Asymptotic Diversity Analysis of Alamouti Transmit Diversity with Quasi-ML Decoding Algorithm in Time-Selective Fading Channels

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Abstract

This paper addresses the asymptotic diversity performance of Alamouti transmit diversity technique especially in time-selective fading channels. In particular, a linear quasi-maximum-likelihood (QML) decoding method is employed for the Alamouti transmit diversity system to solve the error floor problem induced by the conventional linear ML decoding method. By judiciously utilizing the derived asymptotic closed-form formula of symbol pairwise error rate (SPER), it is theoretically verified that the asymptotic diversity orders achieved by the QML decoding algorithm become 2 and 1 in quasi-static and time-selective fading channels, respectively.

Keywords: Alamouti transmit diversity, quasi-maximum-likelihood (QML) decoding, time-selective fading channel, symbol pairwise error rate (SPER)

1. Introduction

Alamouti transmit diversity technique is well-known as an effective method to exploit spatial diversity and thus to mitigate the detrimental effect of fading channels which also has the attractive property in that full-diversity transmission can be achieved along with a low-complexity linear maximum-likelihood (LML) decoding algorithm for a wireless communication system incorporating two transmit antennas and complex signal constellations [1-6]. Accordingly, the Alamouti scheme has been widely adopted in several wireless communication and networking standards such as, IEEE 802.16-2009, IEEE 802.11n, 3GPP LTE, etc.

The aforementioned advantages of Alamouti scheme however, can be easily lost especially in time-selective fading channels (e.g., mobile radio environments), since the orthogonality inherent in the Alamouti scheme is destroyed by the time-selective fading, so that the full-diversity gain obtained from the conventional low-complexity LML decoding cannot be achieved, even showing an error floor of the error rate performance (e.g., bit-error rate (BER), symbol-error rate (SER), etc.) in the high signal-to-noise ratio (SNR) regime, which ultimately leads to the severe performance degradation. In order to resolve this problem, there have been several efforts to design efficient decoding algorithms [4-6]. In particular, a linear QML decoding method for Alamouti transmit diversity technique was proposed in [4] over time-selective fading channels, which has the same decoding complexity as the conventional LML decoding and shows the same error rate performance as the LML decoding in quasi-static fading channels (i.e., time-nonselective fading channels). In [7], the error rate performance assessment was carried out, where a closed-form formula for the symbol pairwise error rate (SPER) and a corresponding union upper bound on the SER are derived for the Alamouti scheme employing the low-complexity QML decoding algorithm. From the derived expressions given in [7], however, it is difficult to obtain analytical insights into the achievable asymptotic diversity orders as the severity of time-selectivity of fading channels. Therefore, in this paper, the researchers present an accurate asymptotic closed-form
approximating formula for the SPER with the aid of the formula derived in [7]. Furthermore, by judiciously exploiting the formula, the researchers also evaluated the asymptotic diversity orders achieved by the QML decoding over both quasi-static and time-selective fading channels.

Throughout this paper, the researchers used the following notation. The superscripts $(\cdot)^*$ and $(\cdot)^H$ denote complex conjugate and Hermitian operations, respectively. $x \sim C \mathcal{N}(m, \sigma^2)$ stands for a circular symmetric complex Gaussian variable $x$ with mean $m$ and variance $\sigma^2$. $Pr(\cdot)$ and $E\{\cdot\}$ denote the probability and expectation, respectively.

2. System Model and Decoding Algorithms

Throughout this paper, the researchers considered the Alamouti transmit diversity system employing two transmit antennas and one receive antenna (i.e., $N_t = 2, N_s = 1$) for brevity. From [1]-[7], the input-output relationship of Alamouti scheme is expressed as

$$
\begin{pmatrix}
    r(1) \\
    r'(2)
\end{pmatrix}
= 
\begin{pmatrix}
    h_1(1) & h_2(1) \\
    -h_1(2) & h_2(2)
\end{pmatrix}
\begin{pmatrix}
    x_1 \\
    x_2
\end{pmatrix}
+ 
\begin{pmatrix}
    v(1) \\
    v'(2)
\end{pmatrix},
$$

(1)

where $r$, $s$, and $v$ are the received signal vector, the transmitted symbol vector, and the noise vector with $v(\cdot) \sim C \mathcal{N}(0, \sigma^2)$, respectively. $H$ is the effective channel matrix, where $h_i(\cdot)$ represents the channel coefficient from the $i$th transmit antenna to the receive antenna, and the index inside $(\cdot)$ stands for the time index. From [4]-[7], the channel coefficient in time-selective fading channels can be approximated as $h_i(t + 1) = \rho h_i(t) + \sqrt{1 - \rho^2} n(t + 1)$, where $\rho = E\{h_i(t)h_i'(t + 1)\}$ with $0 \leq \rho \leq 1$ denotes the fading correlation parameter to characterize the degree of channel time-variation, and $n(\cdot) \sim C \mathcal{N}(0, 1)$. In addition, according to Jakes’ model in [8], the researchers also have $\rho = J_0(2\pi f_d)$, where $J_0(\cdot)$ is the zero-order Bessel function of the first kind and $f_d$ is the relative Doppler frequency.

When the channel is quasi-static, $H$ is orthogonal, which means that by multiplying $H^H$ in (1), the Gramian matrix $G = H^H H$ becomes diagonal. Then, by using the conventional LML decoding algorithm, the two transmitted symbols can be easily decoupled and then decoded. However, when the channel is time-selective, the effective channel matrix is no longer orthogonal and the off-diagonal terms of $G$ are not zeros. Thus, the LML decoding suffers from an error floor in the high SNR region, which leads to the need for more effective decoding methods.

To overcome the aforementioned problem, still maintaining the low-complexity of decoding procedure, the linear QML decoding algorithm was proposed in [4] with the aid of the following orthogonal combining matrix transformation as

$$
Z = \begin{bmatrix}
-z_{12} & z_{11} \\
-z_{21} & -z_{11}
\end{bmatrix}
\text{ for } Z = \begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix},
$$

(2)

Then, the researchers can easily find that the orthogonality at $H^H H$ is achieved through combining $r$ with $H^H$ in (1). Thus, after some straightforward manipulations, the researchers obtained the following linear QML decision metric as

$$
\hat{s}_i = \arg \min_{s \in A} |r(i) - s| 
\text{ for } i = 1, 2
$$

(3)

where $A$ is the constellation of the transmitted symbols, $r(\cdot)$ is an element of $H^H r$, and $\xi = h_1(1)h_1'(2) + h_2(1)h_2'(2)$. The researchers note that the orthogonal combining matrix
given in (2) enables low-complexity linear coding, as shown in (3), however, the elements of $\mathbf{H}^\dagger \mathbf{v}$ are not white-noise (i.e., correlated), so that the corresponding noise enhancement can lead to some performance degradation.

3. Performance Evaluation

3.1. SPER Derivation for QML Decoding Algorithm

Assuming that the channel state information is known at the receiver, the SPER of Alamouti transmit diversity with QML decoding, conditioned on the fading coefficients, is expressed as [7]

\[
P_r(s \rightarrow \tilde{x} | \mathbf{H}) = \mathbb{P}
\left( \frac{1}{\sqrt{2 | h_1(2) |^2 + | h_j(1) |^2}} \xi - \frac{\xi}{| h_1(2) |^2 + | h_j(1) |^2} > \frac{1}{\sqrt{2 | h_1(2) |^2 + | h_j(1) |^2}} \right)
\]

\[
= \mathbb{P}
\left( \frac{1}{\sqrt{2 | h_1(2) |^2 + | h_j(1) |^2}} \xi < | \tilde{\xi} | \right)
\]

\[
= Q \left( \frac{| \tilde{\xi} |}{\sqrt{2 | h_1(2) |^2 + | h_j(1) |^2}} \Lambda^s \right)
\]

where $s \rightarrow \tilde{x}$ stands for a pairwise error event, $Q(\cdot)$ is the Q-function [9], $\Lambda^s = | s - \tilde{x} |^2 / E_x$ represents the normalized squared Euclidean distance, $\gamma = E_x / \sigma^2_j$ is the average SNR at the receiver, and $E_x$ denotes the total transmit power on the two transmit antennas per symbol duration. The researchers noted that the subscripts in $s \rightarrow \tilde{x}$ are dropped due to the symmetric structure of Alamouti transmit diversity. Then, by averaging $P_r(s \rightarrow \tilde{x} | \mathbf{H})$ over $\mathbf{H}$ in (4), the average SPER can be straightforwardly formulated as [7]

\[
P_r(s \rightarrow \tilde{x} ; \gamma, \rho) = \int_{\sqrt{A}} Q(\sqrt{a}) f_{\alpha}(\alpha) d \alpha
\]

\[
= \frac{1 - \rho^2}{\gamma \Lambda^s} F_{\frac{3}{2}} \left( \frac{3}{2}, -1, -1 \right) + \frac{3 \rho^2}{\gamma \Lambda^s} F_{\frac{5}{2}} \left( 2, -3, -1 \right)
\]

where $f_{\alpha}(\alpha)$ is the probability density function (PDF) of $\alpha = \gamma | \xi |^2 \Lambda^s / 2( | h_1(2) |^2 + | h_j(1) |^2)$ and $F_{\cdot, \cdot, \cdot}(\cdot, \cdot, \cdot)$ is the Gauss hypergeometric function [10, (07.23.02.0001.01)], which is implemented in most of the well-known mathematical software packages, such as MATLAB, MATHEMATICA, MAPLE, etc.

It is obvious that the formula in (5) does not provide insightful information for Alamouti transmit diversity with QML decoding, such as achievable diversity orders. Thus, it is needed to evaluate the asymptotic behavior of the SPER in the high SNR regime. To this end, with the aid of $F_{\cdot, \cdot, \cdot}(\cdot, \cdot, 0) = 1$ in [10, (07.23.03.0001.01)] for $\gamma \rightarrow \infty$, the researchers can asymptotically approximate (5) as

\[
P_r(s \rightarrow \tilde{x} ; \gamma, \rho) \approx \frac{1 - \rho^2}{\gamma \Lambda^s} + \frac{3 \rho^2}{\gamma^2 \Lambda^s}
\]

where the fading correlation parameter $\rho$ is bounded $0 \leq \rho \leq 1$ in accordance with the severity of time-selective fading.
For the case of quasi-static fading channel (i.e., $\rho = 1$), the formula and corresponding asymptotic approximate expression of SPER are simplified, respectively, into

$$
\Pr_{s's} (s \rightarrow \bar{x}; \gamma, \rho = 1) = \frac{3}{\gamma^2 \Delta_0} \cdot F\left(2, \frac{5}{2}; 3; -\frac{1}{\gamma \Delta_0}\right),
$$

(7)

$$
\Pr_{s's} (s \rightarrow \bar{x}; \gamma, \rho = 0) = \frac{1}{\gamma \Delta_0^2},
$$

(8)

It is interesting that the formula in (7) is asymptotically equivalent to the formula given in [11, (29)], which is derived by the use of Taylor series expansion technique.

In addition, for the case of fast fading channel (i.e., fading channels are uncorrelated in time with $\rho = 0$), the researchers have

$$
\Pr_{s's} (s \rightarrow \bar{x}; \gamma, \rho = 0) = \frac{1}{\gamma \Delta_0},
$$

(9)

$$
\Pr_{s's} (s \rightarrow \bar{x}; \gamma, \rho = 0) = \frac{1}{\gamma \Delta_0^2},
$$

(10)

Furthermore, by exploiting the derived SPER formulas given in (5)-(10), the union upper and lower bounds on the SER of Alamouti scheme with QML decoding can be obtained as [12]

$$
P_u \leq P_{dul} = \frac{1}{|A|} \sum_{i \in A} \sum_{s \in \tau} \Pr (s \rightarrow \bar{x}_i; \gamma, \rho)
= \max_{s \in \tau} \Pr (s \rightarrow \bar{x}; \gamma, \rho)
$$

(11)

where $|A|$ denotes the cardinality of a constellation $A$.

### 4.2. Asymptotic Diversity Order for QML Decoding Algorithm

The diversity order is known to be generally defined as the magnitude of the slope of the average error rate versus SNR on a log-log scale in the high SNR region. Then, the asymptotic and instantaneous diversity orders can be expressed, respectively, as

$$
d_\infty = \lim_{\gamma \rightarrow +\infty} \frac{-\log \Pr (s \rightarrow \bar{x}; \gamma, \rho)}{-\log \gamma}
$$

(12)

$$
\hat{d} = \frac{\partial \log \Pr (s \rightarrow \bar{x}; \gamma, \rho)}{\partial \log \gamma} = \frac{\partial \Pr (s \rightarrow \bar{x}; \gamma, \rho)}{\partial \gamma}
$$

(13)

where it is clear that $d_\infty = -\lim_{\gamma \rightarrow +\infty} \hat{d}$. Then, substituting the formulas in (5)-(10) into (12) yields the corresponding asymptotic diversity orders for Alamouti transmit diversity system with QML decoding over various fading channels, which can be summarized as
which indicates that the QML decoding can resolve the error floor problem revealed in the conventional LML decoding (i.e., $d_{\text{Alamouti-LML}} = 0$ for $0 \leq \rho < 1$) over time-selective fading channels. It is, however, noteworthy that the QML decoding algorithm still experiences some performance degradation in that full-diversity (i.e., $d_{\text{Alamouti}} = N_f N_r = 2$) is not achieved in time-selective fading channels, which is due to the inherent noise enhancement problem, as mentioned in Section 2.

![Figure 1. Average SER versus SNR for Alamouti transmit diversity with $N_f=2$ and $N_r=1$ and QPSK over Quasi-Static and Time-Selective Fading Channels (i.e., $0 \leq \rho \leq 1$)](image)
4. Numerical Results

In this section, some numerical results are presented to verify the accuracy of the analytical results given in the previous section. Thus, Figure 1 shows the SER vs. SNR curves of the Alamouti transmit diversity system with two transmit antennas and one receive antennas, Gray-coded QPSK modulation, and both LML and QML decoding methods, over various Rayleigh fading channels (i.e., $0 \leq \rho \leq 1$). For the various channel configurations, we consider the time-selective fading channels with $\rho = 0.9911, 0.9755, 0.9522$, which, for example, correspond to the relative Doppler frequency $\Delta f_{\rho} = 0.03, 0.05, 0.07$, respectively, from Jakes’ model. The researchers also consider two further cases such as quasi-static fading channel (i.e., $\rho = 1$) and fast fading channel (i.e., $\rho = 0$).

Specifically, as shown in Figure 1, it is obvious that the QML decoding is more superior to the LML decoding in time-selective fading channels (including the fast fading channel) without showing error floors at high SNRs, which the LML decoding is unable to avoid. Furthermore, from the examination of the slope in the high SNR regions, we can observe that the SER curves from QML decoding shows the diversity orders of 2 and 1 over quasi-static and time-selective fading channels, respectively, which is apparently equivalent to the analytical results given in (14).

![Figure 2. SPER-based Instantaneous Diversity Order Derived in (13) versus SNR for Alamouti Transmit Diversity with $N_t=2, N_r=1$ and QPSK](image)

To further demonstrate the asymptotic diversity orders achieved by the QML decoding against the time-selectivity of fading channels, the researchers depict the instantaneous diversities obtained from (13) versus SNR in Fig. 2. The resultant instantaneous diversity
orders via various $\rho$, especially in the high SNR regime are identical to the analytical results derived in (14), where the researchers can also observe that the convergence speed of the instantaneous diversity order is inversely proportional to the value of $\rho$.

5. Conclusions

The researchers analytically evaluated the asymptotic diversity performance of Alamouti transmit diversity system employing QML decoding algorithm, particularly in time-selective fading channels. By judiciously deriving and utilizing the asymptotic closed-form approximate formula of SPER, the researchers apparently demonstrated that the achievable asymptotic diversity orders become 2 and 1 over quasi-static and time-selective fading channels, respectively, which reveals that the QML decoding is capable of overcoming the error floor effect induced by the conventional LML decoding in the high SNR regime. Finally, the researchers noted that the derived analytical results enable us to efficiently predict the diversity performance of the Alamouti transmit diversity with QML decoding method, especially in time-selective fading environments.

Acknowledgments

This work was financially supported by Hansung University.

References

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