Artificial Neural Network Model for Rainfall-Runoff
-A Case Study

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Abstract

Soft computing models like Artificial Neural Network (ANN) have been widely used to model complex hydrological processes, such as rainfall-runoff and have been reported to be one of the promising tools in hydrology. In this paper, the influences of back propagation algorithm and their efficiencies which affect the input dimensions on rainfall runoff model have been demonstrated. The capability of the Artificial Neural Network with different input dimensions have been attempted and demonstrated with a case study on Sarada River Basin. The developed ANN models were able to map relationship between input and output data sets used. The developed model on rainfall and runoff pattern have been calibrated and validated. The significant input variables for training of ANN models were selected based on statistical parameters viz. cross-correlation, autocorrelation, and partial autocorrelation function. Various combinations were attempted and six combinations were selected based on the statistics of these functions. It was found those models considering rainfall lag rainfall and lag discharge as inputs were performing better than those considering rainfall alone. It was found that the neural network model developed is performing well. It can be inferred from the developed model, Neural Network model was able to predict runoff from rainfall data fairly well for a small semi-arid catchment area considered in the present study.

Keywords: Rainfall-Runoff model, Artificial Neural Network, Cross-correlation, Auto-correlation.

1. Introduction

In recent years, artificial neural network (ANNS) models were widely referred as black box models are being successfully used for modeling complex hydrological processes, such as rainfall-runoff that have been shown as viable tools in hydrology. ANN models are built upon the input and output observations. These models have the capability that even without the detail understanding of the complex physical laws that governs the process under investigation they were able to provide reasonably accurate results. The application of the method was widely adopted in hydrology. Researchers (M.A. Kaltech 2008) have also compared the performance of developed ANN models with other methods successfully and demonstrated their approach.

The merits and shortcomings of this methodology has also been discussed in review by the ASCE task committee on application of ANNs in hydrology (ASCE, 2000a, b). They have indicated that rainfall-runoff modelling has received maximum attention by ANN models. In a preliminary study, Halff et al. (1993) designed a three-layer feed-forward ANN using the rainfall hyetographs as input and hydrograph as output. This study opened up several possibilities for rainfall-runoff application using neural networks. The studies
by Smith and Eli (1995) and Kaltech (2008) may be viewed as a ‘proof of concept’ for the analysis for ANNs in rainfall runoff modelling. Subsequently, number of studies have been reported that employed neural networks for rainfall runoff modeling (Hsu et al., 1995; Tokar and Johnson, 1999; Abrahart and See, 2000). The rainfall runoff process lends itself well to ANN applications. The nonlinear nature of the relationships, availability of long historical records, and the complexity of the physical based models in this regard are some of the factors that have attracted researchers to consider alternative models in which, ANNs have been a one of the viable alternative choice.

1.1 Neural Network Model

Artificial neural networks employs a mathematical simulation approach, that adopts a biological system in order to process the acquired information and derive the output(s) after the network has been trained properly for pattern recognition. The main theme of ANN model is, it considers the brain as a parallel computational device for various computational tasks that were performed relatively poorly by traditional serial computers. The neural network structure in the present study possessed adopts a three-layer learning network consisting of an input layer, a hidden layer and an output layer consisting of output variable(s) as shown in Figure 1. The input nodes pass on the input signal values to the nodes in the hidden layer unprocessed. The values are distributed to all the nodes in the hidden layer depending on the connection weights Wij and Wjk (Najjar, Y., Ali, H., 1998) between the input node and the hidden nodes. Connection weights are the interconnecting links between the neurons in successive layers. Each neuron in a certain layer is connected to every single neuron in the next layer by links having an appropriate and an adjustable connection weight.

![Figure 1. Architecture of the Neural Network Model used in this Study](image)

In the present study, the Feed Forward Back Propagation (FFBP) algorithm was used for training using Levenberg–Marquardt optimization technique. This optimization technique is reported to be more powerful than the conventional gradient descent techniques (Y. Najjar and H. Ali, 1998). The study showed that the Marquardt algorithm is very efficient when training networks which has few hundred weights. Although the computational requirements are much higher in iterations of the Marquardt algorithm its efficiency is higher. This is especially true, when high precision is required. The Feed Forward Back Propagation (FFBP) distinguishes itself by the presence of one or more hidden layers, whose computation nodes are correspondingly called hidden neurons or hidden units. The function of hidden neurons is to intervene between the external input and the network output in useful manner.
1.2 Method of Application of ANN for Rainfall-Runoff Modelling

The runoff from a watershed outlet is a complex phenomenon and mainly related to the current rainfall rate and also may be to the past rainfall and runoff situations and several hydrological processes. In any discrete or lumped hydrological system, rainfall-runoff relationship can be generally expressed as per equation 1. (M.T. Hagan, 1994; S.J. Riad, 2004)

\[ Q(t) = F[R(t), R(t-\Delta t), ..., R(t-n_x\Delta t), Q(t-\Delta t), ..., Q(t-n_y\Delta t)] \]  

(1)

where, \( R \) represents rainfall, \( Q \) represents runoff at the outlet of the watershed, \( F \) is any kind of model structure (linear or nonlinear), \( \Delta t \) is the data sampling interval, and \( n_x \) and \( n_y \) are positive integers numbers reflecting the memory length of the watershed. In this study the Simplex search method is used to find a set of optimum values for those weights used in the ANN, which are denoted by \( W_{ij} \), \( 0 \leq i \leq n_x, 1 \leq j \leq i \) and those by \( W_{jk} \), \( 0 \leq j \leq 1, 1 \leq k \leq 1, 1 \leq k \leq m \). The estimated runoffs, denoted by \( Q(t) \), are determined as a function of those optimum weights of the ANN, which is expressed equation 2.

\[ Q(t) = F[R(t), R(t-\Delta t), ..., R(t-n_x\Delta t), Q(t-\Delta t), ..., Q(t-n_y\Delta t)] W_{ij}, W_{jk} \]  

(2)

When the ANN is implemented to approximate the above relationship between the watershed, average rainfall and runoff there will be a number of \( n = n_x + n_y + 1 \) nodes in the input layer, \( n \leq n_x + n_y + 1 \), and one node in the output, i.e. \( m=1 \).

The database collected for the present study represents ten years daily sets of rainfall-runoff values for the Sarada River Basin. The length of the data used for calibration of any model depends on data sequence length of study area and also several factors depending on the model. In the present study seven years (2001-2008) data was used for calibration and balance three years data was used for validation. The training phase of ANN model will be terminated with the mean squared error (RMSE) and later testing has been performed to achieve mininim error. The runoff flow estimation has been carried out in two steps. Initially, only rainfall data has been employed to the input layer. Later, the previous daily flow value has been incorporated as an input data. It was reported by (A.S. Tokar, 1999) that noticeable improvement in estimation performance has been obtained with the incorporation of flow value into the input layer. In the present study as well, the flow at the precedent day \( Q(t-1) \) has been added as an input to the neural network layer in order to improve the performance.

2. Study Area

The Sarada River Basin is located within 82°13'0” E & 83°05'0” longitude and 17° 25’ 0” & 18° 17’ 0” N and latitude. The total area of the study basin is around 1252.99 km². The Sarada river basin forms a part of Survey of India (SOI) sheets Nos. 65 O/1, 2, 3 and 6 and 65 K/13, 14 and 15 with a scale of 1:50000. The Index map of the study area of Sarada River Basin has been given in Figure 2. After the reconnaissance survey, the watersheds were delineated on the basis of drainage line, land slope and outlet point. Furthermore, on the basis of drainage channels and land topography, the Sarada River Basin is subdivided into five sub basin Viz., K.Kotapadu, Madugula, Chodavaram, Kasimkota and Anakapalli.
3. Model Performance

The selected basin performance has been evaluated with five performance measures to evaluate the model performance. The performance measures are Nash-Sutcliffe coefficient efficiency ($E_{NS}$), root mean square error (RMSE), mean absolute error (MAE), coefficient of determination ($R^2$) and difference in peak (DP). The Nash-Sutcliffe efficiency ($E_{NS}$) was introduced by Nash and Sutcliffe (1970) is one of the most widely used criteria for assessment of rainfall and runoff model performance. $E_{NS}$ provides a measure of the ability of a model to predict values that are different from the mean. It records as a ratio between the observed and modeled datasets and it is reported to be sensitive to differences in the observed and model means and variances, it has been strongly recommended (ASCE, 1993). RMSE and MAE will provide different type of information about the predictive capabilities of the model. The RMSE measures the goodness of fit and it is relevant to high flow values whereas the MAE is not weighted towards higher magnitude or lower magnitude events and reported to be evaluating all the deviations from the observed values, in an equal manner and regardless of sign. The coefficient of determination ($R^2$) shows the relation between observed and predicted values. Goodness of fit index is also referred as difference in peak flow (DP). The mathematical expressions of the goodness of fit indices used in the study are represented in equations 3 to 7.

(i). Nash-Sutcliffe coefficient efficiency ($E_{NS}$): It is expressed as:

$$E_{NS} = 1 - \frac{\sum_{i=1}^{n} (o_i - p_i)^2}{\sum_{i=1}^{n} (o_{\bar{i}} - p_{\bar{i}})^2} \times 100$$

where, $o_i$ and $p_i$ are the observed and predicted value, $\bar{O}_i$ is the mean of the observed flow; $n$ is the number of data points. The value of $E_{NS}$ varies between $-\infty$ to 1. The closer the value is to 1, the better the model performance.

(ii). Root mean square error (RMSE): It is expressed as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (o_i - p_i)^2}$$

(iii). Coefficient of determination: It is expressed as:

$$R^2 = \left[ \frac{\sum_{i=1}^{n} (o_i - \bar{O}_i) (p_i - \bar{P}_i)}{\sum (o_i - \bar{O}_i)^2 \Sigma_{i=1}^{n} (p_i - \bar{P}_i)^2} \right]^2$$
(iv). Mean absolute error (MAE): It is expressed as:

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |o_i - p_i|$$  \hspace{1cm} (6)

(v). Difference in Peak: It is expressed as:

$$\text{DP} = (o_i) - \text{Max}(p_i)$$  \hspace{1cm} (7)

Where, \(o_i\) is the observed value, \(\overline{o}_i\) is the mean of the observed runoff values, \(p_i\) is the estimated runoff value, \(\overline{p}_i\) is the mean of the estimated runoff values.

### 3.1 Mean Areal Rainfall

Thiessen polygon weightage method has been adopted for the analysis of rainfall data and runoff data by considering basin as five sub basins. The runoff data available at Anakapalli gauging station was used to make a rainfall-runoff relationship. The runoff data is up to Anakapalli was used to identify the rain gauge station that contributes to mean annual rainfall of the basin. The Arc-GIS software has been used to develop polygons (Figure 3) and to calculate the area of polygons for better accuracy. The Thiessen weightage for each raingauge station was calculated and used to calculate mean areal rainfall over the area. The statistic of Thiessen polygon of Sarada River Basin is presented in Table 2.

![Thiessen Polygon for the Study Area](image)

**Figure 3. Thiessen Polygon for the Study Area**

**Table 1. Thiessen Polygon Statistics**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Stations Name</th>
<th>Area (Km(^2))</th>
<th>Weightage Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Chodavaram</td>
<td>346.6</td>
<td>0.276</td>
</tr>
<tr>
<td>2</td>
<td>Madugula</td>
<td>88.54</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>K.Kotapadu</td>
<td>379.1</td>
<td>0.302</td>
</tr>
<tr>
<td>4</td>
<td>Anakapalli</td>
<td>243.24</td>
<td>0.194</td>
</tr>
<tr>
<td>5</td>
<td>Kasimkota</td>
<td>195.51</td>
<td>0.156</td>
</tr>
</tbody>
</table>
4. Results and Discussions

All the networks selected were calibrated with different combinations of input. Daily rainfall and runoff values are used as input to ANN models. The length of the data used for calibration is 7 years (2556 days) and for validation is 3 years (1096 days). Networks are tested with different number of hidden neurons and the model structures with least root mean square error (RMSE) are considered as the best structure. The ANN structure was tested for 1 to 6 hidden neurons. It can be observed that on adding hidden neurons RMSE decreases up to a certain value and again increases. Accordingly, selection of hidden neurons is done by comparing RMSE of the network. A total of six combinations of input variables were investigated for the Sarada river basin. Simulated runoff has been compared with that of observed values using performance functions like as Nash Sutcliffe efficiency and RMSE as per the equation 3.

The Goodness-of-fit statistic of each of the six developed ANN models during calibration and validation is presented in Table 2 and Table 3 respectively. The performance of the developed ANN models in terms of RMSE and Nash-Sutcliffe efficiency is depicted in Figure 4 and Figure 5 for calibration and validation respectively. It was observed from the figure that adding lag rainfall in the input with the day rainfall has improved efficiency marginally. It is also observed that considering lag rainfall and lag discharge together in input has considerably improved the performance of model (Model D compared to Model C). The best performance is achieved for model F during calibration. This model C resulted in Nash Sutcliffe efficiency of 85.9% during calibration and 68.0% during validation. It was also observed that model E resulted in Nash Sutcliffe efficiency of 78.4% during calibration and 76.7% during validation. Model E has better Nash Sutcliffe efficiency than Model F during validation period. Therefore Model E was selected for daily runoff prediction of Sarada river basin.

<p>| Table 2. Goodness-of-fit Statistics for the Observed and Predicted Daily Runoff for Gauging Station of Sarada River Basin during Calibration Period (2001-2007) |
|-----------------|-------|-------|       |       |       |</p>
<table>
<thead>
<tr>
<th>MODEL</th>
<th>ENS</th>
<th>RMSE</th>
<th>R²</th>
<th>MAE</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9.202</td>
<td>40.603</td>
<td>0.093</td>
<td>20.557</td>
<td>475.210</td>
</tr>
<tr>
<td>B</td>
<td>13.916</td>
<td>39.535</td>
<td>0.196</td>
<td>24.777</td>
<td>444.227</td>
</tr>
<tr>
<td>C</td>
<td>22.616</td>
<td>37.484</td>
<td>0.288</td>
<td>16.979</td>
<td>342.672</td>
</tr>
<tr>
<td>D</td>
<td>77.644</td>
<td>20.148</td>
<td>0.783</td>
<td>7.733</td>
<td>146.131</td>
</tr>
<tr>
<td>E</td>
<td>78.373</td>
<td>19.816</td>
<td>0.815</td>
<td>8.465</td>
<td>153.577</td>
</tr>
<tr>
<td>F</td>
<td>85.868</td>
<td>16.019</td>
<td>0.877</td>
<td>7.941</td>
<td>158.418</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ENS</th>
<th>RMSE</th>
<th>$R^2$</th>
<th>MAE</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7.116</td>
<td>16.837</td>
<td>0.096</td>
<td>9.070</td>
<td>148.684</td>
</tr>
<tr>
<td>B</td>
<td>9.631</td>
<td>16.608</td>
<td>0.159</td>
<td>8.954</td>
<td>93.858</td>
</tr>
<tr>
<td>C</td>
<td>11.921</td>
<td>16.396</td>
<td>0.129</td>
<td>9.988</td>
<td>139.033</td>
</tr>
<tr>
<td>D</td>
<td>71.700</td>
<td>9.128</td>
<td>0.754</td>
<td>5.040</td>
<td>48.112</td>
</tr>
<tr>
<td>E</td>
<td>76.684</td>
<td>8.236</td>
<td>0.783</td>
<td>4.407</td>
<td>33.818</td>
</tr>
<tr>
<td>F</td>
<td>67.997</td>
<td>9.983</td>
<td>0.729</td>
<td>4.827</td>
<td>41.077</td>
</tr>
</tbody>
</table>

Figure 3. Performance Indices during Calibration of ANN Models with Different Input Vector for Sarada River Basin

Figure 4. Performance Indices during Validation of ANN Models with Different Input Vector for Sarada River Basin
Figure 5. Comparison of Correlation Coefficients between Observed Runoff and Predicted Runoff ANN Model. (a) Calibration Period (2001-2007) (b) Validation Period (2008-2010)

The scatter plots of the observed and predicted daily runoff for basin during calibration and validation for Model E are shown along with 1:1 line in Figure 6(a) and 6(b) respectively. The simulated values for low and high runoff are slightly on the lower side of the 1:1 line during training and shows under prediction of runoff but major portion of the scatter plot is well distributed about the 1:1 line. The value of coefficient of determination ($R^2$) is found to be 0.815 during calibration and 0.783 during validation. The high $R^2$ values indicate a close relationship between the observed and predicted daily runoff by selected ANN model. It was observed from the Figure 6(b) that the predicted runoff by ANN model has fairly matched well and the trend of the observed runoff and sometimes showing slightly higher value and lower values were observed during validation period (2008-2010), but it is an acceptable deviation range. The comparison of observed and predicted values of daily runoff for the basin during validation for year 2008, 2009 and 2010 is also presented in Figure 7 for better visualization. It was observed from these figures that predicating daily runoff values for model E and F match well with the observed runoff values during validation period but model E predicted daily runoff very close to observed daily runoff.

Figure 7. Comparison of Runoff Values between Observed and ANN Model during Validation Period
5. Conclusion

The ANN model simulated daily runoff has fairly matched with the observed values. Statistical analyses have also been performed to compare the simulated daily runoff with its measured counterpart. The high coefficient of determination ($R^2$) values of 0.815 and model efficiency of 78.37% shows the close agreement between the measured and simulated runoff value during the calibration period. The coefficient of determination ($R^2$) values of 0.783 and model efficiency of 76.68% also shows the close agreement between the measured and simulated runoff value during the validation period. The low RMSE value for selected ANN model during calibration and validation also shows better prediction of peak runoff value. In this study, the results obtained show clearly that the artificial neural networks are capable of model rainfall-runoff relationship in the small semi-arid catchments in which the rainfall and runoff are very irregular, thus, confirming the general enhancement achieved by using neural networks in many other hydrological fields. The ANN approach could provide a very useful and accurate tool to solve problems in water resources studies and management.

References


