Image Edge Detection Based on Anti-Symmetrical Biorthogonal Wavelet Filter Banks with the Same Even Length

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Abstract

With the development of the theory of wavelet transform, biorthogonal wavelet filter banks with linear phase and compact support characteristics are widely used in signal and image processing. We study the multiscale edge detection using anti-symmetrical biorthogonal wavelet filter banks with the same even length based on the analysis of the properties of this wavelet filter banks. The steps of multiscale edge detection based on anti-symmetrical biorthogonal wavelet are introduced in detail. Experimental results on test images show that compared with the anti-symmetrical biorthogonal wavelet, the proposed anti-symmetrical biorthogonal wavelet filter banks with the same even length have better performance in terms of image edge detection. Therefore, the anti-symmetrical biorthogonal wavelet filter banks with the same even length are more suitable for image edge detection.

Keywords: Image Processing, Multiscale Edge Detection, Filter Banks, Anti-symmetrical Biorthogonal Wavelet

1. Introduction

Edge defines the boundaries between regions in an image, so edge detection is an important content of image processing. Due to excellent local time-frequency and multiresolution analysis characteristic, wavelet transform especially biorthogonal transform is widely used in signal and image processing, such as signal denoising [1], image compression [2], multiscale edge detection [3], singularity detection [4], image segmentation [5], etc., Yang, et al., proposed an improved method of image edge detection based on wavelet transform [6]. Zhang, et al., proposed a novel edge detection method based on the fusion of wavelet transform and mathematical morphology [7].

On the basis of analyzing Mallat algorithm [8], Hou, et al., [9] presented two kinds of boundary extension methods, point-symmetric extension (PSE) and edge-symmetric extension (ESE) commonly used in finite length signal processing. They preserve the perfect reconstruction while keeping the signal length unchanged.

In the past decades, the construction of biorthogonal wavelet had been studied and many kinds of wavelets were constructed [10-12]. Liu, et al., [13] constructed a new kind of biorthogonal wavelet filter banks, named anti-symmetrical biorthogonal wavelet filter banks with the same even length. Design formulas and steps were described in detail. Particularly,
the properties of this wavelet filter banks were studied and the differential operator property was analyzed. In this paper, we explore its application in image edge detection, and compare it with other anti-symmetrical biorthogonal wavelet [14].

The rest of this paper is organized as follows. In Section 2, the anti-symmetrical biorthogonal wavelet filter banks with the same even length and the differential operator property are presented. Section 3 introduces the Mallat algorithm for finite length signal. In Section 4, the process of multiscale edge detection based on anti-symmetrical biorthogonal wavelet is introduced in detail. Experimental results of multiscale edge detection using anti-symmetrical biorthogonal wavelet and anti-symmetrical biorthogonal wavelet filter banks with the same even length are presented in Section 5. Finally, conclusions are given in Section 6.

2. Anti-symmetrical Biorthogonal Wavelet Filter Banks with the Same Even Length

2.1. Differential Operator Property

Supposed the support interval of sequence \( \{ q_k \} \) is \( [ -K + 1, K ] \), \( K > 0 \), i.e.

\[
\{ q_k \} = \{ q_{-K+1}, \ldots, q_0, q_1, \ldots, q_K \} \tag{1}
\]

and

\[
q_k = -q_{-k} , \quad k = 1, 2, \ldots, K . \tag{2}
\]

We define a new sequence \( \{ c_k \} \)

\[
c_k = \sqrt{2} \sum_{j=-K+1}^{K} q_j , \quad k \in [-K, K - 1] . \tag{3}
\]

Because of \( \sum_{k=-K+1}^{K} q_k = 0 \), we get

\[
c_k = \sqrt{2} \sum_{j=-K+1}^{K} q_j = 0 . \tag{4}
\]

Therefore, the support interval of sequence \( \{ c_k \} \) is \( [- (K - 1), K - 1] \) with length of \( 2K - 1 \).

And as a result of

\[
\sum_{j=-K+1}^{K} q_j = 0 , \quad \forall k \in [- (K - 1), K ] , \tag{5}
\]

we have

\[
c_{-k} = \sqrt{2} \sum_{j=-K+1}^{K} q_j = \sqrt{2} \sum_{j=-K+1}^{K} q_j + \sqrt{2} \sum_{j=-K+1}^{K} q_{j+1} = \sqrt{2} \sum_{j=-K+1}^{K} q_j = c_k . \tag{6}
\]

So the sequence \( \{ c_k \} \) is even symmetrical about \( k = 0 \) [13].

According to Eq. (3), the definition of sequence \( \{ c_k \} \), we know \( \sqrt{2} q_k = c_{k-1} - c_k \). Using the two-scale relation

\[
\psi(t) = \sqrt{2} \sum_{k=0}^{\infty} \tilde{q}_k \phi(2t - k) , \quad \psi(t) = \sqrt{2} \sum_{k=0}^{\infty} q_k \phi(2t - k) . \tag{7}
\]

we have
\[ \psi(t) = \sqrt{2} \sum_{k=-(K-1)}^{K} q_k \varphi(2t - k) = \sum_{k=-(K-1)}^{K} (c_{k+1} - c_k) \varphi(2t - k) \]

\[ = \sum_{k=-(K-2)}^{K-1} c_{k+1} \varphi(2t - k) - \sum_{k=-(K-3)}^{K-2} c_k \varphi(2t - k) \]

\[ = \sum_{k=-(K-1)}^{K-1} c_k \varphi(2t - k - 1) - \sum_{k=-(K-2)}^{K-1} c_k \varphi(2t - k). \]  

(8)

Because that the value of \( |q_k| \) decreases with the increase of \(|k|\) and that negative and positive values of sequence \( \{c_k\} \) just cancel each other out in the sum of Eq. (3), the absolute value of central element \( c_0 \) in sequence \( \{c_k\} \) is obviously greater than absolute value of others. Therefore, we can keep only the central part of Eq. (8), thus \( \psi(t) \) can be approximately expressed as

\[ \psi(t) \approx c_0 \varphi(2t - 1) - \varphi(2t) \]  

(9)

If \( \varphi(t) \) is smooth enough, as for Eq. (9), in virtue of Lagrange mean value theorem, it becomes

\[ \psi(t) \approx c_0 \frac{d}{dt} \varphi(2t - 0.5) \]  

(10)

This means that \( \psi(t) \) possesses the property of differential operator. According to duality, we know that \( \tilde{\psi}(t) \) has the property of differential operator too. The functions \( \psi(t) \) and \( \psi(t) \) are suitable for image edge detection.

2.2. Other Properties

(a) \( \{\tilde{p}_k\}, \{p_k\}, \{\tilde{q}_k\} \) and \( \{q_k\} \) have the same support interval \( \text{supp} = [-K + 1, K], \) \( K > 0; \)

(b) \( \{\tilde{p}_k\} \) and \( \{p_{-k}\} \) are symmetric about 0.5, i.e. \( \tilde{p}_k = p_{-k}, \) \( p_{-k} = p_k, \) \( k = 1, 2, \cdots, K; \)

(c) \( \{q_k\} \) and \( \{q_{-k}\} \) are anti-symmetric about 0.5, i.e. \( q_k = -q_{-k}, \) \( q_{-k} = -q_k, \) \( k = 1, 2, \cdots, K; \)

(d) \( \tilde{\psi}(t), \psi(t), \tilde{\psi}(t) \) and \( \psi(t) \) have the same support interval \( \text{supp} = [-K + 1, K], \) \( K > 0, \) \( \tilde{\psi}(t) \) and \( \psi(t) \) are symmetric about 0.5, while \( \tilde{\psi}(t) \) and \( \psi(t) \) are anti-symmetric about 0.5.

3. Mallat Algorithm for Finite Length Signal

In the derived process of Mallat algorithm, the length of input sequence \( \{c_{i,k}\} \) is assumed to be infinite. The decomposition algorithm is

\[
\begin{align*}
\{c_{j-1,k}\} &= \sum_{k \in \mathbb{Z}} \tilde{p}_{i-2k} e_{j-1,k} = \sum_{k \in \mathbb{Z}} \tilde{p}_{i-2k} e_{j-1,k}, \\
\{d_{j-1,k}\} &= \sum_{k \in \mathbb{Z}} \tilde{q}_{i-2k} e_{j-1,k} = \sum_{k \in \mathbb{Z}} \tilde{q}_{i-2k} e_{j-1,k}.
\end{align*}
\]  

(11)
The reconstruction algorithm is
\[
    c_{j,k} = \sum_{i \in \mathbb{Z}} \left[ p_{k-2i} c_{j-1,i} + q_{k-2i} d_{j-1,i} \right]
\]
\[
    = \sum_{i \in \mathbb{Z}} \left[ p_{k-2i} c_{j-1,i} + q_{k-2i} d_{j-1,i} \right] + \sum_{i \in \mathbb{Z}} \left[ p_{k-2i} \cdot 0 + q_{k-2i} \cdot 0 \right]
\]
\[
    = \sum_{i \in \mathbb{Z}} \left[ p_{k-2i} c_{j-1,i} + q_{k-2i} d_{j-1,i} \right],
\]
where \( \{c_{j-1,i}'\}, \{d_{j-1,i}'\} \) are obtained from \( \{c_{j-1,i}\}, \{d_{j-1,i}\} \) by adding zero in odd index. In Eqs. (11) and (12), \( \{c_{j-1,i}\} \) and \( \{d_{j-1,i}\} \) are infinite sequences, where \( k \in \mathbb{Z} \). However, we can only acquire the finite sample of continuous function \( f(t), t \in \mathbb{R} \) or discrete sequence \( \{f_k\}, k \in \mathbb{Z} \) in applications [9].

The flow chart of Mallat algorithm for finite length sequence is shown in Figure 1. The decomposition algorithm in Eq. (11) can be rewritten as
\[
    c'_{N-1,k} = \sum_{i=-N}^{N} p_i c_{N,k+i}, \quad d'_{N-1,k} = \sum_{i=-N}^{N} q_i d_{N,k+i},
\]
\[
    c_{N-1,k} = c'_{N-1,k}, \quad d_{N-1,k} = d'_{N-1,k},
\]

The decomposition and reconstruction flow charts of \( \{c_{N,k}\} \) obtained from image Lena are shown in Figure 2.

4. Multiscale Edge Detection Algorithm

The wavelet transform of image at each scale provides much edge information. Multiscale edge detection realizes the combination of edge information at each scale in order to get ideal edge. The algorithm of multiscale edge detection is introduced in detail as follows.

(1) Anti-symmetrical biorthogonal wavelet is applied to wavelet decomposition process, four areas are derived: horizontal and vertical low frequency information (LL) called
approximate component; horizontal high frequency and vertical low frequency information (HL); horizontal low frequency and vertical high frequency information (LH); horizontal and vertical high frequency information (HH) [15]. Then the same decomposition process is performed on horizontal and vertical low frequency information.

![Diagram](image)

**Figure 2. An Example of ESE with Image Data (L = 8): (a) The Decomposition Process, (b) The Reconstruction Process**
Based on the pyramidal decomposition data of image, we compute modulus image $M_{2^j}(x,y)$ and angle image $A_{2^j}(x,y)$ at each scale $2^j$, $j = -1, -2, \ldots, -J$. The modulus of the gradient vector is obtained by

$$M_{2^j}(x,y) = \sqrt{W_{2^j}^1 f(x,y)^2 + W_{2^j}^2 f(x,y)^2}. \quad (15)$$

The angle of the gradient vector is computed by

$$A_{2^j}(x,y) = \arctan \left( \frac{W_{2^j}^1 f(x,y)}{W_{2^j}^2 f(x,y)} \right), \quad (16)$$

where $W_{2^j}^1 f(x,y)$ is the horizontal high frequency and vertical low frequency information at each scale $2^j$, and $W_{2^j}^2 f(x,y)$ means the horizontal low frequency and vertical high frequency information at each scale $2^j$.

For each scale $2^j$, we detect the local maxima of $M_{2^j}(x,y)$ along the direction given by angle image $A_{2^j}(x,y)$ [16]. There are four possible directions for gradient vector, respectively, $0^\circ$, $90^\circ$, $45^\circ$ and $135^\circ$. For any point $(m,n)$ of $M_{2^j}(x,y)$, if the modulus of it is the local maxima of the ones of three points on the direction of gradient vector, we record $(m,n)$ as candidate edge point. All candidate edge points constitute edge image $B_{2^j}(x,y)$.

(a) If the direction of gradient vector is $0^\circ$, we compare the following three points: $(m-1,n)$, $(m,n)$ and $(m+1,n)$.

(b) If the direction of gradient vector is $90^\circ$, we compare the following three points: $(m,n-1)$, $(m,n)$ and $(m,n+1)$.

(c) If the direction of gradient vector is $45^\circ$, we compare the following three points: $(m-1,n-1)$, $(m,n)$ and $(m+1,n+1)$.

(d) If the direction of gradient vector is $135^\circ$, we compare the following three points: $(m+1,n-1)$, $(m,n)$ and $(m-1,n+1)$.

Due to the presence of noise and fine texture, there are many pseudo-edge points in the set of edge points. Based on the property of smaller modulus of pseudo-edge point, we can adopt the threshold method to eliminate those points whose moduluses are smaller than the specified threshold from edge image.

We synthesize the obtained edge information in order to get the precise and single pixel wide edge according to the set of edge points at each scale $2^j$. The process of multiscale combination is introduced in detail as follows.

Step 1. At the scale $2^j$, $j = -J$, we link those points with similar moduli and angles from the thresholded edge image $B_{2^j}(x,y)$. Then the length and average modulus of each chain are computed. For each chain, if the length of it is shorter than the specified chain length threshold $L_{2^j}$ or the average modulus is smaller than the specified link modulus threshold $T_{2^j}$, we remove it. Therefore the single pixel wide edge image $E_{2^j}(x,y)$, $j = -J$ is obtained.

Step 2. For each edge point in $E_{2^j}(x,y)$, we search the corresponding area in modulus maxima image $B_{2^j}(x,y)$ with size of $3 \times 3$ to find all possible edge points.
These edge points constitute edge image. Those points with similar moduli and angles in obtained edge image are linked. The length and average modulus of each chain are analyzed. For each chain, if the length of it is shorter than the specified chain length threshold $L_{2,m}$ or the average modulus is smaller than the specified chain modulus threshold $r_{2,m}$, we remove it. In this way, we get the single pixel wide edge image $E_{2,m} f(x, y)$.

Step 3. The Step 2 is repeated until $j = -1$.

5. Experimental Results

Any anti-symmetrical biorthogonal wavelet which meets the conditions of the differential operator can be used to image edge detection. Simulation experiments are conducted using MATLAB 7.8. To test the edge detection performance of the anti-symmetrical biorthogonal wavelet filter banks with the same even length, Lena (8bits/pixel, 512 x 512) is used in simulation experiments as test image. In the following, anti-symmetrical biorthogonal wavelet filter banks with the same even length Bior3.1, Bior5.1 and Bior7.1 are tested, and compared with anti-symmetrical biorthogonal wavelet Bior1.3 which has best performance in edge detection [14]. In multiscale edge detection algorithm, we set wavelet decomposition parameter $J = 3$.

The coefficients of Bior5.1 are shown as follows:

$$\{ \hat{p}_0 \} = \{ \hat{p}_{-2} \sim \hat{p}_1 \} = \{0.0442, 0.2210, 0.4419, 0.4419, 0.2210, 0.0442\};$$

$$\{ \hat{q}_0 \} = \{ \hat{q}_{-2} \sim \hat{q}_1 \} = \{0.2652, -1.3258, -1.7678, 1.7678, -1.3258, -0.2652\};$$

$$\{ p_0 \} = \{ p_{-2} \sim p_1 \} = \{0.2652, -1.3258, 1.7678, 1.7678, -1.3258, 0.2652\};$$

$$\{ q_0 \} = \{ q_{-2} \sim q_1 \} = \{0.0442, -0.2210, 0.4419, -0.4419, 0.2210, -0.0442\}.$$

The coefficients of Bior7.1 are shown as follows:

$$\{ \hat{p}_0 \} = \{ \hat{p}_{-2} \sim \hat{p}_1 \} = \{0.0110, 0.0773, 0.2320, 0.3867\};$$

$$\{ \hat{q}_0 \} = \{ \hat{q}_{-2} \sim \hat{q}_1 \} = \{0.2210, 1.5468, 4.0217, 3.4030\};$$

$$\{ p_0 \} = \{ p_{-2} \sim p_1 \} = \{-0.2210, 1.5468, -4.0217, 3.4030\};$$

$$\{ q_0 \} = \{ q_{-2} \sim q_1 \} = \{-0.0110, 0.0773, 0.2320, 0.3867\};$$

The coefficients of Bior1.3 are shown as follows:

$$\{ \hat{p}_0 \} = \{ \hat{p}_{-2} \sim \hat{p}_1 \} = \{-0.0884, 0.0884, 0.7071, 0.7071, 0.0884, -0.0884\};$$

$$\{ \hat{q}_0 \} = \{ \hat{q}_{-2} \sim \hat{q}_1 \} = \{-0.7071, 0.7071\};$$

$$\{ p_0 \} = \{ p_{-2} \sim p_1 \} = \{0.7071, 0.7071\};$$

$$\{ q_0 \} = \{ q_{-2} \sim q_1 \} = \{-0.0884, -0.0884, 0.7071, -0.7071, 0.0884, 0.0884\}.$$

The multiscale edge detection results of the Lena image using Bior1.3, Bior3.1, Bior5.1 and Bior7.1 are shown in Figure 3. From the experimental results, we can see that compared with Bior1.3, the multiscale edge detection performances of Bior3.1,
Bior5.1 and Bior7.1 are all better in terms of the continuity of edge and edge positioning accuracy.

Figure 3. Edge Detection Results of Lena with Different Wavelets: (a) Lena, (b) Bior1.3, (c) Bior3.1, (d) Bior5.1, (e) Bior7.1

Figure 4(a) shows the image Lena with Gaussian noise whose mean value is 20 and variance is 36. While Bior1.3, Bior3.1, Bior5.1 and Bior7.1 applying on multiscale edge detection of noised Lena, the corresponding experimental results are shown in Figure 4(b)−(e). Apparently the multiscale edge detection results of noised Lena using Bior3.1, Bior5.1 and Bior7.1 are good, but the one using Bior1.3 is bad. Therefore the edge detection based on anti-symmetrical biorthogonal wavelet filter banks with the same even length has good anti-noise performance especially Bior5.1 and Bior7.1.

6. Conclusion

In this paper, anti-symmetrical biorthogonal wavelet filter banks with the same even length are introduced. Based on the analysis of properties of anti-symmetrical biorthogonal wavelet filter banks with the same even length, the multiscale edge detection algorithm using this wavelet filter banks is achieved. The experiments to typical test images are done in MATLAB. Compared with anti-symmetrical biorthogonal wavelet, the proposed algorithm using anti-symmetrical biorthogonal wavelet filter banks with the same even length has better performance in terms of edge detection. And the multiscale edge detection algorithm based on anti-symmetrical biorthogonal wavelet filter banks with the same even length shows good anti-noise performance.
Figure 4. Edge Detection Results of Noised Lena with Different Wavelets:
(a) Noised Lena, (b) Bior1.3, (c) Bior3.1, (d) Bior5.1, (e) Bior7.1

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