Research on Different Representation Methods for Classification

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Abstract

Under today’s big data environment, with the rapid development of computer network technology and information technology, data mining is becoming more and more important in computer science. Classification is one of the most important aspects in data mining research field. Recently, representation methods, such as sparse representation and low rank representation, have been much concerned. They both have wide applications in scientific and engineering fields. However, sparse representation and low rank representation include many methods, although these methods have their own characteristics, they are all effective for handling classification problems. This paper focuses on the performance comparison of different representation methods currently used in handling classification problems and views other conventional methods that can be applied in this field.

Keywords: Sparse representation, Low rank representation, Classification

1. Introduction

Data mining is a hot topic in the field of artificial intelligence and machine learning. Data mining is a process, which reveals the implicit, previously unknown and potentially useful information from the large amount of data in the database. With the rapid development of information technology and computer network, Internet data and resources show massive features. In order to manage these massive information effectively, data mining is becoming a hot research field increasingly. However, data mining techniques include many aspects. Classification is one of the most important aspects in data mining.

Given some training samples from multiple classes, the aim of classification task is to assign one of the class labels to a test sample. Classification has been widely used in scientific and engineering fields, such as pattern recognition, data mining, computer vision, etc.

There are also many conventional methods for handling classification problems, such as Nearest Neighbor, Nearest subspace classifier, Linear SVM, etc., Recently, there has been an increasing interest in representation theory. Representation includes many methods, such as sparse representation, low rank representation and collaborative representation, etc., although these methods have their own characteristics, they are all effective for handling classification problems.

However, representation methods also include many algorithms. These algorithms also have advantages and disadvantages. The aim of this paper is to compare the state of the art algorithms in representation theory. We try to clarify the similarities among different representation algorithms and reveal the differences of them.

Wright, et al., [1] proposed the sparse representation classifier (SRC) method for classification. SRC is a classical method, the solution of SRC can be obtained by using $l_1$
norm minimization. SRC boosts the research of sparsity. Many application problems are also solved by sparse representation methods.


The low rank representation (LRR) was proposed by Liu [6], which is different from the sparse representation. The aim of sparse representation is to obtain the sparsest solution of each test sample respectively. However, unlike sparse representation, the aim of low rank representation is to find the lowest rank representation of all the test samples jointly. Low rank representation can be also used to handle the classification problems.

This paper is organized as follows. In section 2, sparse representation methods are reviewed. In section 3, collaborative representation methods are reviewed. In section 4, some conventional methods used for classification are reviewed. In section 5, low rank representation methods are reviewed. In section 6, we review the advantages and disadvantages of these methods. In section 7, some experiments on digit recognition and face recognition have been done in order to compare these representation methods. Section 8 concludes this paper.

2. Sparse Representation Methods for Classification

Sparse representation is based on the concept of mathematical norm, which has a long history. With the rapid development of norm minimization methods, such as OMP [7], BP [8] and many other algorithms Sparse representation has been much concerned for many years. Recently, sparse representation has obtained many applications in signal processing, image feature extraction, pattern recognition, image denoising, etc., [9-25].

Pati proposed Orthogonal Matching Pursuit (OMP) method [7], which is an optimization method by using $l_0$ norm minimization. Chen proposed Basis Pursuit (BP) method [8], which is an optimization method by using $l_1$ norm minimization. Wright, et al., [1] proposed SRC for classification. SRC is a classical method, it is an optimization method by using $l_1$ norm minimization. Based on SRC, some papers proposed many other methods. Elhamifar et al. [2, 3] proposed a Block-Sparse representation. Chi, et al., [4] proposed a Collaborative Representation Optimized Classifier (CROC). Yang and Zhang [5] proposed a kind of Collaborative representation classification (CRC RLS) [5] method, by using $l_2$ norm minimization.

2.1. Multi-class Classification

If there are $K$ classes, and there are $n_i$ training data from the $i$th class formed a matrix as $A_i = [a_{i1}, a_{i2},...,a_{in_i}] \in \mathbb{R}^{m \times n_i}$. $A$ is denoted by the collection of all training samples: $A = [A_1, A_2,...,A_K]$. If given a test sample $y \in \mathbb{R}^m$, the aim of multi class classification is to identify $y$ belongs to which class [1, 4, 5, 11].
2.2. Orthogonal Matching Pursuit

Orthogonal Matching Pursuit (OMP) is a well known algorithm, which was proposed by Pati in 1993 [7]. It is a sparse representation method to find the approximate solution of $l_0$ norm minimization. The steps of OMP algorithm are as follows [11]:

**Task:** Find the approximate solution of $P_\mathcal{O}: \min_x \|x\|_0$ s.t. $y = Ax$

1) **Input:**
A matrix concatenated by training samples $A = [A_1, A_2, \ldots, A_k] \in \mathbb{R}^{mn}$ for $k$ classes, a test sample $y \in \mathbb{R}^m$, an error threshold $\varepsilon_0$.

2) **Initiation:**
Initialize $k=0$, and set: The initial solution $x^0 = 0$. The initial residual $r^0 = y - Ax = y$.

The initial solution support $S^0 = \text{Support}(x^0) = \emptyset$.

3) **Iteration:**
   a) Compute the errors $\varepsilon(j) = \min_{z_j} = \|a_jz_j - r^{k-1}\|_2^2$ for all $j$ using the optimal choice $z_j^* = a_j^T r^{k-1} / \|a_j\|_2^2$.
   b) Find a minimizer $j_0$ of $\varepsilon(j): \forall j \notin S^{k-1}, \varepsilon(j) \leq \varepsilon(j_0)$, update $S^k = S^{k-1} \cup \{j_0\}$.
   c) Compute $x^k$, the minimizer of $\|Ax - y\|_2$ subject to Support $\{x\} = S^k$.
   d) Compute $r^k = y - Ax^k$.
   e) If $\|r^k\|_2 < \varepsilon_0$, stop. Otherwise, perform another iteration.

4) **Output:** The solution is $x^k$ after $k$ iterations.

The aim of OMP is to obtain the approximate solution of $l_0$ norm minimization. However, OMP is a greedy algorithm, so its computational complexity is high. Furthermore, it is sensitive to noise.

2.3. Sparse Representation-based Classification

The $l_0$ norm optimization is NP-hard, a convex relaxation of it can be obtained by replacing them $l_0$ with $l_1$ norm. Sparse representation-based classification (SRC) is just a classical method by using $l_1$ norm minimization. SRC was proposed by Wright, et al., [1]. It is a classical method for classification. Based on SRC, some papers proposed many another methods. The steps of SRC algorithm are as follows [1]:

**Task:** Find the solution of $P_1: \min_x \|x\|_1$ s.t. $y = Ax$

1) **Input:**
A matrix concatenated by training samples $A = [A_1, A_2, \ldots, A_k] \in \mathbb{R}^{mn}$ for $k$ classes, a test sample $y \in \mathbb{R}^m$.

2) **Solve the $l_1$ norm minimization problem:**
   
   $x_i = \arg \min_x \|x\|_1$ s.t. $y = Ax$

3) **Compute the residuals:**
\( r_i(y) = \|y - A\delta_i(x_i)\|_2^2 \), for \( i=1,2,...,k \).

4) Output:

\( \text{id}entity(y) = \arg \min r_i(y) \).

The SRC method looks for the sparsest representation of a test sample by using \( l_1 \) norm minimization. The classification results of SRC are good. However, the sparsest representation does not mean obtaining the best classification results. Furthermore, from \( l_1 \) norm minimization, the SRC cannot obtain closed form solution, so its computational complexity is high.

2.4. Structured Sparse Representation

The dictionary of the training samples has a structure; it means data from each class forming a few blocks of the dictionary. However, the SRC method only looks for the sparsest representation of a test sample; it does not take into account of the similarity of these samples. The solution of \( l_1 \) norm minimization does not indicate the sparse distribution feature of the samples. Thus, there still remain some problems about multi-class classification using SRC method. Elhamifar proposed structured sparse representation method (SSR) [2, 3], the structured sparse representation looks for a representation, which the test sample involves the minimum number of blocks from the dictionary. The non-convex optimization programs are as follows [2, 3]:

\[
P_{q\|_b} : \min \sum_{i=1}^{n} J(\|D[i]\|_q) > 0 \text{ s.t. } y = Ax ,
\]

and

\[
P_{q\|_b}^\prime : \min \sum_{i=1}^{n} J(\|A[i]x[i]\|_q) > 0 \text{ s.t. } y = Ax ,
\]

\( J(\cdot) \) is the indicator function, \( q \geq 1 \), \( x[i] \in \mathbb{R}^m \) are the entries of \( x \) corresponding to the \( i \)-th block of the dictionary. However, the optimization program \( P_{q\|_b} \) is NP-hard, a \( l_1 \) relaxation of it can be given as follows:

\[
P_{l_1\|_b} : \min \sum_{i=1}^{n} \|x[i]\|_1 \text{ s.t. } y = Ax ,
\]

and

\[
P_{l_1\|_b}^\prime : \min \sum_{i=1}^{n} \|A[i]x[i]\|_1 \text{ s.t. } y = Ax .
\]

Output: \( \text{id}entity(y) = \arg \min _{i} \|y - B[i]c^*[i]\|_2 \)

3. Collaborative Representation Methods for Classification

3.1. Collaborative Representation based on Methods for Classification

The solution of sparse representation methods can be only obtained by the \( l_1 \) norm minimization. However, it cannot obtain the closed form solution from \( l_1 \) norm minimization. The computational complexity of \( l_1 \) norm minimization is a little high. Thus, some authors
proposed regularized least square method using $l_2$ norm minimization. Collaborative representation classification (CRC_RLS) is proposed by Zhang and Yang [5], which is a typical method by $l_2$ norm minimization. The steps of CRC_RLS algorithm are as follows [5]:

Task: Find the solution of $P_i : \min_x \|x\|_2$ s.t. $y = Ax$

1) Input:
A matrix concatenated by training samples $A = [A_1, A_2, \ldots, A_k] \in \mathbb{R}^{m \times n}$ for $k$ classes, a test sample $y \in \mathbb{R}^m$.

2) Solve the $l_2$ norm minimization problem:

$$x_2 = \arg \min_x \|x\|_2 \text{ s.t. } y = Ax$$

3) Compute the residuals:

$$r_i(y) = \|y - A\delta_i(x_i)\|_2^2 \quad \text{for } i = 1, 2, \ldots, k.$$  

4) Output:

$\text{identity}(y) = \arg \min_{i} r_i(y)$.

From $l_2$ norm minimization, a closed form solution can be obtained, which gives $x = (A^T A + \lambda I)^{-1} A^T y$. The classification results of CRC_RLS are good. Furthermore, from the $l_2$ norm minimization, its computational complexity is low. However, the obtained solution is not sparse.

3.2. Collaborative Representation Optimized Classifier

Combined the Nearest Subspace Classifier (NSC) [26] and the Collaborative Representation based Classifier (CRC), Chi and Porikli proposed a collaborative representation optimized classifier (CROC), which depends on the trade-off between the NSC and CRC. The residual of CROC for each class is calculated as:

$$r_i(y) = r_i^{\text{NS}} + \lambda r_i^{\text{CR}}, \quad \text{for } i = 1, \ldots, k,$$

where $\lambda \geq 0$. If the $i$th residual is the minimal, then CROC assigns the test sample to the $i$th class.

4. Conventional Methods used for Classification

4.1. Nearest Neighbors

Nearest neighbors (NN) was first proposed by Cover and Hart for classification [27]. It was developed to be K nearest neighbor classifier subsequently [28]. This method is a conventional method, which is familiar to us. It can be used in digital image processing, pattern recognition, data mining etc., Zhang and Yang combined the idea of NN and SRC, they presented KNN-SRC method [28]. The idea of NN is very simple, the steps of NN algorithm are as follows [28]:

With a test sample $y$, for $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$, compute the residuals

$$r_{ij}(y) = \|y - a_{ij}\|_2^2.$$  

If a residual $r_{ij}$ is the smallest, the NN will judge the test sample $y$ belongs to the $i$-th class.
NN is a very simple method used for classification; its computational complexity is low. However, the classification results of NN are poor.

4.2. Nearest Subspace Classifier

Nearest subspace classifier (NSC) was proposed by Lee [26]. Chi and Porikli also utilized this method in their paper. The steps of NSC algorithm are as follows [26]:

For $i = 1, 2, ..., k$, there are $K$ classes, there are $n_i$ training data from the $i$-th class formed a matrix as $A_i = [a_{i1}, a_{i2}, ..., a_{im}] \in \mathbb{R}^{m \times n_i}$. $A_i$ span a subspace. Compute the residual $r_{i}^{NS} = \min \| y - A_i x_i \|_2$. If the $i$-th residual is the smallest, NSC assigns the test sample $y$ to the $i$-th class.

5. Low Rank Representation Methods

Liu, et al., [6] proposed the low rank representation method, which is different from sparse representation. The aim of the sparse representation is to obtain the sparsest solution of each test sample respectively. However, unlike sparse representation, the aim of low rank representation is to find the lowest rank representation of all the test samples jointly. Low rank representation can be also used to handle the classification problems.

With a set of sample vectors $X = [x_1, x_2, ..., x_n]$, every column is a sample. Each sample can be represented by the linear combination of the basis in a dictionary $A = [a_1, a_2, ..., a_m]$. That is, $X = AZ$. $Z = [z_1, z_2, ..., z_n]$ is the coefficient matrix, each $z_i$ is the representation of $x_i$. Our aim is to capture the global structure of $X$. However, sparse representation cannot capture the global structure of $X$, low rank representation is a more appropriate criterion for capturing the global structure of $X$. In other words, our aim is to look for a representation $Z$ by solving the following problem:

$$\min_Z \text{rank}(Z) \quad \text{s.t.} \quad X = AZ,$$

(1)

however, due to the discrete property of the rank function, the problem (1) is hard to solve. Thus, the problem (1) can be substituted for solving the problem (2):

$$\min_Z \| Z \|_* \quad \text{s.t.} \quad X = AZ.$$

(2)

For problem (2), $\| \cdot \|_*$ is the nuclear norm of a matrix, the definition of the nuclear norm is the sum of the singular values of the matrix.

The problem (2) can be solved using Augmented Lagrange Multiplier (ALM) algorithm [29-33], which is a classical method for solving the low rank representation (LRR) problem. LRR can also handle supervised classification problems as SRC. However, there are some problems with LRR. First, it cannot obtain closed form solution from the ALM algorithm. Second, there are too many parameters with the ALM algorithm. Third, the convergence property of ALM cannot be analyzed in detail.

6. Experiments

In this section, some experiments on face recognition and digit recognition are presented to show the accuracy of classification. We focus on the comparison of different representation methods mentioned above. Three databases, including Extended-YaleB [1, 4, 5], AR [1, 4, 5] and MNIST Handwritten Digits database [4], are used to test the performance of some
methods, including SRC, CRC_RLS and NN. Our experiments focus on the performance comparison of different methods.

6.1. Face Recognition

These methods are tested for comparing the recognition rate. Recognition rate is a percentage, which denotes how many test samples can be classified correctly for all the test samples. Higher recognition rate means the performance of this method is better. In our experiments, the Eigenface is used as preprocessing in feature extraction.

1) Extended Yale-B database: The Extended Yale-B database contains 2414 frontal face images of 38 individuals [1, 4, 5]. The images were cropped and normalized to 54x48. A few images of Extended Yale-B database are shown in Figure 1. Table 1 shows the recognition rates versus feature dimension by SRC, CRC_RLS and NN.

\[\text{Table 1. The Recognition Results of Different Methods on the Extended Yale-B Database}\]

<table>
<thead>
<tr>
<th>Dimension</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>69.24%</td>
<td>71.78%</td>
<td>72.96%</td>
<td>74.05%</td>
<td>75.41%</td>
</tr>
<tr>
<td>SRC</td>
<td>96.01%</td>
<td>96.37%</td>
<td>96.41%</td>
<td>96.5%</td>
<td>97.19%</td>
</tr>
<tr>
<td>CRC-RLS</td>
<td>94.74%</td>
<td>95.04%</td>
<td>95.92%</td>
<td>96.28%</td>
<td>97.01%</td>
</tr>
</tbody>
</table>

2) AR database: The AR database contains about 4000 frontal images for 126 individuals [1, 4, 5]. These images are captured under different illuminations, expressions and facial disguises. The images are cropped to size 60 x 43. A few images of AR database are shown in Figure 2. Table 2 shows the recognition rates versus feature dimension by SRC, CRC_RLS and NN.

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Table 2. The Recognition Results of Different Methods on the AR Database

<table>
<thead>
<tr>
<th>Dimension</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>49.07%</td>
<td>54.08%</td>
<td>58.23%</td>
<td>60.52%</td>
<td>61.80%</td>
</tr>
<tr>
<td>SRC</td>
<td>38.05%</td>
<td>50.36%</td>
<td>58.08%</td>
<td>64.95%</td>
<td>70.96%</td>
</tr>
<tr>
<td>CRC-RLS</td>
<td>19.46%</td>
<td>36.34%</td>
<td>45.92%</td>
<td>56.51%</td>
<td>64.38%</td>
</tr>
</tbody>
</table>

6.2. Digit Recognition

The MNIST handwritten digits database is used to test the property of these methods. The dimension of each image is 28 × 28. Every image, which is a 8 bit gray scale image from 0 to 9 [4].

For the MNIST handwritten digits database, which has a training set of 60,000 samples, and a test set of 10,000 samples of each class? For our experiment, 10 training samples are randomly selected from each class, 10 test samples are also randomly selected from each class. A few images of MNIST database are shown in Figure 3. Table 3 shows the recognition rates versus feature dimension by SRC, CRC RLS and NN.

![Image of some training samples from MNIST database]

(a)

![Image of some test samples from MNIST database]

(b)

Figure 3. (a) Some Training Samples from the MNIST Database; (b) Some Test Samples from the MNIST Database

Table 3. The Recognition Results of Different Methods on the MNIST Database

<table>
<thead>
<tr>
<th>Dimension</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRC</td>
<td>61%</td>
<td>66%</td>
<td>55%</td>
<td>61%</td>
<td>62%</td>
</tr>
<tr>
<td>CRC-RLS</td>
<td>59%</td>
<td>58%</td>
<td>58%</td>
<td>59%</td>
<td>57%</td>
</tr>
</tbody>
</table>

7. The Comparison of Different Methods

As mentioned above, there are many representation methods used for handling the classification problems. However, these methods all have their own characteristics, they also have advantages and disadvantages. The reviews of these representation methods are listed in Table 4.

Table 4. The Reviews of Different Representation Methods used for Classification

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMP</td>
<td>Approximate solution of (l_0) norm minimization</td>
<td>Greedy algorithm, computational complexity is high, sensitive to noise</td>
</tr>
<tr>
<td>SRC</td>
<td>Classification results are good</td>
<td>Cannot obtain closed form solution, computational</td>
</tr>
</tbody>
</table>
8. Conclusions

Under big data environment, data mining technologies are increasingly becoming a hot research field. However, data mining technologies include many aspects; classification is one of the most important aspects of data mining. Representation methods are very effective for handling classification problems. However, these methods all have their own characteristics; they also have advantages and disadvantages. In the future work, we should combine the advantages of currently used methods. It means that, the improved representation methods, which the classification results should be good and the computational complexity should be low. These methods are expected to apply in classification area.

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References

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