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On Application to Partial Differential Equations of Warranty Reclaims

Lee Sang-Hyun[1], Chun Dong-Joon[2*] and Moon Kyung-Il[3]

[1,3] *Dept. of Computer Engineering, Honam University, Korea*
[2] *Dept. of Mechanical Engineering, Chonbuk National University, Korea*
{leesang64, kimoon}honam.ac.kr, djchun@jbnu.ac.kr

Abstract

The different types of warranty policy have been established in order to fulfill the demand of manufacturers and the requirement of buyers so that a win-win situation could be acquired. However, when considering these warranty policies, an important concept to keep in mind is warranty reclaims. Warranty reclaims extend the scope of warranty activities beyond the walls of a single company to encompass suppliers, manufacturers, OEMs, distributors, dealers, repair centers, policy carriers, and customers. The present work is designed for a novel method to solve the nonlinear warranty reclaims in the form of diffusion equations. The main motivation for this work is that the warranty reclaims can be represented as the diffusion equations, and the diffusion equation is a partial differential equation which describes density dynamics in a material undergoing diffusion. The diffusion equation is also used to describe processes exhibiting diffusive-like behavior such as warranty reclaims. The approximate solution of this problem is calculated in the form of finite differences with easily computable terms. To represent the capability and reliability of the method, some automobile warranty cases have been illustrated.

Keywords: *diffusion equation, finite difference, partial differential equation, warranty reclaim*

1. Introduction

Warranty data consists of claims data and supplementary data. Claims data are the data collected during the servicing of claims under warranty and supplementary data are additional data (such production and marketing related, items with no claims, *etc.*,) that are needed for effective warranty management. Analyzing such data can therefore be of benefit to manufacturers in identifying early warnings of abnormalities in their products, providing useful information about failure modes to aid design modification, estimating product reliability for deciding on warranty policy, and forecasting future warranty claims needed for preparing fiscal plans. An accurate prediction of optimal warranty period and warranty costs is often counted by the manufacturer. A warranty period may be unprofitable for the manufacturers if the choice of duration given is either too short or too long. Same thing goes to warranty cost which is an underestimation or overestimation of the warranty cost may have a high influence to the

[*] Corresponding author: Chun Dong-Joon, Ph.D.
Dept. of Mechanical Engineering, Chonbuk National University, Korea
E-mail: djchun@jbnu.ac.kr

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manufacturers. A manufacturer may want to know if warranty claims are a result of inadequate design, defects generated during component manufacturing, or errors in the assembly process. The manufacturer response or corrective actions for these various failure types are quite different. Thus, most of manufacturers provide customers with a basic two-attribute warranty that quantifies lifetime with two metrics: time and usage.

When we consider the warranty period and costs, an important concept to keep in mind is warranty reclaim, say extended warranty. The concept of the warranty reclaim is currently emphasized in the warranty issues because of its effect on increasing the scope of warranty cost savings. Extended warranties are not usually provided through the manufacturer but are extended through independent administrators. In some circumstances it may work to the consumer's benefit having an assurance to the product from a company outside of place of purchase and/or service. For instance, when an auto warranty is provided through a car dealership, it's usually a sub-contracted warranty (often from the retailer with the lowest offer), where vehicle repairs are negotiated to a lower rate, often compromising the service, labor and parts to a lower standard. Many times these types of warranties require an unexpected out-of-pocket expense at the time of repair, such as: - unexpected services provided outside of the warranty terms - uncovered parts and labor rates - paying the full balance while a reimbursement is arranged through dealership/warranty claims offices. Some mechanics and dealer service centers might put off, or defer the needed repair until the dealership's warranty has expired so that their warranty will no longer be bound to cover the cost of repair.

Singpurwalla and Wilson develop a two-attribute failure model for automobile warranty data indexed by time and usage [14]. They derive the two marginal failure distributions and present a method for predicting the warranty claims per thousand vehicles reported cumulatively by month in service, using a log-log model. Moskowitz and Chun suggest using a two-attribute Poisson model to predict claims for a two-attribute warranty by fitting the cumulative Poisson parameter with various functions of time and mileage [8]. Kim and Rao try to find expected warranty cost of two-attribute free replacement warranties based on a bivariate exponential distribution [4]. But, these approaches are not suitable due to some statistical assumptions. Also, there are a lot of uncertainty and many gaps in the data that can only be filled by qualitative assessments by warranty experts. The model can be extended or altered to fit in with the heuristic situation [16]. In this context, Lolas, *et al.*, discussed an approach used in the construction of the fuzzy logic knowledge base for a new reliability improvement expert system, whose main objectives are to be able to improve the reliability of new vehicle systems [6]. In regard to this warranty policy, Rai and Singh discussed a method to estimate hazard rate from incomplete and unclear warranty data [11]. Lee and Moon presented a new sets-as-points geometric view of fuzzy warranty sets under two-dimensional warranty policy [5]. Majid *et. al.*, mention that there are several studies in warranty problem specifically by using soft computing method [7].

This paper is interested to discuss about the problems on warranty reclaims in the form of typical diffusion equations. The diffusion equation is a partial differential equation which describes density dynamics in a material undergoing diffusion. The diffusion equation is also used to describe processes exhibiting diffusive-like behavior. In general, the diffusion equation is used to describe a number of transport phenomena such as heat transport through matter. The diffusion equation also has a passing resemblance to the Schrodinger wave equation for a particle moving in free space. The main difference from the diffusion equation is that the time derivative term is imaginary. Amazingly, this complex modification changes the behavior from diffusive to wave-like. The approximate solution of this problem is calculated in the form of finite differences with easily computable terms. To represent the capability and reliability of the method, some automobile warranty cases are demonstrated.

2. Background

Automobile manufacturers provide customers with a basic two-dimensional warranty that quantifies vehicle lifetime with two metrics: time and mileage. However, many manufacturers model field performance in the time domain due to the uncertainty associated with mileage accumulation. Thus, it is defined $R(t)$-claims per thousand vehicles reported cumulatively by month in a statistical model that automobile manufacturers use to track warranty performance. Such a dynamic linear predictive model can use data from multiple model years of a given vehicle. Its solution is obtained by plotting $R(t)$ on log-log paper and fitting a line to the observed data [13]. Another method is to develop a two-attribute failure model for automobile warranty data indexed by time and mileage. It derives the two marginal failure distributions and presents a method for predicting $R(t)$ using a log-log model. In general, manufacturers also use probability models for automobile warranty data. Poisson model may be used to predict automobile warranty claims in the time domain [1]. In particular, a two-attribute Poisson model is used to predict claims for a two dimensional warranty by fitting the cumulative Poisson parameter with various functions of time and mileage.

The warranty modeling techniques cited above fit warranty data using the probability models. To make inference on product features or design changes, the researchers suggest stratifying data or making model parameters a function of two attributes. Unfortunately, none of these models differentiate between failure types or provide feedback or assessment of supporting business processes. For example, a manufacturer may want to know if warranty claims are a result of inadequate design, defects generated during component manufacturing, or errors in the assembly process. The response or corrective actions of the manufacturers for these various failure types are quite different. Automobile manufacturers sell their products to automobile dealers, who in turn sell them to the end customer, which defines time zero for warranty duration. Some vehicles are repaired under warranty in the interval between the two sales, generating a pre-delivery claim or a claim with a negative lifetime. These negative values invalidate many probability models that reliability engineers fit to lifetime data. Many manufacturers remove pre-delivery claims from the population to allow fitting data with traditional methods, rather than developing techniques to accommodate this unconventional data that regularly occurs in automobile warranty claims. Until now, standard models have not been available for universal warranty claims processing. Most systems for processing warranty claims have been custom-developed solutions that only cope with individual processes and have no connection to external systems.

This paper develops a general model that assumes warranty claims represent a combination of product reliability issues (usage related failures) and quality problems (manufacturing defects). A usage related failure assumes that the product functioned properly when purchased. Then, after some amount of customer usage, the product experienced a failure. It is common to model these usage related failures with lifetime functions. Thus, we consider the one-dimensional time dependent usage-diffusion equation. Basically, the technique for calculating the solutions of equations of motion is: given an equation for the time derivative of quantity that can be evaluated at time t and its value at that instant, one can compute the quantity's value at some short time interval Δt in the future by using a finite difference approximation[3-4]:

$$w(t+\Delta t) = w(t) + \Delta t \left.\frac{\partial w}{\partial t}\right|_{t+\Delta t/2} \quad (1)$$

In practice, this equation is used to advance a solution to the equations of motion given non-linear forcing terms that would have otherwise been difficult at best to solve analytically. In this study, we will further develop our techniques for using finite differences to approximate differential equations, and discover that these techniques can be applied for solving partial differential equations. This technique will be then be used in a program to solve an important equation in warranty reclaim analysis, namely the warranty diffusion equation. The following warranty diffusion equation can be used to describe a number of warranty reclaim phenomena.

$$\frac{\partial w}{\partial t} = D\frac{\partial^2 w}{\partial x^2} \quad (2)$$

The function *w* might represent the warranty reclaims as a function of time and mileage. Here, *D* is a constant (known as a diffusion ratio) [12].

We are going to need a technique for computing the derivatives from a set of values that representative a continuous function. Suppose we are given values of a function at a series of discrete points representing mileage, say $w_j = w(j\Delta x)$ and want to compute the derivative of the function at the point *j*. The way this is done is to compute the derivative using finite differences and to average the derivative of the function for the interval before and after the point in question:

$$\left.\frac{\partial w}{\partial x}\right|_j = \frac{1}{2}\left[\frac{w_j - w_{j-1}}{\Delta x} + \frac{w_{j+1} - w_j}{\Delta x}\right] = \frac{w_{j+1} - w_{j-1}}{2\Delta x} \quad (3)$$

We will also need the expressions for the second derivative of a function. In some ways, this is easier than doing the derivative because the second derivative is simply the derivative of the first derivative. We don't need the derivative at the points themselves, and we can just compute the second derivative from the difference of the derivative between points:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_j = \frac{\left[\frac{w_j - w_{j-1}}{\Delta x} + \frac{w_{j+1} - w_j}{\Delta x}\right]}{\Delta x} = \frac{w_{j+1} - 2w_j + w_{j-1}}{(\Delta x)^2} \quad (4)$$

3. Solving Warranty Diffusion Equation

We are going to have to find solutions which are not only functions of time, but also functions of space *w*(*x*, *t*). Let's start by considering the diffusion equation. To determine the value of some function at some future instant in time, all we need is information about the spatial derivative at the present time. Using Equations (1) and (2),

$$w(t+\Delta t) = w(t) + \Delta t D \left.\frac{\partial^2 w}{\partial x^2}\right|_t \qquad (5)$$

The equation for the time derivative is given in terms of mileage derivatives at the present time. Clearly, what needed is some technique and framework for computing that mileage derivatives at a given time. Here is how it works.

First, we need to define a grid on which we will determine the value $w(x, t)$. We will approximate the continuous function $w(x, t)$ by a set of discrete values separated both in time and mileage. These values are usually written as

$$w_i^j = w(x_0 + j\Delta x, t_0 + i\Delta t) \qquad (6)$$

So the indices i and j represent the temporal and usage parts of a grid. The solution to the differential equation will be represented by a two-dimensional table. The goal here is to completely fill in this table with values. The solution will satisfy the differential equation in all interior points, and it will satisfy initial conditions and boundary conditions on the edges of this table.

The boundary values and initial conditions are quantities that are inputs to the problem and represent warranty conditions. For example, suppose our warranty diffusion problem represents warranty reclaims along a rod of length l_m and we want to determine the warranty reclaims in the range $0< x < l_m$, and the warranty reclaims at each end is fixed by being in contact with a warranty reservoir such that the warranty reclaims $w(x=2, t)=2$ and $w(x=1, t)=5$. In addition, we need to know the initial conditions of the warranty reclaims. Suppose we choose an initial condition of $w(x, t=0)$. The initial conditions and boundary conditions then let us immediately fill in the boundaries of the grid with values. To fill in the interior points, we need the finite difference equations approximating the differential equation. For the diffusion Equation, this becomes

$$\frac{w_j^{i+1} - w_j^i}{\Delta t} = D \frac{w_{j-1}^i - 2w_j^i + w_{j+1}^i}{(\Delta x)^2} \qquad (7)$$

Here the time derivative can be compute from the spatial derivative at the present time. After a little rearranging, this equation becomes

$$w_j^{i+1} = w_j^i + \frac{D\Delta t}{(\Delta x)^2}\left[w_{j-1}^i - 2w_j^i + w_{j+1}^i\right] =$$
$$\frac{D\Delta t}{(\Delta x)^2} w_{j-1}^i + w_j^i\left(1 - \frac{2D\Delta t}{(\Delta x)^2}\right) + \frac{D\Delta t}{(\Delta x)^2} w_{j+1}^i \qquad (8)$$

The constant $D\Delta t/(\Delta x)^2$ is determined by the grid spaces and the magnitude of the diffusion D. The steps solving the warranty diffusion equation are the following.

1) Set up a grid ranging from $0 < x < b$ with a grid spacing of Δx;
2) For boundary conditions assume the endpoints are fixed to $w(x=0, t)= w(x=b, t)=0$ (fill these in now);

3) For the initial conditions assume $w(x, t=a)=f(x)$ (fill these values in now);

4) Fill in the finite difference equations to advancing the solution in time to fill out the rest of the grid. Simulate the diffusion equation for $a<t<d$, with a time step of Δt for D.

5) How does the solution depend upon Δt? Observe that the time step needs to be smaller than a critical value for this numerical technique to work.

6) Next, change the initial condition to $w(x, t=a)$, but change the boundary conditions to $w(x=0, t)$ and $w(x=b, t)$.

The extended warranty generally does not progress in a smooth linear fashion as regards its choice for the initial condition function. The diffusion rate is defined as the amounts at which some warranty system adopt the green warranty. Warranty diffusions make two key assumptions regarding the phenomenon. First, the existing number of warranty claims positively drives the rate of warranty growth. Second, the difference between the potential number of warranty claims at the saturation level and the number of existing warranty also influences the rate of growth. Traditionally, diffusion has been specified by three basic models: internal-influence, external-influence, and mixed-influence. Mixed influence model represents both internal and external influences in the growth process, $dW_t/d_t=(p+qW_t)(k-W_t)$, where k is the potential number of warranty claims, W_t is the cumulative number of warranty claims at time period t, p is the coefficient of external influence, and q is the coefficient of internal influence. Logistic model leads to the form, $W_t =1/(k+ab^t)$. For $a>0$ and $0<b<1$, W_t is an increasing S-curve which reaches the upper bound or the saturation point of 1/k as time t approaches its theoretical limit of infinity. This curve reaches its inflection point at $W_t=k/2$. That is, the inflection point occurs when W_t reaches 50% of its saturation level.

4. Case Study

In this research, the historical warranty data is drawn from South Korea automobile company. The data that we concerns is warranty reclaims which is project name S1-2000cc degree. There are about 3000 samples of vehicles adopted in this study. The warranty information and status are recorded once a maintenance service is made. There are about 255 warranty reclaims recorded from the 3000 samples. Some elements that can be found in this warranty data are the date of sales, the date when the claims made, vehicle mileage, number of failure and defect and cost of every inspection. We consider the mileage and age as warranty reclaims data. Since the age and mileage increase accordingly, the data of failure and defect also we sort cumulatively. Table 1 denotes a summary table for vehicle mileage and time. Although it is a particular vehicle type the warranty reclaims ratio is very high. Currently, the warranty reclaims have been increasing because of extended warranty such as environmental concerns. Considering that some companies have total warranty costs equal to their R&D spending, achieving savings in this often overlooked area can result in significant improvements to the bottom line. The area of warranty costs is ripe for savings, yet has been neglected until now.

Table 1. Summary Table of Warranty Reclaims

Mileage (10^4*km)*	*Time (year)* 1	2	3	4	5	6	7	Total
0.50~1.00		1	1					2
1.00~1.50	2	2	1		3			8
1.50~2.00	3	2	2	2	4	3		16
2.00~2.50	4	7	5	4	3	3		26
2.50~3.00	3	6	7	5	6	4	1	32
3.00~3.50	3	6	10	8	8	1		36
3.50~4.00	1	3	11	8	5	5	3	36
4.00~4.50		2	5	5	5	3	4	24
4.50~5.00	2	3	3	9	3	3	2	25
5.00~5.50		2	3	4	4	3	2	18
5.50~6.00	1	1	3	2	2	4		13
6.00~6.50		1		1	1	2	1	6
6.50~7.00					3	1		4
7.00~7.50						1		1
7.50~8.00					2		3	5
8.00~					1		1	2
Total	19	36	51	48	50	33	17	254

When we do input data design, we have to filter the data because there are some missing data and also sometimes there are data that do not meet with the model we want to use.

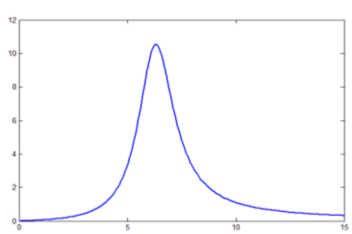

Figure 1. Initial Condition with Typical Diffusion

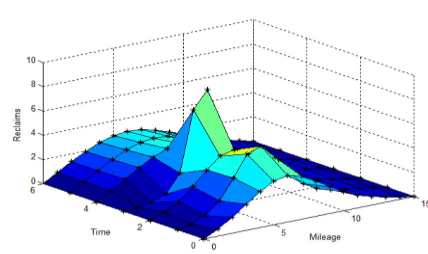

Figure 2. Fitting Surface using Typical Diffusion Function

Firstly, we choose an initial condition of $w(x, t{=}3)$, since this condition is very high frequencies from Table 1. Let's start by considering a typical diffusion equation, $f(x)=p*x/(1+(x-q)^2)$.

Matlab supports basic curve fitting through the basic fitting interface. Using this interface, we can quickly perform many curve fitting tasks within the same easy-to-use environment. The coefficients p and q that best fit the vectored value function $f(x)$ are 1.682 and 6.23, respectively (see Figure 1). The initial condition and boundary conditions then let us immediately fill in the boundaries of the grid with values. To fill in the interior points, we need the proposed algorithm approximating the differential equation.

Figure 2 denotes a fitting surface for our warranty reclaim data. The root mean square error (RMSE) of fitting the warranty reclaims with the typical diffusion differential equation is 19.8866.

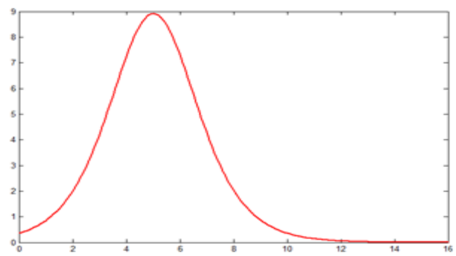

Figure 3. Initial Condition with Logistic Diffusion

Figure 3 denotes an initial condition function with a logistic diffusion. The coefficients k, a, and b that best fit the logistic function are 1.011, 0.985 and 0.4108, respectively.

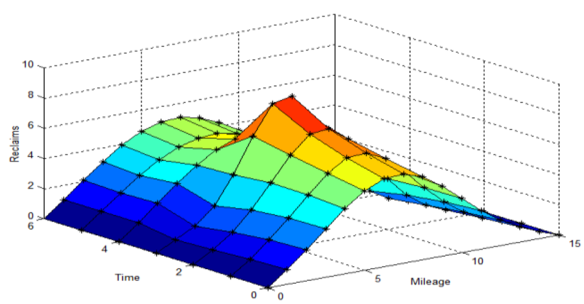

Figure 4. Fitting Surface using Logistic Typical Diffusion Function

Figure 4 represents a fitting surface for the warranty reclaim data using the initial condition, $f(x) = (0.3531 * 0.4108^x) / (0.011+0.985*0.4108^x)^2$. The root mean square error of fitting the warranty reclaims is 4.5864. The logistic condition has the much smaller RMSE than that of the typical diffusion equation in the warranty reclaim data. It is evident that the logistic forecasts are superior to any diffusion equation.

5. Conclusion

This paper makes a contribution to the warranty reclaim research by suggesting a novel approach to model its diffusion. The warranty reclaims can be represented as a diffusion equation, and it is a partial differential equation which describes density dynamics in a material undergoing diffusion. The diffusion equation describes the diffusion of warranty claims starting at a specific time, with an initial usage distribution and progressing over time. The approximate solution of this problem was calculated in the form of finite differences with easily computable terms. To represent the capability and reliability of the proposed method, some automobile warranty cases were illustrated. The traditional diffusion models are variants of S-curves. Warranty reclaims such as extended warranty are largely ignored when it comes to modeling. This is perhaps because external effects can be domain-sensitive or occur at any stage of the product life cycle. The mathematical models that fit S-curves to the warranty reclaim data thus essentially treat external perturbations. These random errors reduce the accuracy of the forecasts of these models. Our approach can be basic to most other extended warranty modeling studies in the future as long as there is some theoretically informed argument that external influence is suspected.

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References

[1] J. D. Kalbfleisch, J. F. Lawless and J. A. Robinson, "Methods for the analysis and prediction of warranty claims", Technometrics, vol. 33, **(1991)**, pp. 273-285.
[2] S. Kumar and R. Singh, "A mathematical model to solve reaction diffusion equation using homotopy perturbation method and a domain decomposition method", VSRD-TNTJ, vol. 2, no. 4, **(2011)**, pp. 186-201.
[3] S. H. Lee, S. J. Lee and K. I. Moon, "A two-attribute green warranty model for automobile warranty data", Information Journal, vol. 15, no. 5, **(2012)**.
[4] S. Lolas, O. A. Olatunbosun and J. Buckingham, "Fuzzy logic knowledge base construction for a reliability improvement expert system", Proceedings of the World Congress on Engineering, **(2007)**.
[5] H. A. Majid, N. H. Kasim, N. I. Jamahir and A. A. Samah, "Soft computing methods in warranty problems: review and recent applications", International Journal of Computer Science Issue, **(2012)**, pp. 190-196.
[6] H. Moskowitz and Y. H. Chun, "A poisson regression model for two-attribute warranty policies", Naval Res Logistics, vol. 41, **(1994)**, pp. 355-376.
[7] K. B. Rai and N. Singh, "Reliability analysis and prediction with warranty data", U.S: CRC, **(2009)**.
[8] B. Rahul, A. K. Singh and D. P. Singh, "A mathematical model to solve reaction diffusion equation using differential transformation method", International Journal of Mathematics Trends and Technology, vol. 2, no. 2, **(2011)**.
[9] J. A. Robinson and G. C. McDonald, "Issues related to field reliability and warranty data", Data quality control: theory and pragmatics, New York: Marcel Dekker, **(1991)**.
[10] S. H. Lee, D. J. Chun and I. M. Moon, "On Application to Partial Differential Equations of Warranty Reclaims", Proceedings of the 1th International Symposium, ISAAC 2013 in conjunction with ICACT 2013 Seoul, Korea, Revised Selected Papers, **(2013)** November.

Authors

Sang- Hyun Lee, he received the BS and MS in Department of Computer Engineering from Honam University in 2002 and 2004, respectively. He received Ph.D. degrees in Computer Science from Chonnam National University in 2009. He has been a professor at Honam University since 2012. His research interests include artificial intelligence, Software Engineering, Early Warning System, claim analysis, intelligence automotive.

Dong-Joon Chun, he received the BS in department of mechanical engineering from Chonnam National University in 1987. He received PH.D. degree in Kyunghee University in 1999. His research interests include automotive engineering, computer aided engineering, clean diesel engine and energy plant system.

Kyung-li Moon, he received a Ph.D. Ph.D, is a professor at the Department of Computer Engineering, Honam University in Gwang-Ju, Korea. His theoretical work began at Seoul University as a statistical computing scientist, and then expanded into complexity science, chaos theory, and cognitive science generative sciences.