical realizable decoders and work much below the Shannon limit. In this scheme, turbo structure can be used to fill the gap in Shannon limit and PAPR improvement of 7 dB is achieved.

2.3.5. Low Density Parity Check (LDPC): LDPC codes were first introduced by Gallanger [66-67]. These codes dominated the forward error correction codes in terms of error correction capabilities with reasonable complexity of encoders they gave performance near to Shannon’s limit. In [68-69], LDPC codes are investigated by researchers for PAPR reduction.

3. Conclusions
Multicarrier transmission such as OFDM is one of the most attractive techniques for both wired and wireless applications due to its high data rates, robustness to multipath fading and spectral efficiency. However, it has a major drawback of generating high peak-to-average ratio. Lots of PAPR reduction techniques are proposed in literature and discussed in this review paper.

In clipping technique, clipping is done around the peaks but at the cost of increased distortion. In probabilistic scrambling techniques, the input data block is scrambled and sequence with lowest PAPR is transmitted. These techniques are selective mapping (SLM), partial transmit sequence (PTS), interleaved OFDM, Tone Reservation (TR) and Tone Injection (TI), which do not suffer from the out-of-band power but the spectral efficiency decreases and the complexity increases as the number of sub-carriers increases. Coding techniques are also used for PAPR reduction but in this approach PAPR reduction is achieved at higher complexity and lower bandwidth efficiency.

All of proposed schemes have the potential to reduce PAPR substantially but at the cost of loss in data rate, transmit signal power increase, BER increase, computational complexity increase and so on. Thus, the PAPR reduction technique should be carefully chosen according to various system requirements.

Acknowledgments
Authors are highly thankful to the reviewers for their valuable suggestions to improve the paper.

References


Authors

Mamta Bisht received her B.Tech degree in Electronics & Communication Engg. in 2011 from H.N.B. Garhwal University, Uttarakhand, India and M.Tech. degree in Digital Communication in 2015 from B.T.K.I.T, Dwarahat (Uttarakhand Technical University), India. Her research interest includes OFDM systems and various modulation schemes.

Dr. Alok Joshi received B.E. degree in Electronics & Communication Engg. in 2001 from G.B. Pant Engg. College (H.N.B. Garhwal University), Uttarakhand, India and M.tech.in Digital Communication in 2006 from U.P.T.U. Lucknow, India. He has teaching experience of more than 12 years in various technical universities in India. Currently working with Jaypee Institute of Information Technology, Noida, India. His research interests are coded OFDM systems and received Ph.D. degree in same domain in 2015.