MR Image Segmentation Using Graph Cuts Based Geodesic Active Contours

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Abstract

In this paper, present a graph cuts based geodesic active contours (GAC) approach to object segmentation problems. Our method is a combination of geodesic active contours and the optimization tool of graph cuts and differs fundamentally from traditional active contours in that it uses graph cuts to iteratively deform the contour. Consequently, it has the following advantages. 1. It has the ability to jump over local minima and provide a more global result. 2. Graph cuts guarantee continuity and lead to smooth contours free of self-crossing and uneven spacing problems. Therefore, the internal force which is commonly used in traditional energy functions to control the smoothness is no longer needed, and hence the number of parameters is greatly reduced. 3 Our approach easily extends to the segmentation of three and higher dimensional objects. In addition, the algorithm is suitable for interactive correction and is shown to always converge. Experimental results and analyses are provided.

Keywords: Graph Cuts, Geodesic Active Contours, ACM, MRI, Segmentation

1. Introduction

MRI is a kind of more unobtrusive way of imaging. And short axis image for detection has high value in global and regional cardiac function, in order to effectively analysis heart function and 3-D reconstruction to accurately segmentation, left ventricular and right ventricular is necessary.

At present, there are multiple segmentation method of left ventricle, mostly based on variation method, such as Paragios [1] using variation method to realize automatic segmentation of left ventricle, ChenQiang [2] improved the Cemers [3] model to partition left ventricle contour. Variation method is, however, by solving the energy functional minimization to partition image, only get local minimum value, so this method is more sensitive to initial contour, robustness and stability are poor; Based on ASM and AAM [4] are all belong to statistical deformable model, its advantage is that it allows a degree of shape changes and can constraint on behalf of one kind of shape, but its target search is depending on the initial point set, also easily trapped in local extreme. So for cardiac MR images uneven gray, left and right ventricle are very close with other organizations around the grayscale, has weak edge, edge fracture and noise caused edge blur etc. Phenomenon, these methods all exist edge leakage phenomenon, segmentation accuracy are affected. Geometric active contour [5] based on graph theory methods to map image for weighted undirected graph, the pixel as a node, after the initial contour is determined, according to the initial contour strip, the target contour segmentation problem is converted to find the problem of minimum cut sets in the area of the strip. Image segmentation problem can be converted to the global optimization problem in the method, in order to get the global optimal solution, but because of the MR image gray unequal, weak edge, edge fracture and noise phenomenon etc., for improve the segmentation
accuracy, we introduce the shape statistics method to govern the evolution of the edge curve.

Based on the detailed analysis of the left ventricle MR image segmentation problem, in this paper, we propose the method that is geometric active contour based on graph theory combine with shape statistical.

Geometric active contour model (GAC) is an effective target automatic segmentation method. It through some constraint conditions in high dimensional space of the outline of the initial iterative evolution, to realize the image segmentation. But as a result of echocardiogram heart valves edge and texture feature is not very prominent, and it's complicated structure, even if the geometric active contour segmentation is also very difficult. A kind of effective solution is of according to the specific object segmentation, make full use of prior knowledge of the target, and under the guidance of prior knowledge to partition MR image. For the intervention of prior knowledge, make by fuzzy, shade and noise interference and is difficult to deal with the image goal, specific target segmentation also reached an unprecedented accurate and efficient [6].

This method on the basis of point distribution model through training form registration and mode analysis was carried out on the, with shapes statistics to constraints, the active contour model based on graph theory left ventricle MR image segmentation, effectively deal with curve evolution when the edge of the leakage problems, improve the segmentation accuracy.

In addition, the method not only can choose a different target or different initial contour of the same target, but also provides interactive segmentation result changes.

2. Approach

2.1. Related Graph Theory

Graph-theoretic (14) description of s-t minimum cut can be found in many graph theory textbooks. The minimum cut considered in this paper is required to separate multiple source nodes from multiple sink nodes. A simple operation on a graph G called node identification identifies a set of nodes \((V_i, V_2, ..., V_n)\) into a single new node \(V\), deleting selfloops, if any merging parallel edges, as shown in Figure 1. For \(V\) multi-source multi-sink s-t minimum cut problem, we have the following theorem.

Theorem 1: (Minimum Cut with multiple sources and multiple sinks): The minimum cut of graph G which separate a source set \(\{s_1, s_2, ..., s_n\}\) and a sink set \(\{t_1, t_2, ..., t_n\}\) is exactly the s-t minimum cut of the result graph after identifying \(\{s_1, s_2, ..., s_n\}\) to a new source s and identifying \(\{t_1, t_2, ..., t_n\}\) to a new sink t.

The proof follows as there is a one to one mapping between a cut \((S, T)\) in the original graph G that separate \(\{s_1, s_2, ..., s_n\}\) from \(\{t_1, t_2, ..., t_n\}\) and s-t cut \((S', T')\) in the result graph \(G'\) after identifying the source set to s and the sink set to t, where \(S = S' - s + \{s_1, s_2, ..., s_n\}\) and \(T = T' - t + \{t_1, t_2, ..., t_n\}\). The capacities of the corresponding cuts are same since node identification only deletes self loops which will not be on the cuts. So if a cut \((S', T')\) is a s-t minimum cut in \(G'\), its corresponding cut \((S, T)\) is a minimum cut of G that separate the source set and the sink set.

With this theorem, we can use s-t minimum cut algorithms to solve the multi-source multi-sink minimum cut problem by simply identifying the multi-source as a single source and multi-sink as a single sink respectively.

Theorem 2: (Convergence Theorem) Within finite element data set, the graph cuts based active contour will either converge or oscillate between several results with same capacity after limited iterations.
Proof: Let $C_i$ be the capacity of the result boundary $R_i$ at the $i$th time iteration, then, $C_{i+1} \leq C_i, i = 1, 2, \ldots$. Finite data yields finite number of different results $R^u, 0 < u < N$. Since the dilation process and the edge weights are well defined, we should have $R_{j+k} = R_{j+k}$, for $k = 1, 2, \ldots$ if $R_j = R_j$. If the algorithm doesn’t converge, after $N$ iterations, at least one result $R^u$ will show up twice and then the sequence between this two $R^u$ will oscillate. Also, $C_{i+1} \leq C_i, i = 1, 2, \ldots$ the capacity of each $R_i$ within this sequence will be the same.

So if one boundary shows up twice in different iterations, we can terminate the algorithm.

### 2.2 Graph Cuts based Geodesic Active Contours

The graph cuts theory discussed above provides us with a method to compute the globally optimal partition of an image after we transform it into an edge capacitated graph $(V, E)$. One such transformation is as follows. Each pixel within the image is mapped to a vertex $v \in V$. If two pixels are adjacent, there exists an edge $(u, v) \in E$ between the corresponding vertices $u$ and $v$. The edge weight $c(u, v)$ is assigned according to some measure of similarity between the two pixels: the higher the edge weight, the more similar they are [7].

Each contour is that partitions the image into two parts $S$ and $T$ corresponds to a cut $(S, T)$ on the graph. The cut corresponding to a desired object contour is in general not the global minimum among all possible cuts on the graph (for example, the contour of another, smaller object might correspond to a cut with smaller capacity). However, as explained in section 1, the desired object contour is a global minimum within its contour neighborhood (CN). Since there might be many this kind of global minima in the image, an initial contour is required to distinguish them, and the objective of our approach is to find the closest contour that is a global minimum within its contour neighborhood. Given an initial contour, our algorithm consists of the following steps:

1. Represent the image as an adjacency graph $G$.
2. Dilate current boundary into its CN with an inner boundary and an outer boundary (Figure 1).
3. Identify all the vertices corresponding to the inner boundary as a single source $s$ and identify all the vertices corresponding to the outer boundary as a single sink $t$.
4. Compute the $s$-$t$ minimum cut to obtain a new boundary that better separates the inner boundary from the outer boundary.
5. Return to step 3 until the algorithm converges.

![Figure 1. Extract Inner and Outer Contour of the CN using Dilation](image1.png)

The inner and outer boundaries are treated as the sources and sinks, respectively, in the corresponding graph.

### 2.3 Dilation

The dilation process in our approach consists of a few single binary dilation steps, whose structuring element is a $3 \times 3$ matrix with all entries set to 1 in the 2D case or a 3
\( \times 3 \times 3 \) tensor of 1 in the 3D case. Other structuring element scans can also be used. The number of single dilations in each step (also referred to as step size in this paper), determines the size of the CN. The step size is a very important parameter and is selected based on two factors: the size of the object to be segmented and the amount of noise in the data. Large size objects may have large step size. For noisy images, a large step size is required to make the active contour break away from the many local minima near the object contour. However, if the step size is too large, the active contours may skip the real object contour. Point A is the desired minimum point, and point B is another local minimum, having smaller energy than point A. If we select a small step size, the active contours will find point A for a large range of initializations. However, for the energy function, the geodesic active contours will probably get stuck at one of those local minima. On the contrary, if we select a very big step size for both cases, point B will be found, which is not desired. In our implementation, we select the step size manually according to the size of the object we want to segment and the noisiness of the data.

3. GAC Model based on Prior Shape of the Sample Registration and Model Analysis

Because there are gray uneven in the left ventricle MR images, weak edge, edge fracture and the phenomenon such as noise, when the GAC [8] based on graph theory division the left ventricle MR image will occur edge of the leak. In order to prevent leakage of curve edge, in this section, introduced statistics constraint curve shape.

In this paper, the shape of the target knowledge representation into velocity field, embedded into the level set iterative equation, drive the zero level set evolve to the ideal contour. In this paper, the main work is in Caselles V geometric active contour model based on,

\[
\frac{\partial \varphi}{\partial t} = u(x)(k + v_0)|\nabla \varphi| + \nabla u \cdot \nabla \varphi \quad u(x) = -|\nabla G_{\sigma} * I| \quad (1)
\]

\( v_0 \) does not depend on outward expansion force contour evolution of the image; \( k \) is contour curvature; \( u \) is image gradient. Under the level set, surface function \( \varphi \) in the elastic force, expansion force and image gradient force, driving the zero level evolution to the target edges.

Controlled by a priori knowledge is to add a new item in the equation of the contour evolution. New item prior knowledge with the original form new force, internal force and the external force driving outline to the ideal goal evolution. Additional prior knowledge of the force can be divided into two levels, one is the low level of regional restrictions, the zero level evolution in specific areas; another is the shape of the high level constraints, make zero convergence in specific prior shape. Hence the new iteration equation is obtained:

\[
\frac{\partial \varphi}{\partial t} = u(x)(k + v_0)|\nabla \varphi| + \nabla u \cdot \nabla \varphi + \sum F_i |\nabla \varphi| \quad (2)
\]

\( F_i \) is the target of a priori knowledge. The kind of direct, efficient speed force, but prior knowledge representation into can directly drive the evolution of zero velocity field is the key to the problem. Due to noise, motion blur and object and the background gray similarity, makes the segmentation error is difficult to avoid. In order to improve the segmentation accuracy, sometimes need to use in the process of segmentation of left ventricle prior shape knowledge. The velocity field of force near the target point had a trend to move to the target. For prior shape outline, define the point (including contour inside and outside points) to the \( C \) distance \( \varepsilon \) :

\[
\varepsilon(X) = \min(|X - X_f|), \quad X_f \in C \quad (3)
\]

Velocity field \( \varepsilon \) :
\[ F_{shape}(X) = f_s(\varepsilon) \frac{\nabla \varepsilon}{|\nabla \varepsilon|} \] (4)

Prior shape velocity field force of nature is very important. There are two kinds of the nature of the force, a kind of similar to the elastic force, more far away from the prior shape, the stronger the field force; another kind similar to the electric field force, the closer distance prior shape, the greater field force. Hope \( F_{shape} \) only to the neighbour, points to work, and the closer distance, the greater the field force. Set farthest effectively distance:

\[
f_s(\varepsilon) = \begin{cases} 
    k(\delta - \varepsilon), & \varepsilon \leq \delta \\
    0, & \varepsilon > \delta 
\end{cases} \tag{5}
\]

In the end, based on the prior shape of geometric active contour model

\[
\frac{\partial \phi}{\partial t} = u(x)(k + v_o) | \nabla \phi | + \nabla u \cdot \nabla \phi + f_s(\varepsilon) \frac{\nabla \varepsilon}{|\nabla \varepsilon|} \cdot \nabla \phi \tag{6}
\]

Through the following a variegated images based on transcendental GAC partition in the shape of the effectiveness of specific objectives. \( \delta = 15 \). In figure 2, under the constraint of prior shape, obtain more ideal target contour. Among them, target segmentation, \( \delta = 15 \).

![Figure 2: Based on the Transcendental Geometric Active Contour in the Shape of the Target Segmentation](image)

**Figure 2. Based on the Transcendental Geometric Active Contour in the Shape of the Target Segmentation**

Figure 2 Circle segmented by shape-prior-based GAC (a) circle stained by a bar and salt & pepper noise; (b) circle segmented by GAC; (c) prior circle shape; (d) circle segmented by shape-prior-based GAC.

Active contours based on graph theory method to divide the entire black, this segmentation method is only interested in the elliptical target zone. Shape due to join the a priori constraints, makes the target by shade can also be a good segmentation, target shape of translation, rotation and scaling has no impact on the segmentation results, and the method also is not sensitive to noise [9].

![Figure 3: MR Left Ventricular Internal and External Training Samples](image)

**Figure 3. MR Left Ventricular Internal and External Training Samples**

![Figure 4: Shape Constraints on the Result of Segmentation](image)

**Figure 4. Shape Constraints on the Result of Segmentation**
Figure 4 shows a method of active contours based on graph theory and methods in this chapter in the same initial contour segmentation of cardiac MR image segmentation, which reflects a priori shape plays a role in cardiac MR image segmentation. Figure 4 (a) active contours based on graph theory method of left ventricular contour segmentation, Improved methods segmentation.

Active contour segmentation method based on graph theory, edge leakage occurred when MR images of left ventricle (Figure 4 (a)), it is necessary to add shapes in the segmentation process statistical constraints [10].

3.1 Based on Graph Theory and Shape Constraint Algorithm for Segmentation of Left Ventricle MR

Based on the above training in the shape of a registration and change pattern analysis, this section proposes combining graph theory with the shape statistics of the left ventricle MR image segmentation algorithm is proposed [11, 12]. Namely in the process of evolution curve, must also will curve projection to allow space shape constraints imposed by the shape. Due to the boundary of the training sample is expressed with $N$ points, and the curve of target segmentation active contour $C$ is not necessarily the $N$ points, So when calculating the shape change vector can't directly do bad. $d\bar{x}_i = d(\bar{x}_i, \mathcal{O})$, as the shape change of the point.

3.2 Algorithm Steps of Segmentation

Step1: Select $M$ image structure training set as a sample cardiac MR images of the training focus, one by one manually split left ventricular outline, boundary shape point set $x = (x_1, x_2, ..., x_M)$ (7)

Step2: In accordance with the methods described in section 3.3 registration points set changes, calculate average shape and $\bar{x} = \sum_{i=1}^{M} \bar{x}_i$ shape change vector,

$$dx = (dx_1, dx_2, ..., dx_M), \quad dx_i = \bar{x}_i - \bar{x}; \quad (8)$$

Step3: A principal component analysis of shape vector, find the feature vector $p_x$;

Step4: According to the image constructs a weighted undirected graph and initialize contour $C$;

Step5: On the contour $C$ expands operations to form bands, network maximum flow algorithm is used to obtain the new object contour $C'$;

Step6: firstly registration $C'$ and average shape calculate $b_j = \frac{3\sqrt{\lambda_i}}{\lambda_j}$, $d\bar{x}'_j = (dx'_1, dx'_2, ..., dx'_N)$, judgment whether or not the allowable range, then $b_j > \frac{3\sqrt{\lambda_i}}{\lambda_j}$, $b_j < -\frac{3\sqrt{\lambda_i}}{\lambda_j}$, so $b_j = \frac{3\sqrt{\lambda_i}}{\lambda_j}$;

Step7: if contour curves converge, and $b_j$ in the allowable range, so calculated $C = A^{-1}(\bar{x} + p_xb_j)$ as the segmentation result; otherwise, jump to step 5

3.3 Interactive Modification of Segmentation Results

Image segmentation depends to a large extent on the subjective understanding of the image, in order to better address segmentation must also rely on other mechanisms, such as interacting with the user. Therefore, the interaction modification results are necessary. Methods this chapter can provide interactive modification of the segmentation results.
These interests in active contours have the flexibility to update the source point and the meeting point [13].

![Figure 4. Interactive Modification of Segmentation](image)

If the final result does not meet user needs, users can modify multiple times until it reaches the user satisfaction. This interactive modifications greatly facilitated human-computer interaction, improve the segmentation accuracy.

4. Experimental Methods

Figure 5 for the method and document [14] methods of segmentation results. This method effective divided the left ventricle and capture larger area. Contour segmentation results can be seen from the left ventricle, the method with high robustness in this paper.

![Figure 5. Comparison of the Method and Document](image)

We compare our results with those obtained using the traditional active contours (Figure 6), specifically, the GVF snakes, which have a large capture range and can move into boundary concavities. We use the GVF implementation available at http://iacl.ece.jhu.edu/projects/gvf/. We select the parameters of the GVF snakes as follows: $\mu = 0.1$, $\alpha = 0.05$, $\beta = 0$, $\gamma = 1$, $\kappa = 0.6$, $D_{\text{min}}=2$, and $D_{\text{max}}=4$. Shows the results of our approach and GVF Snakes approach for the same real images. Note that the initial contours provided for GVF Snakes are more accurate than these for improved GAC.

![Figure 6. The Method Comparison with GVF](image)

In this paper, some faults in a cardiac cycle of cardiac MR image segmentation. Figure 7 for left ventricular internal profile contour segmentation results, Figure 8 for left ventricular external profile contour segmentation results. Experimental show, the method in our paper has a better result for segmentation MR image.
5. Performance Analysis

In order to accurately evaluate the accuracy of this segmentation method, comparing the results with physician segmentation results by hand.

<table>
<thead>
<tr>
<th>Method</th>
<th>average</th>
<th>variance</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>HybridAAM</td>
<td>1.06</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Shape particle filtering</td>
<td>1.10</td>
<td>0.3</td>
<td>1.90</td>
</tr>
<tr>
<td>Multi-view AAM</td>
<td>1.40</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>ASM</td>
<td>1.70</td>
<td>1.1</td>
<td>3.51</td>
</tr>
<tr>
<td>Extended AAM</td>
<td>1.06</td>
<td>1.34</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. MAD Evaluation Results with Methods in the Paper (Pixels)

<table>
<thead>
<tr>
<th>Method</th>
<th>average</th>
<th>variance</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left ventricular external profile</td>
<td>0.80</td>
<td>0.11</td>
<td>1.05</td>
</tr>
<tr>
<td>Left ventricular internal profile</td>
<td>0.84</td>
<td>0.10</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Results from evaluation: method in the paper to achieve high levels of accuracy, the left ventricular internal and external contours with an average error of less than 1 pixel. The literature of [15] between the segmentation accuracy is 1-2 pixels and good stability than other methods in this chapter, segmentation error is the standard deviation of less than 0.2.

6. Conclusion

Active contours based on graph theory method is a new method of image segmentation, it maps the image for weighted undirected graph, as nodes pixels, converted in the vicinity of the initial contour segmentation problem solving min-cut problem within the band. This method converts the image segmentation problem for global optimization problems, so can get global optimal solution, but due to the presence in MR images of left ventricle papillary muscle disturbances, gray balance, weak, edge fracture and the noise fuzzy edges and other phenomena, still be marginal leakage.
phenomenon, in order to improve the segmentation accuracy statistics must also introduce shapes to constrain the evolution of curves.

For MR images of left ventricle segmentation problems, this chapter presents a combined active contour and shape of graph theory and statistics interactive segmentation of MR images of left ventricle. Taking into account active contour model based on graph theory methods split edges leak occurs when using statistics to constrain a shape based on graph theory, active contour segmentation of MR images of left ventricle. Point distribution model is used to describe the shape and training registration and change patterns of shapes. In the course of evolution curves, will Project to shape allows space to impose constraints. Prior shape constraints are due to join, making translation, rotation and scaling of the target shape has no effect on results, but not sensitive to noise. Experiments show that this method is valid for inner and outer contour segmentation of the left ventricle.

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References

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