

# Simple PSO Algorithm with Opposition-based Learning Average Elite Strategy

Bing AI, Ming-Gang DONG \* and Chuan-Xian JANG

*College of Information Science and Engineering, Guilin University of Technology,  
Guilin 541004, China  
d2015mg@qq.com*

## Abstract

*Due to the slow convergent speed of particle and easily get trapped in the local optima, a novel simple PSO algorithm with opposition-based learning average elite strategy is proposed. In this algorithm, a particle updating formula of the simplified swarm optimization (sPSO) algorithm is adopted. Moreover, the opposition-based learning elite strategy and Gaussian disturbance are exerted on the personal best particles and then replace personal best particle of sPSO with the average of elite opposite solutions with Gaussian disturbance of personal best particles. The adjustment of inertia weight is based on setting a threshold and then the inertia weight selects each mode adaptively according to its current state. A set of experimental results on benchmark functions demonstrate that the proposed PSO algorithm is an effective and efficient approach for optimization problems. Furthermore, the T-test analysis shows that the proposed algorithm is able to achieve better results.*

*Keywords: particle swarm optimization; elite opposition-based learning; Gaussian disturbance*

## 1. Introduction

The particle swarm optimization (PSO) algorithm is a population-based optimization technique proposed by Kennedy and Eberhart in 1995[1]. Because the main advantage of PSO is the relative simplicity of implementation and has very few parameters to be adjusted, and has already been empirically shown to perform well on many optimization problems since its advent[2][3]. However, PSO algorithm is likely to get stuck in local optimum, especially in the aspect of dealing with complex high-dimensional multi-peak optimization problems[4]. Meanwhile, PSO algorithm is diversity weakened rapidly with the increase of the iteration times, which make the convergence rate decrease considerably in the late evolution period.

In response to these circumstances, many researchers have tried to do a lot of improvements and changes on the PSO algorithm from different perspectives in order to enhance the performance of the algorithm. Shi and Eberhart introduced particle swarm optimization algorithm with linear descend inertia weight, which uses the dynamic inertia weight that decreases linearly in the light of iterative generations increasing. In[5], the opposition-based learning(OBL) was employed on the personal best positions to reconstruct them, which is helpful to enhance convergence speed. Tsai *et al.* combined the gravity search methods and PSO algorithm and gave a central role to the global optimal particle, which can improve the global convergence of the algorithm in[6]. Zhou *et al.* put forward to an improved particle swarm optimization combined with differential evolutionary mutation and elite opposition-based learning in[7].

Even though the search performance of the modified PSO algorithm has been improved to some extent, the improvement strategy of the algorithm has a certain

complexity. According to the famous No Free Lunch (NFL) theorem[8], it is impossible to design an efficient and omnipotent algorithm to solve all kinds of optimization problems in reality. Hu *et al.* analyzed the particle velocity of the standard PSO algorithm and drew a conclusion that the evolutionary process of PSO had nothing to do with particle velocity, and proposed the simplified particle swarm optimization algorithm (sPSO)[9]. The sPSO discards the particle velocity and reduces the PSO from the second order to the first order difference equation and has been proved to be an effective optimization method from literatures. But there are still some problems existed in sPSO such as this algorithm does not make full use of the characteristic of personal best particle, which can cause a loss of valid information.

OBL concept was first proposed by Tizhoosh and was successfully applied to a variety of optimization algorithms. Considering elite learning opposition-based can generate more and better opposite solutions towards global optimal point than generalized opposition-based learning. And a large amount of research work has been conducted to show that EOBL strategy is able to reflect better performance in terms of dealing with optimization problems compared with the generalized OBL [7][10]. On the basis of the advantages of elite opposition-based learning strategy, EOBL is introduced to the simple PSO algorithm and personal best positions are regarded as elite individuals. In this paper, the new algorithm referred to as the simple PSO algorithm with opposition-based learning average elite strategy (OLAE-SPSO) is established. The capability of increasing the diversity is got by performing EOBL concept for generating elite opposition solutions of personal best particle and introducing a novel Gaussian disturbance scheme. In order to fully excavate the useful information of personal best particles, the proposed algorithm replaces personal best particle of sPSO with the average of elite opposite solutions with Gaussian disturbance of personal best particles in this paper.

In addition, different methods of adjusting inertia weight will lead to different optimized results. The proposed algorithm divides states of the inertia weight into adjustment state and normal state by setting a threshold and then the inertia weight selects each mode adaptive according to its current state during the evolution process, which can reduce the calculation time of the inertia weight during a course of run, thereby remarkably enhance execution efficiency of the algorithm. Experimental results demonstrate that the proposed algorithm can effectively speed up the convergence rate, significantly enhance the robustness and accuracy as well.

The remainder of this paper is organized as follows. In Section 2, briefly describes the simple PSO algorithm. Section 3 presents our proposed OLAE-SPSO algorithm in detail, and the computation complexity of the proposed algorithm is also discussed. Section 4 performs the comprehensive experiments using benchmark functions. The experimental results are also analyzed in this section. Finally, the conclusion is drawn in Section 5, with future work also highlighted.

## 2. Basic Description of Simple PSO

The sPSO is one of the improved PSO algorithms, similarly to the process of PSO algorithm, sPSO algorithm is initialized with a population of random solutions and searches for optima by updating generations. Assume that the searching area of particle swarm which is consisted of  $NP$  particles in  $D$  dimensional space, then the best previous position along the  $d$  th dimension of particle  $i$  at iteration  $t$  is  $p_{id}^t$  and the best previous position along all the particles along the  $d$  th dimension of particle  $i$  at iteration  $t$  is  $p_{gd}^t$ . Each particle has a corresponding position and velocity. But sPSO

discards the particle velocity of the particle updating formula to help improves the convergence velocity and precision in the evolutionary optimization. Therefore, the particles are manipulated according to the following equations:

$$x_{id}^t = \omega x_{id}^{t-1} + c_1 r_1 (p_{id}^t - x_{id}^{t-1}) + c_2 r_2 (P_{gd}^t - x_{id}^{t-1}) \quad (1)$$

where  $i = 1, 2, \dots, NP$  and  $d = 1, 2, \dots, D$  and  $t$  represents the current iterative time and  $x_{id}^t$  is the

position of the  $d$ th vector of particle  $i$  in the  $t$ th iteration and  $\omega$  is the inertia weight;  $c_1$  and  $c_2$  are acceleration coefficients;  $r_1$  and  $r_2$  are evenly distributed random number in  $[0,1]$ .

### 3. Design of OLAE-SPSO

#### 3.1. The Introduction of Elite Opposition-Based Learning Strategy

As mentioned earlier, it has observed that elite opposition-based learning is capable of yielding much better search performance than the generalized opposition-based learning strategy. Therefore, according to the concept of elite opposition-based learning, by constructing the suitable search space of the elite opposition-based learning, we can search more effective area to maintain the diversity of population and provide more chance of guidance algorithm to approach the global optimal value and avoid the premature convergence phenomenon. In this paper, the personal best position is chosen as an object of the elite opposition-based learning strategy to obtain elite opposite solutions of personal best positions, so the definition of elite opposite solutions of personal best particles can be given as follows:

**Definition 1** Assume that  $p_{id}^t$  and  $op_{id}^t$  denote respectively the  $d$  th vector of the  $i$  th member of the personal best positions and their elite opposite solutions in the swarm at iteration  $t$ . For  $p_{id}^t \in [a_d^t, b_d^t]$ , the elite opposite point of  $p_{id}^t$ ,  $op_{id}^t$ , which can be expressed as follows:

$$op_{id}^t = \lambda(a_d^t + b_d^t) - p_{id}^t \quad (2)$$

where  $\lambda$  is called as generalized coefficient and is random number uniformly distributed in the range  $[0,1]$ . Accordingly,  $\lambda$  is employed to generate more opposite solutions of elite individuals. And  $a_d^t$  and  $b_d^t$  are lower and upper bounds of search area and can be obtained by using the following equation

$$\begin{cases} a_d^t = \min(x_{id}^t) \\ b_d^t = \max(x_{id}^t) \end{cases} \quad (3)$$

where  $\min(\cdot)$  and  $\max(\cdot)$  are minimize function and maximize function respectively.

In addition, to overcome the situation that the elite opposite solution  $op_{id}^t$  may jump out of the border of search space to become an infeasible solution, inspired by the concept of elite opposition-based learning, we use the reset method for  $op_{id}^t$  in this paper to enhance the performance of algorithm so as to obtain better optimization results, that is,  $op_{id}^t$  is truncated into an interval  $[2 * a_d^t - op_{id}^t, 2 * b_d^t - op_{id}^t]$  in implementations, as shown in

$$op_{id}^t = \begin{cases} 2 * a_d^t - op_{id}^t, & op_{id}^t > x_{\max} \\ 2 * b_d^t - op_{id}^t, & op_{id}^t < x_{\min} \end{cases} \quad (4)$$

where  $x_{\max}$  and  $x_{\min}$  represent the lower and upper bounds of search space, respectively.

### 3.2. The Addition of Improved Gauss Disturbance

In Reference[11], Gauss disturbance operator is integrated into the personal best positions, which is able to avoid falling into premature convergence, the ability to jump out of local optima be enhanced. On the basis of the above literature, Gaussian disturbance is added in the elite opposite-individual optimal position solutions to give them random variation, thus the new elite opposite-individual optimal position with Gaussian disturbance can be obtained, namely the elite opposite-individual optimal position solutions containing Gaussian disturbance.

One of the core problems in this section is that how to establish a suitable disturbance function. Through directly setting disturbance with a constant can help PSO find better solutions rather more quickly than do not take into account disturbance factors, there is also a potential disadvantage that the performance of convergence may decrease due to the disturbance is kept unchanged. In view of this, thus through an analysis of the properties of Gauss disturbance variables, the absolute value of individual optimal position is used as the standard deviation, this process can be written as

$$\psi = r_3 r_4 \text{gaussian}(0, |op_{id}^t|) \quad (5)$$

where the physical meaning of  $r_3$  and  $r_4$  are the same as  $r_1$ , which are responsible for the magnitude of disturbance amplitude.  $| \cdot |$  is a symbol, namely the absolute value operation.

In general, at the early stage of evolution, Gauss disturbance should have less amplitude to provide an effective global exploration of the search space in pursuit of a better solution. As the optimization progresses, the amplitude of  $\psi$  should be weakened gradually to guarantee more accurate solutions can be obtained. Especially, near of the end of iterations, small disturbance of its amplitude tends to facilitate the nearly stable exploration towards individual optimal position, so as to further improve the convergence speed of the algorithm. Therefore, according to the above description, we further update Gauss disturbance as follows

$$\psi^* = (1 - \frac{t}{T_{\max}}) r_3 r_4 \text{gaussian}(0, |op_{id}^t|) \quad (6)$$

where  $T_{\max}$  is the maximum number of iterations that the PSO is allowed to continue.

### 3.3. The Simplification of The Particle Updating Formula

The key step of PSO algorithm is the updating formula of the particle. In this procedure of structuring the updating formula of the particle, considering that particles of the swarm are mutual influence between individual during the search process. In order to fully excavate the effective search information of particles and further strengthen the interchange and sharing of information between particles in each dimension, we replace personal best particles with the average of elite opposite solutions with Gaussian disturbances of personal best particles in each iteration, which can not only maximize the absorption of the useful search information of the elite opposition solutions, but also further take full use of the advantages of the elite opposition-based learning strategy. So  $op(t)$  can be formulated as:

$$op(t) = \frac{\sum_{i=1}^{NP} [op_{id}^t + (1 - \frac{t}{T_{\max}}) r_3 r_4 \text{gaussian}(0, |op_{id}^t|)]}{NP} \quad (7)$$

According to the prior knowledge, sPSO can find a globally optimal solution with better convergence accuracy and higher evolution velocity than PSO since sPSO discards the particle velocity and reduces the PSO from the second order to the first order difference equation. Therefore, a novel method called the average elite opposition-based simple particle swarm optimization algorithm is proposed in this paper by means of the

combination of  $op(t)$  and sPSO, which make use of both the performance of enhancing the global exploration of the elite opposition-based learning strategy and excellent calculative ability of sPSO. On the basis of the analysis above, the particles are manipulated according to the following equations:

$$x_{id}^t = \omega x_{id}^t + c_1 r_1 (op(t) - x_{id}^t) + c_2 r_2 (P_{gd}^t - x_{id}^t) \quad (8)$$

### 3.4. Decreasing Inertia Weight Based On Cosine Function

It is an accepted fact that the performance of PSO can be affected considerably by the choice of inertia weight strategies. However, so far as we know, in the literature, there is no fixed inertia weight setting that achieves the best performance for all types of problems. In addition, it is clear that because the actual search process of particles is in a state of nonlinear and highly complicated, the traditional inertia weight adaptation mechanism such as the inertia weight is only adjusted in accordance with the iteration number of the overall algorithm is not a proper choice that is able to effectively reflect the actual optimization search process. Recent studies on inertia weight strategies have shown that the value of the inertia weight is dynamically regulated based on the conditions of the particles is useful in improving the performance of PSO. Therefore, in this paper, a threshold is employed to estimate the current state of the inertia weight, that is, if the difference between the current inertia weight and its minimum value is less than the threshold, then this run of the algorithm is regarded as invalidation and the current inertia weight will be set to be the minimum value. Otherwise, it is considered the state of adjustment. Meanwhile, the current inertia weight will be calculated according to time-varying inertia weight strategy in which the value of the inertia weight is determined based on the cosine function [12] with iteration number that meets the rule of PSO with inertia weight that a relatively large inertia weight is preferred to provide an effective global exploration of the search space while a relatively small inertia weight results in a faster convergence. Hence the inertia weight update rule is given as

$$\omega = \begin{cases} [(\omega_{\max} - \omega_{\min}) / 2] \cos(\pi / T_{\max}) + (\omega_{\max} + \omega_{\min}) / 2 & \Delta \geq \text{limit} \\ \omega_{\min} & \text{otherwise} \end{cases} \quad (9)$$

where  $\omega_{\min}$  and  $\omega_{\max}$  are the minimum value and the maximum value of  $\omega$  respectively and set  $\omega_{\min} = 0.4$  and  $\omega_{\max} = 0.9$  in this paper and  $t$  denotes the current iteration of the algorithm.  $\Delta = \omega - \omega_{\min}$  and  $\text{limit}$  is a threshold, which is set as 0.001.

It can be seen that the advantage of the set of a threshold is to give the algorithm a better ability to rapidly search and thus lead to a positive effect on the promotion of computation efficiency from the perspective of the analysis of time complexity.

### 3.5. Procedure of the OLAE-SPSO

According to the basic idea above, the calculation steps of the OLAE-SPSO algorithm are expressed as follows:

Step1: Initialize the particle swarm and set the maximal iterative times, and initialize the position and speed of particles within the prescribed limits.

Step2: Compute the fitness value of each particle with evaluation function. And  $p_{best}$  is kept as the previous best position of the particle swarm,  $g_{best}$  is kept as the global best position of the particle swarm.

Step3: Execute improved Gauss disturbance of personal best particles in accordance with the steps of Section 3.1 and 3.2, and obtain elite opposition-based solutions of personal best positions and use Eq.(9) calculate the inertia weight factor and update particles according to Eq.(8).

Step4: For every particle, compare its fitness value with its global optimum and choose the better one as the current global optimum.

Step5: Calculate the value of the objective function and if  $p_{best}$  is superior to  $g_{best}$ , then replace  $g_{best}$  with  $p_{best}$ .

Step6: Judge whether the updating reaches the stop criterion that the maximum iteration number is reached. If the satisfaction is got, then the search of particles of swarm is completed, output the global optimal position and its corresponding function value; otherwise, go back to Step3, until the satisfaction is got.

### 3.6. Runtime Complexity of the OLAE-SPSO

The runtime complexity of a classic PSO algorithm is  $O(NP \cdot D \cdot T_{max} \cdot C)$ , where  $C$  is the runtime complexity of process of the PSO algorithm and includes 5 times multiplication operations and 5 times operations. However, the proposed OLAE-SPSO algorithm requires 21 times multiplication operations and 21 times addition operations to accomplish. Hence, over generations, the overall runtime complexity is  $O(NP \cdot D \cdot T_{max} \cdot C')$ . Our proposed does not significantly increase the overall complexity of the original PSO algorithm anymore.

Based on the analysis, we can see that OLAE-SPSO does not impose any serious burden on the runtime complexity.

## 4 Algorithm Simulation and Analysis

### 4.1. Test Functions

In order to better evaluate the solving performance and efficacy of the proposed OLAE-SPSO algorithm, in this section, we select a suit of test functions such as Sphere, Schwefel's P2.22, Quadric, Griewank, Rastrigin and Ackley from reference[13] and [14], which can be expressed as  $f_1, f_2, f_3, f_4, f_5, f_6$ , respectively. The first three functions ( $f_1 - f_3$ ) are unimodal optimization functions which are used to investigate the convergence speed and optimization precision of the algorithms while the rest of the functions ( $f_4 - f_6$ ) are multimodal optimization functions which are used to evaluate the abilities of jumping out of local optimum and seeking the global excellent result. The unimodal optimization functions only have one extreme value point within a given search area while the multimodal optimization functions have more than one local one extreme value points within a given search area.

Considering the difference between the elite opposition-based learning strategy in the selected learning object will lead to different PSO algorithms, based on basic PSO algorithm, this paper carry out elite on personal best position and global best position respectively in order to obtain two different modified PSO algorithms and are recorded as EPSO1 and EPSO2 respectively. Compare OLAE-SPSO with EPSO1, EPSO2 and the simplified PSO based on stochastic inertia weight (SIWSPSO) in reference[16] to verify the advantage of the OLAE-SPSO algorithm proposed in this paper.

### 4.2. Experimental Parameters Settings

Considering that the different settings of algorithm parameters may also affect the comparability of the algorithms. For a fair comparison among all the improved algorithms, this paper adopts the same criteria. The number  $NP$  of particles in the swarm is fixed to 40 corresponding to the dimension 30 and the maximum allowed number of iterations is 300. Other parameters are set as follows: in the EPSO1 algorithm and EPSO2 algorithm,  $\omega = 0.9 - (0.9 - 0.4)t/300$  and  $c_1 = c_2 = 2$ ; the parameters of SIWSPSO

algorithm are set based on the recommended values in reference[16];For OLAE-SPSO,  $\omega_{\max} = 0.9$  and  $\omega_{\min} = 0.4$  and  $c_1 = 2 - (2 - 0.5)t / 300$  and  $c_2 = 0.5 + (2 - 0.5)t / 300$ , respectively.

### 4.3. Experimental Results and Discussion

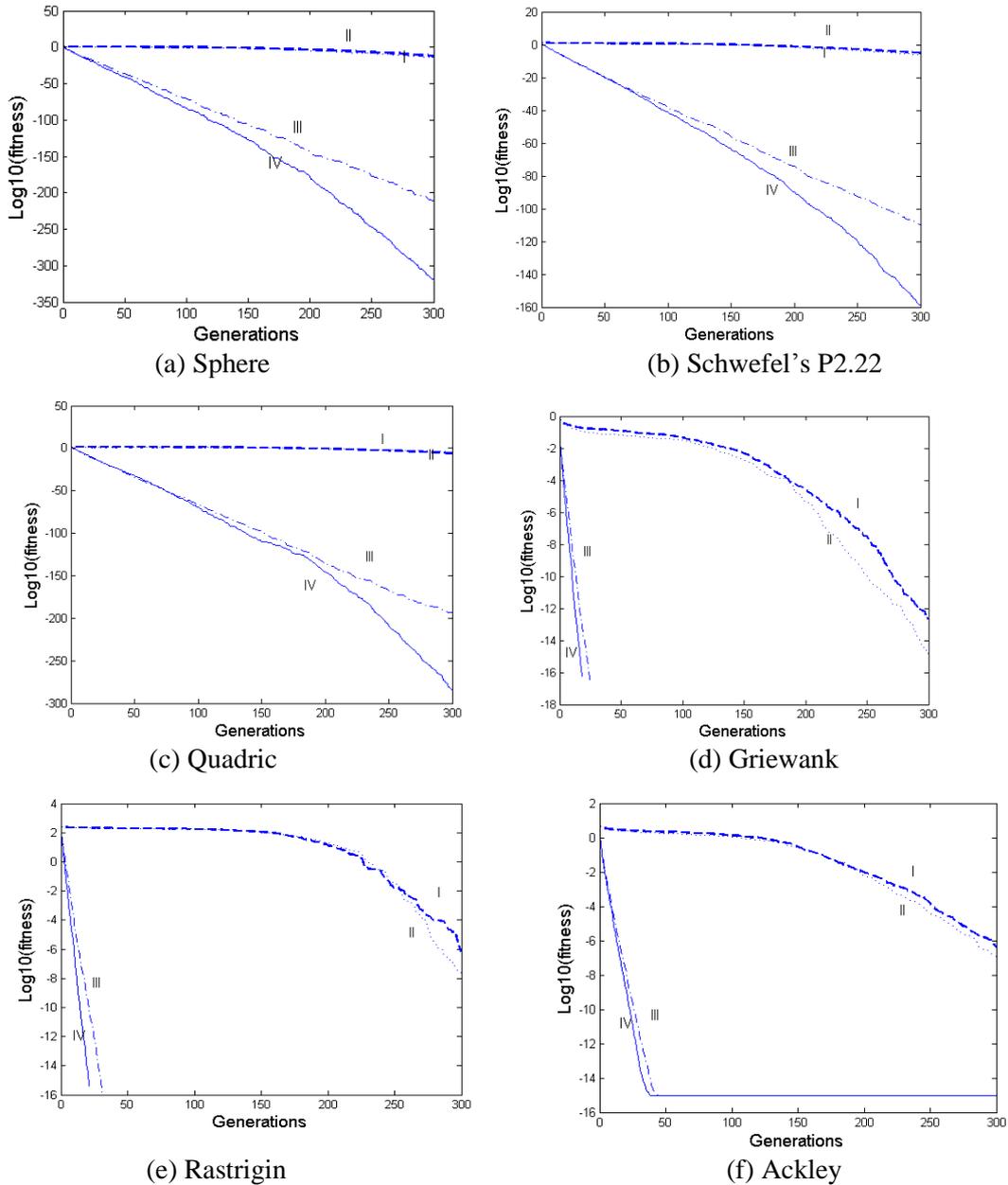
In this section, we present and discuss the experimental results obtained by the EPSO1, EPSO2 SIWSPSO and OLAE-SPSO, described in Table 2 and Figure 6.

For the above given functions of Table 1, in order to eliminate stochastic discrepancy, thereby further assuring the comparisons to be fair, each experiment is run 50 times independently and the results are averaged. The experiments are made on a 2.10 GHz Core(TM) i3-2310M CPU personal computer with 2.0 GB memory under professional Microsoft Windows 7. The best solutions, worst solutions, mean best solutions as well as variance of the best solutions are detailed in Table 2.

**Table 2. Testing Results for the Benchmark Functions Given In Table 1**

| Functions | Optimization Methods | Best Solutions | Worst Solutions | Mean Best Solutions | Variance    |
|-----------|----------------------|----------------|-----------------|---------------------|-------------|
| $f_1(x)$  | EPSO1                | 9.6723e-020    | 5.6013e-011     | 2.7183e-012         | 1.0924e-022 |
|           | EPSO2                | 2.5081e-022    | 5.8179e-014     | 2.2100e-015         | 7.8756e-029 |
|           | SIWSPSO              | 4.6250e-256    | 6.0139e-206     | 1.2569e-207         | 0           |
|           | OLAE-SPSO            | 0              | 7.9268e-319     | 1.5894e-320         | 0           |
| $f_2(x)$  | EPSO1                | 1.9709e-010    | 2.5595e-004     | 1.1326e-005         | 2.5311e-010 |
|           | EPSO2                | 1.2423e-010    | 1.8367e-006     | 1.3693e-007         | 1.9427e-012 |
|           | SIWSPSO              | 1.0406e-120    | 1.9205e-110     | 5.6113e-112         | 7.5467e-222 |
|           | OLAE-SPSO            | 3.3835e-187    | 2.6111e-157     | 5.223e-159          | 1.3363e-315 |
| $f_3(x)$  | EPSO1                | 6.2502e-016    | 1.1290e-004     | 6.0830e-006         | 5.0174e-010 |
|           | EPSO2                | 1.5021e-019    | 0.0020          | 4.0447e-005         | 7.5089e-008 |
|           | SIWSPSO              | 1.2200e-244    | 9.4025e-198     | 1.8808e-198         | 0           |
|           | OLAE-SPSO            | 0              | 3.7090e-307     | 7.4185e-309         | 0           |
| $f_4(x)$  | EPSO1                | 0              | 7.5184e-012     | 2.2482e-013         | 1.1215e-024 |
|           | EPSO2                | 0              | 3.8458e-013     | 7.9381e-015         | 2.8960e-027 |
|           | SIWSPSO              | 0              | 0               | 0                   | 0           |
|           | OLAE-SPSO            | 0              | 0               | 0                   | 0           |
| $f_5(x)$  | EPSO1                | 0              | 1.4242e-005     | 6.7723e-007         | 6.4668e-012 |
|           | EPSO2                | 0              | 9.8421e-007     | 4.2067e-008         | 2.3510e-014 |
|           | SIWSPSO              | 0              | 0               | 0                   | 0           |
|           | OLAE-SPSO            | 0              | 0               | 0                   | 0           |
| $f_6(x)$  | EPSO1                | 2.8089e-011    | 7.1540e-006     | 3.9171e-007         | 1.3799e-012 |
|           | EPSO2                | 3.8804e-011    | 9.0486e-007     | 8.8266e-008         | 3.8057e-014 |
|           | SIWSPSO              | 8.8818e-016    | 8.8818e-016     | 8.8818e-016         | 0           |
|           | OLAE-SPSO            | 8.8818e-016    | 8.8818e-016     | 8.8818e-016         | 0           |

From the analysis of Table 2, as we can see, it is obvious that the OLAE-SPSO algorithm in this paper can find the global optimal solutions for all these six benchmark functions. For the unimodal functions  $f_1 - f_3$ , the OLAE-SPSO algorithm arrives at the global optimum value of approximating to zero and demonstrates a far better average convergence precision than the EPSO1, EPSO2 and SIWSPSO. For multimodal optimization functions  $f_4 - f_6$ , the OLAE-SPSO algorithm outperforms the EPSO1 and EPSO2 and also yields the same compelling results in terms of average convergence precision and stability and robustness as SIWSPSO. Hence, the performance of the proposed OLAE-SPSO algorithm is superior to the other algorithms and can be considered as one of the most efficient improved PSO algorithms.



**Figure 1. The Mean Best Solution Evolution Curves On Six Benchmark Functions**

Figure 1 clearly illustrates that the optimization processes of six benchmark functions calculated by the proposed algorithm and the results are consistent with those shown in Table 2. It is apparent that the OLAE-SPSO algorithm has a strong ability to get away from the local optima, and it can effectively prevent the premature convergence and significantly enhance the rate and accuracy in the evolutionary process, especially during the optimization process of the complex multimodal optimization functions, it also obtain better results. Therefore, it can be concluded that the OLAE-SPSO algorithm can be well applied in the optimization problems, rapidly yielding results with sufficient accuracy.

#### 4.4. T-Test Analysis

In the previous experiments, we have verified the effectiveness of our proposed OLAE-SPSO algorithm. In this section, the T-test analysis is used to study the superiority between the results achieved by the OLAE-SPSO algorithm and the improved PSO algorithms for comparison. The statistical analysis results are provided in Table 3.

**Table 3. T-Test Result Comparisons Among Improved PSO Algorithms**

| Functions | EPSO1 | EPSO2 | SIWSPSO |
|-----------|-------|-------|---------|
| $f_1(x)$  | +     | -     | +       |
| $f_2(x)$  | +     | +     | +       |
| $f_3(x)$  | +     | +     | +       |
| $f_4(x)$  | +     | +     | +       |
| $f_5(x)$  | +     | +     | +       |
| $f_6(x)$  | =     | =     | +       |
| w/t/l     | 7/1/0 | 6/1/1 | 8/0/0   |

Values of t that are large than 1.697 imply that the proposed algorithm is significantly better, whereas t are smaller than -1.697 means significantly worse results. “+”, “-”, “=” indicate our approach is respectively better than, worse than, or similar to its competitor according to the T-test with significance level equals 0.05.

In Table 3, according to the t-test, the results are summarized as “w/t/l”, which denotes that our proposed OLAE-SPSO algorithm wins in w functions, ties in t functions, and loses in l functions, compared with EPSO1, EPSO2 and SIWSPSO. We can see that OLAE-SPSO algorithm outperforms EPSO1 algorithm in 5 functions, outperforms EPSO2 algorithm in 4 functions, and outperforms SIESPSO in all functions.

From the above comparison and analyses, we can say that, the T-test results further indicate the OLAE-SPSO can achieve better performance than other improved PSO algorithms.

## 5 Conclusion

In this paper, the OLAE-SPSO algorithm is proposed by the combination of the simple PSO algorithm. This algorithm employs the elite opposition-based learning strategy and Gaussian disturbance to personal best position to achieve better search of the solution space in pursuit of a better solution and strengthen the swarm diversity in the late evolution period, so the global convergence ability of the proposed algorithm is reinforced consequentially.

One of the crucial factors in successful implementation of the proposed algorithm is the proper choice of inertia weight. The proposed algorithm divides states of the inertia weight into adjustment state and normal state by setting a threshold and then the inertia weight adaptively selects each mode according to its current state during the evolution process, thereby improve the convergence precision and speed of the algorithm.

Experimental results and T-test analysis in Section 4 clearly demonstrate the superiority of the OLAE-SPSO algorithm over other improved PSO algorithms in terms of computational efficiency and robustness as well as the convergence rate, especially when is used to solve complex high-dimensional multimodal functions, the algorithm shows stronger global exploration and local search capacity, which provides an effective method for solving high-dimensional multimodal problems. The application of the OLAE-SPSO for dealing with complex real-world problems is also an interesting research content in the later research work.

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## Authors



**AI Bing**, He was born in 1990, M. S. candidate. His current main research interests include differential evolution algorithms, multi-objective optimization particle swarm optimization and their applications to real-world problems.