Research on an Improved Quantum Particle Swarm Optimization and its Application

Lei Wang

Department of Mathematics, Binzhou University, Binzhou Shandong 256600 China

Abstract

The quantum particle swarm optimization (QPSO) algorithm exists some defects, such as premature convergence, poor search ability and easy falling into local optimal solutions. The adaptive adjustment strategy of inertia weight, chaotic search method and neighborhood mutation strategy are introduced into the QPSO algorithm in order to propose an improved quantum particle swarm optimization (AMCQPSO) algorithm in this paper. In the AMCQPSO algorithm, the chaotic search method is employed to promote the quality of initial population. The adaptive adjustment strategy of inertia weight is used to adjust the global search ability and local search ability of particles in the running process of QPSO algorithm. The neighborhood mutation strategy is used to increase the diversity of population and avoid premature convergence. Finally, in order to evaluate the performance of the AMCQPSO algorithm, several well-known benchmark functions are selected in this paper. The experiment simulations show that the proposed AMCQPSO algorithm can effectively improve the quality of solutions, and takes on powerful optimizing ability and more quickly convergence speed.

Keywords: quantum particle swarm optimization; chaotic search; adaptive adjustment of inertia weight; neighborhood mutation; optimization function

1. Introduction

Particle swarm optimization (PSO) algorithm is a kind of optimization algorithm, which is proposed by Kennedy and Eberhart in 1995[1]. The thought of PSO is to originate the study on the behavior of birds. The PSO algorithm is more simple, easy implementing, less adjusting parameters than genetic algorithm(GA)[2] and ant colony optimization(ACO) algorithm[3]. So the PSO algorithm has been successfully applied to structural design, combinatorial optimization, numerical optimization, data mining, clustering, electromagnetic field and task scheduling and other engineering optimization problems. But the PSO algorithm has some disadvantages in the process of evolution, such as premature convergence and poor local searching ability and so on. Many researchers have done a lot of work for these shortcomings. The main improvements have three following strategies: (1) the improvements are based on parameter selection. (2) The improvements are based on update rules of the position and velocity of particle. (3) The improvements are based on combining or fusing other algorithms, such as chaotic PSO, fuzzy PSO, multi-subpopulation PSO, simulated annealing PSO and so on.

Quantum evolutionary algorithm (QEA) is a newly developed probabilistic evolutionary algorithm based on some concepts and theories of quantum computing, such as quantum bits and quantum superposition states and so on. Sun et al. [4] proposed a quantum particle swarm optimization (QPSO) algorithm in order to improve the search capability and optimization efficiency, avoid premature convergence. But it is similar to other evolutionary algorithms, the QPSO algorithm has the same disadvantages of premature convergence and low search ability in solving complex optimization problems. So many researchers proposed a lot of improved QPSO
algorithms in recent years. Luo et al. [5] proposed an improved Quantum Particle Swarm Optimization (QPSO) based on Gaussian mutation operator to overcome its shortcoming of falling into local convergence. Zhang et al. [6] proposed a diversity-guided modified quantum-behaved particle swarm optimization (DGMQPSO) algorithm for solving complex problems. Zhu et al. [7] proposed a hybrid QPSO algorithm for LS-SVM parameter selection to improve the learning performance and generalization ability of LS-SVM model. Tokgo and Li [8] proposed a modified natural selection based quantum behaved particle swarm optimization (SelQPSO) algorithm for the path planning of mobile robot vehicles. Li and Feng [9] proposed an adaptive subgroup collaboration QPSO algorithm is proposed to optimize the parameters of the fuzzy controller. Wu et al. [10] proposed a dual-group interaction quantum-behaved particle swarm optimization (DIR-QPSO) algorithm based on random evaluation by constructing the master-slave sub-groups with different potential well centers, which avoids the rapid disappearance of swarm diversity and enhances the global searching ability through collaboration between sub-groups. Pradhan and Patra [11] proposed a new algorithm based on hybridizing the notion of QPSO and BFA for tradeoff between local and global search. Leandro dos Santos [12] proposed a novel Quantum-behaved PSO (QPSO) using chaotic mutation operator. The application of chaotic sequences is a powerful strategy to diversify the QPSO population and improve the performance of QPSO in preventing premature convergence to local minima. Leandro Santos and Piergiorgio [13] proposed a new quantum-behaved approach using a mutation operator with exponential probability distribution. The simulation results demonstrate good performance of the proposed algorithm in solving a significant benchmark problem. Xi et al. [14] proposed an improved quantum-behaved particle swarm optimization with weighted mean best position (WQPSO) according to fitness values of the particles. Sun et al. [15] proposed a modified quantum-behaved particle swarm optimization method(QPSO-DM) based on combining the QPSO algorithm with differential mutation operation to solve the economic dispatch (ED) problem in power systems, whose objective is to simultaneously minimize the generation cost rate while satisfying various equality and inequality constraints. Leandro dos Santos [16] proposed a novel quantum-behaved approach using a mutation operator with Gaussian probability distribution. Two case studies are described and evaluated in this work. Lu et al. [17] proposed a modified quantum-behaved particle swarm optimization (QPSO-DM) algorithm for short-term combined economic emission scheduling (CEES) of hydrothermal power systems with several equality and inequality constraints. Qu et al. [18] proposed a chaotic quantum particle swarm optimization (CQPSO) based on making use of the randomness, regularity and ergodicity of chaotic variables to improve the quantum particle swarm optimization algorithm. Farzi and Dastjerdi [19] proposed a modified quantum-behaved particle swarm optimization based on adding a leaping behavior to avoid falling into the local optimum. Jau et al. [20] proposed a modified QPSO (MQPSO) algorithm based on applying the concept of the GA to improve the convergent speed and conquer the phenomenon of premature. Sun et al. [21] proposed a diversity-maintained quantum-behaved particle swarm optimization (DMQPSO) algorithm based on the analysis of QPSO and integrates a diversity control strategy to enhance the global search ability of the particle swarm. Then, Sun et al. [22] proposed a modified version of quantum-behaved particle swarm optimization (QPSO) algorithm, known as the Multi-Elitist QPSO (MEQPSO) model. Li et al. [23] proposed a cooperative quantum-behaved particle swarm optimization (CQPSO). This CQPSO, a particle firstly obtaining several individuals using Monte Carlo method and these individuals cooperate between them. Gao et al. [24] proposed a novel cultural quantum-behaved particle swarm optimization algorithm (CQPSO) to improve the performance of the quantum-behaved PSO (QPSO). Niu et al. [25] proposed an improved quantum-behaved particle swarm optimization (SQPSO) based on combining QPSO with a selective probability operator to solve the

In this paper, in order to avoid premature convergence and falling into local optimal solutions, improve search ability of QPSO, an improved quantum particle swarm optimization (AMCQPSO) algorithm is proposed in this paper. The chaotic search method is employed to promote the quality of initial population. The adaptive adjustment strategy of inertia weight is used to adjust the global search ability and local search ability of particles in the running process of QPSO algorithm. The neighborhood mutation strategy is used to increase the diversity of population and avoid premature convergence. Several well-known benchmark functions are selected to evaluate the performance of the AMCQPSO algorithm.

The rest of this paper is organized as follows. Section 2 briefly introduces the particle swarm optimization (PSO) algorithm. Section 3 briefly introduces quantum particle swarm optimization (QPSO) algorithm. Section 4 proposed an improved quantum particle swarm optimization (AMCQPSO) algorithm based on the adaptive adjustment strategy of inertia weight, chaotic search method and neighborhood mutation strategy. Section 5 gives the describing of AMCQPSO algorithm. Section 6 applies the AMCQPSO algorithm to solve benchmark functions. Finally, the conclusions are discussed in Section 7.

2. Particle Swarm Optimization (PSO) Algorithm

The particle swarm optimization (PSO) algorithm is a population-based search algorithm based on the simulation of the social behavior of birds within a flock. The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredictable choreography of a bird flock, with the aim of discovering patterns that govern the ability of birds to fly synchronously, and to suddenly change direction with a regrouping in an optimal formation. The search behavior of particle is affected by other particles within the population. The position of particles within the search space are changed based on the social-
psychological tendency of individuals in order to delete the success of other individuals. The consequence of modeling for this social behavior is that the search is processed in order to return toward previously successful regions in the search space. Namely, the velocity ($v$) and position ($x$) of each particle will be changed by the particle best value ($pB$) and global best value ($gB$). The velocity and position updating of the particle is shown by the followed expression:

$$v_{ij}(t+1) = w v_{ij}(t) + c_1 r_1 (pB_{ij}(t) - x_{ij}(t)) + c_2 r_2 (gB_{ij}(t) - x_{ij}(t))$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)$$

Where $v_{ij}(t+1)$ is velocity of particle $i^{th}$ at iterations $j^{th}$, $x_{ij}(t+1)$ is position of particle $i^{th}$ at iterations $j^{th}$. $w$ is inertia weight to be employed to control the impact of the previous history of velocities. $c_1$ is the cognition learning factor, $c_2$ is the social learning factor, $r_1$ and $r_2$ are random numbers uniformly in [0, 1].

3. Quantum Particle Swarm Optimization (QPSO) Algorithm

In order to make the particles to better meet the global convergence, Sun et al. [4] introduced quantum theory and quantum evolutionary algorithm into particle swarm optimization algorithm in order to a quantum particle swarm optimization (QPSO). Because the particles in the space of quantum meet the entirely different characters of aggregation state, the movements of particles do not determine trajectory, this will make the particles to explore and find the global optimal solution in the feasible solution space. So the global search ability of QPSO algorithm is far superior to the classical PSO algorithm.

$$X_i = (X_{i1}, X_{i2}, X_{i3}, \cdots, X_{iD})$$ is the position of the $i^{th}$ particle, $P_i = (P_{i1}, P_{i2}, P_{i3}, \cdots, P_{iD})$ is the individual optimal position of the $i^{th}$ particle, $P_g = (P_{g1}, P_{g2}, P_{g3}, \cdots, P_{gD})$ is the global optimal position. Clerc and Kennedy proved that each particle in the PSO algorithm converged to its local attractor $P = (P_1, P_2, P_3, \cdots, P_D)$. The component $P_d$ of $P$ with the $d^{th}$ can be expressed as follows:

$$P_d = (\phi_1 P_{d1} + \phi_2 P_{d2}) / (\phi_1 + \phi_2)$$

In the QPSO, because the velocity and position of particle can not be determined at the same time, the velocity and position of particle can not be used to express the state of particle. The wave function $\psi(x)$ is used to express the particle state. Sun et al. used the Delta trapping quantum particle with the center of $P$ to converge to the local $P$ point. In the Delta trapping, the wave function of particle is expressed as follow:

$$\psi(x) = \frac{1}{L} \exp(-\| P - x \| / L)$$

The expression is obtained by using Carlo Monte method:

$$X(x) = P \pm \frac{L}{2} \ln(\frac{1}{n})$$

Sun et al. introduced the average optimal position $mBest$ into QPSO algorithm.

$$mBest = \frac{1}{M} \sum_{i=1}^{M} P_i = (\frac{1}{M} \sum_{i=1}^{M} P_{i1}, \frac{1}{M} \sum_{i=1}^{M} P_{i2}, \cdots, \frac{1}{M} \sum_{i=1}^{M} P_{iD})$$

The value of the parameter $L$ is given:

$$L(t+1) = 2 \times \beta \times |mBest - X(t)|$$

The iterative formula of QPSO is described:
\[
X(t + 1) = P \pm \beta \cdot |mBest(t) - X(t)| \cdot \ln \frac{1}{u}
\] (8)

Where \( M \) is population size, \( u \) a random number between 0 and 1, \( \beta \) is the contraction expansion coefficient. Experiment result shows that the algorithm can achieve better result by reducing the value of \( \beta \) from 1.0 to 0.5.

4. Improved Quantum Particle Swarm Optimization (AMCQPSO) Algorithm

4.1. Chaotic Search Method

Chaos is a kind of nonlinear phenomenon in nature. The chaotic motion can traverse all the states in a certain range according to its own rule. Chaotic search method uses the random, ergodic, regularity of these chaotic variables to optimize and search in the solution space in order to jump out the local optimal solution. And chaotic search method is significant in the small search space. But in the large search space, the effect is not satisfactory. At present, there is no strict definition for chaos, the random motion state is obtained by using the deterministic equation. Logistic mapping is a typical chaotic system, and the iterative formula is given as follows:

\[
x_{i+1} = \mu x_i (1 - x_i) \quad i = 1, 2, 3, 4, \ldots n = 2, 4
\] (9)

Where \( \mu \) is control parameter, when \( \mu = 4.0 \) and \( 0 \leq x_i \leq 1 \), Logistic is complete in a state of chaos. In this paper, the chaotic characteristics of \( \mu = 4.0 \) is used.

4.2. Adaptive Adjustment Strategy of Inertia Weight

In the QPSO, the inertia weight \( w \) has important role to determine the convergence of the algorithm. It makes the particle to keep motion inertia. The large value of \( w \) is beneficial to global search and fast convergence speed, but it is not easy to obtain the exact solution. The small value of \( w \) is beneficial to local search and obtaining more accurate solution, but it takes on the slow convergence speed. In the original version, the value of \( w \) is a constant, then the linear reducing inertia weight \( w \) with the iteration and dynamical adjusting inertia weight \( w \) based on fuzzy rule are proposed, but they can not be widely used due to the more complex implementation. So adaptive adjustment strategy of inertia weight \( w \) strategy is proposed to improve the QPSO algorithm. This strategy divides the population into three subpopulations according to the different fitness values of individuals, then each subpopulation uses the different adaptive operation. The particles with smaller weight are used for local optimization and accelerate convergence of the QPSO algorithm. The particles with larger weight are used for global optimization and jump out local optimum in the later stage. The specific strategies are described as follows:

Strategy 1: \( f_i > f_{avg} \). If the particle meets this condition, then the particle is better particle and is close to the global optimum. The small weight is given in order to strengthen the local search.

\[
w = w_{max} - \frac{f_i - f_{avg}}{f_{max} - f_{avg}} \times (w_{max} - w_{min})
\] (10)

Strategy 2: \( f_{avg} < f_i = f_{avg} \). If the particle meets this condition, then the particle is general particle in the population and takes on the better global search ability and local search ability.
\begin{equation}
\begin{aligned}
1 + \cos \frac{t_{her}}{T_{\text{max}}} - 1 \cdot \pi \\
&= w_{\text{min}} + \frac{T_{\text{max}}}{2} \times (w_{\text{max}} - w_{\text{min}})
\end{aligned}
\end{equation}

\text{Strategy 3: } f_i < f_{avg}^i. \text{ If the particle meets this condition, then the particle is poor particle. The weight is adjusted by using the control parameters } k_1 \text{ and } k_2.

w = 1.5 - \frac{1}{1 + k_1 \times \exp(-k_2 \times \mid f_g - f_{avg}^g \mid)}

\text{Where } f_i \text{ is the fitness value of the } t^\text{th} \text{ particle, } f_g \text{ is optimal fitness value of population, } f_{avg}^i \text{ is average fitness value of all particles, } f_{avg}^g \text{ is average fitness value of these particles with superior } f_{avg}. \ t_{her} \text{ is current iteration, } T_{\text{max}} \text{ is the maximum number of iteration. } \mid f_g - f_{avg} \mid \text{ is used to evaluate the premature convergence of population. If the value is smaller, then the particle is more premature convergence.}

\subsection{4.3. Neighborhood Mutation Strategy}

The particle closely moves to the optimal position of population and gradually gathers to a smaller area in the evolution process. This will reduce the diversity and search ability of population. If the global optimal position of population is local optimal solution, the premature convergence phenomenon is easily occurred. In order to improve the search efficiency of the QPSO algorithm, the optimal individual of population is randomly mutated in the generational reducing neighborhood range in order to locally fine search. If the fitness value of new individual is increased by using the neighborhood mutation strategy, the global optimal individual is replaced by new individual. Otherwise, the individual in the population is randomly replaced according to the certain probability. Set the variable } Y \text{ is mutated to get } Y'. \text{ The calculation formula is described as follows:}

\begin{equation}
Y' = Y + R_k (2r_4 - 1)
\end{equation}

\begin{equation}
R_k = (R - \overline{R}) \times (\overline{k} - k) \times \overline{k}^{-1} + \overline{R}
\end{equation}

\text{Where } R_k \text{ is the radius of neighborhood search of } k^\text{th} \text{ iteration, } \overline{R} \text{ and } R \text{ respectively are the upper bound and lower bound of radius of neighborhood search. } r_4 \text{ is uniformed random number on } [0,1].

\section{5. The Describing of AMCQPSO Algorithm}

In order to improve search ability, avoid premature convergence and falling into local optimal solutions, the chaotic search method, adaptive adjustment strategy of inertia weight and neighborhood mutation strategy are introduced into the QPSO algorithm in order to propose an improved quantum particle swarm optimization(AMCQPSO) algorithm. The chaotic search method is employed to promote the quality of initial population, enhance the search efficiency, reduce the number of blind search and improve the search efficiency. The adaptive adjustment strategy of inertia weight is used to adjust the global search ability and local search ability of particles in the running process of QPSO algorithm. The neighborhood mutation strategy is used to increase the diversity of population and avoid falling into local extremum. The steps of AMCQPSO algorithm are described as follows:
Step 1. Set the related parameters, the terminated condition, initialize the position of particles.

Step 2. Calculate the fitness value of each particle according to the objective function. Determine the convergence criterion of AMCQPSO algorithm. If the result meets the terminated condition, go to Step 9. Otherwise, Step 2 is executed.

Step 3. According to their fitness, the optimal position of individual (pB) and the optimal position of population (gB) are updated.

Step 4. The position of each particle is updated and a new population is obtained.

Step 5. The average optimal position mBest is executed chaotic optimization. The fitness value of each passed feasible solution for chaotic variables is calculated in the original solution space in order to obtain the feasible solution with better performance.

Step 6. The feasible solution is used to replace random particle in the population.

Step 7. Neighborhood mutation search is executed for the global optimal position of population.

Step 8. The corresponding strategy is used to adaptively adjust the inertia weight according to the different fitness value of particle. Then set \( t = t + 1 \) and return to Step 2.

Step 9. Output global optimal position (gB) and its fitness value.

6. Experiment Results and Analysis

In order to test the performance of proposed AMCQPSO algorithm for solving complex problem, the PSO algorithm, QPSO algorithm, CQPSO algorithm and four Benchmark functions are selected. The experiment environments are: Matlab2012b, the Pentium CPU 2.30GHz, 2.0GB RAM. The values of parameters of these algorithms could be a complicated problem itself, the change of parameters could affect the optimum value. So the selected ones are those that gave the best computational results concerning both the quality of the solution and the run time needed to achieve this solution. The obtained initial values of these parameters are: population size \( m = 30 \), learning factor \( c_1 = c_2 = 2.0 \), inertia weight \( w = 0.8 \), the inertia weight \( w \) in the AMCQPSO algorithm is adaptively adjusted in the evolution. The max iteration \( T_{\text{max}} = 500 \), mutation probability \( P_m = 0.1 \).

1. Rosenbrock function

\[
f(x) = \sum_{i=1}^{n} 100(x_{i+1} - x_i^2)^2 + (x_1 - 1)^2 \quad x \in [-100,100]
\]

where the optimal state and optimal value \( \min f_1(x^*) = f(1,1,\cdots,1) = 0 \).

2. Griewank function

\[
f_2(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 + \prod_{i=1}^{n} \cos(x_i / \sqrt{i}) + 1 \quad x \in [-600,600]
\]

where the optimal state and optimal value \( \min f_2(x^*) = f(0,0,\cdots,0) = 0 \)

3. Rastrigrin function

\[
f_3(x) = \sum_{i=1}^{n} [x_i^2 - 10 \cos(2\pi x_i) + 10] \quad x \in [-5.12,5.12]
\]

where the optimal state and optimal value \( \min f_3(x^*) = f(0,0,\cdots,0) = 0 \)
(4) Sphere function

\[ f_s(x) = \sum_{i=1}^{n} x_i^2, \quad x \in [-100,100] \]

Where the optimal state and optimal value \( \min f_s(x^*) = f(0,0,0,\ldots,0) = 0 \)

The test functions are taken as the fitness function for the PSO algorithm, QPSO algorithm, CQPSO algorithm and AMCQPSO algorithm. In order to eliminate the accidental factors, each function is independently run 30 times. The average optimal adaptive value is used as the basis of performance comparison. Finally, the experiment results are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. The Experimental Tested Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>( f_1 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( f_2 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( f_3 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( f_4 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

From Table 1, we easily discover that the proposed AMCQPSO algorithm can find the best solution for Rosenbrock function, Griewank function, Rastrigrin function and Sphere function by observing the experiment results. The AMCQPSO algorithm takes on better solving performance than the PSO algorithm, QPSO algorithm and CQPSO algorithm for Rosenbrock function, Griewank function, Rastrigrin function and Sphere function. So the experiment shows that the AMCQPSO algorithm is more capable to research for the global optimization solution and overcome the premature phenomenon for the high-dimensional function problems. So the proposed AMCQPSO algorithm is superior to the PSO, QPSO and CQPSO algorithms for function optimization problems.
7. Conclusion

The QPSO is a particle swarm optimization algorithm with quantum behavior based on the classical particle swarm optimization algorithm. It exists premature convergence, poor search ability and easy falling into local optimal solutions in solving complex optimization problem. In order to improve the quantum particle swarm optimization algorithm, an improved quantum particle swarm optimization (AMCQPSO) algorithm based on the adaptive adjustment strategy of inertia weight, chaotic search method and neighborhood mutation strategy is proposed this paper. In the AMCQPSO algorithm, under protecting the information of the global optimal particle in this iteration, the optimal particle is executed chaotic search method in order to quickly find search near the optimal particle area and quickly locate the optimal solution. Computer simulation experiments are executed, the simulation results show that the proposed AMCQPSO algorithm can greatly improve the convergence speed and accuracy. And the optimization performance is better than the PSO, QPSO and CQPSO algorithms.

Acknowledgement

The program for the initialization, study, training, and simulation of the proposed algorithm in this article was written with the tool-box of MATLAB 2010b produced by the Math-Works.

References


Authors

Lei Wang. Lecturer, received the Master degree in applied mathematics from Shandong University in 2010, Jinan, China. The main research directions: Artificial intelligence, algorithms.