Multivariate Statistical Kernel PCA for Nonlinear Process Fault Diagnosis in Military Barracks

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Abstract

Because of the nonlinear characteristics of monitoring system in military barracks, the traditional KPCA method either have low sensitivity or unable to detect the fault quickly and accurately. In order to make use of higher-order statistics to get more useful information and meet the requirements of real-time fault diagnosis and sensitivity, a new method of fault detection and diagnosis is proposed based on multivariate statistical kernel principal component analysis (MSKPCA), which combines statistic pattern analysis framework (SPA) and kernel principal component analysis (KPCA). First, the transformation of multivariate statistics and kernel function are conducted in which technology of moving time window is used. Then, PCA is executed to analysis the kernel function obtained from the first step. Moreover, the statistics of T^2 and SPE and the control limits of them are calculated. Finally, simulations on a typical nonlinear numerical example show that the proposed MSKPCA method is more effective than PCA and KPCA in terms of fault detection and diagnosis.

Keywords: fault diagnosis, multivariate statistics, PCA, kernel function

1. Introduction

With the development of military modernization, a large number of intelligent equipment has been used. Instead of the traditional ones, these equipment such as water meters, power meters, calorimeters can be monitored and controlled with remote computers, which has brought very much convenience to managers [1]. Many precautions are made to improve the engineering of these devices to prevent them from malfunction. However, despite all of these efforts, failures of the devices are inevitable, especially in a long-term use process [2]. These equipment failures are harmful in at least two ways: (1) They could lead the whole monitoring system to an unstable state, and have a strong impact on system normal operation; and (2) get invalid (even error) information, end up doing the wrong kind of system analysis, and finally draw wrong conclusions in fault diagnosis.

In general, the on-line fault diagnosis of devices in military barracks undergoes two stages. The first stage is manual inspection. From the late 1990s to 2008, the development of military barracks digitization is in the early discovery phase. Manual meter reading was widely used at that time. Fault diagnosis of field devices relies on inspection and maintenance of fixed time, fixed location and fixed orientation. The second stage is intelligent diagnosis phase in which the development of military barracks digitization is from pilot phase to extending application phase. How to detect and diagnose the faults in military barracks devices become an emphasis and a nodus. At this moment, the current diagnosis methods of military barracks center on using internal alarm modules without considering from a systems perspective. Therefore, the analysis, detection and diagnosis of field devices in military barracks using systematic method appears especially important.

Remote monitoring and fault diagnosis (RM-FD) attracts an increasing attention during
the past several decades [3]-[5]. A RM–FD system can be successfully designed by a large number of standard approaches. One criterion used in the past was to classify these methods into three general categories. They are quantitative model-based methods, qualitative model-based methods, and process history based methods [6]. The last one, because of need no form of model information and rely only on historic process data, has been an active research field in the control community [7].

Multivariate Statistical Process Fault Diagnosis (MSPFD) method, which is one of the most common process history based methods, because of its simplicity and effectiveness, has been extensively applied in power supply system, machinery, process monitoring and fault diagnosis in the last decade. Facing the process data, which are stored in RM-FD system, due to its ability to discriminate among classes of data, MSPFD has been widely used in the field of process fault detection.

Principal Component Analysis (PCA) is one of the effective fault diagnosis methods [8][10]. By calculating the eigenvalues of covariance matrix of the original data, PCA linearly transforms a high-dimensional input vector into a much lower one whose components are uncorrected. That is to say, by using this method, a few aggregate variables are proposed instead of the original data, in which the most information of original ones is covered. However, the linearity of original data is the prerequisite for the use of PCA. As a matter of fact, the data of RM-FD system in military barracks always contains both linear part and nonlinear part. By using this method, false alarming and missed alarming are unavoidable.

In order to solve the problem of nonlinearity, kernel-based learning method is applied to RM-FD system in recent years [11]. The kernel method is originally used for Support Vector Machine (SVM). Later, it has been generalized into many algorithms having the term of dot products such as PCA. Specifically, KPCA firstly maps the original data into a high-dimensional feature space using the kernel method and then calculates PCA in the high-dimensional feature space. Therefore, the nonlinear part of original data space can be transformed into a linear part in the high-dimensional feature space. Researches on Kernel Principal Component Analysis (KPCA) received great success when it comes to solving the problem of nonlinear problems in RM-FD [12][14]. However, the high order statistics information has been missing while using this method.

This paper mainly focuses on the transformations of data spaces, which accomplished by statistical theory and kernel function concept. Based on transform twice of raw data, a new nonlinear fault detection method called multivariate statistical kernel principal component analysis (MSKPCA) is proposed. By combining the traditional KPCA with Statistical Process Analysis (SPA), MSKPCA firstly maps the original data space into statistical sample space. Then, the second transformation is conducted by mapping statistical sample space into high dimensional kernel space. Figure 1 shows sample spaces that involved in feature extraction when using different fault detection methods. As we can see from Figure 1, MSKPCA offers two major benefits: retaining the high-order information that exist in raw data through the transformation of statistical sample space and solving nonlinear problem in statistical samples by nonlinear kernel transformation. Finally, the proposed approach will be applied to fault detection and diagnosis in a nonlinear numerical example.
2. Multivariate Statistical Sample Space Constructing

For continuous process monitoring of devices in military barracks, consider a given data matrix $X \in \mathbb{R}^{n \times m}$ which represents $m$ columns of measured variables at $n$ rows of sample points. Suppose $X_k$, which has a sliding window width of $\omega$, is the sub-sample of $X$, in which $k$ is the sequence number of samples:

$$X_k = \left[ x_1, x_2, \ldots, x_m \right] = \begin{bmatrix} x_1(k-\omega+1) & x_2(k-\omega+1) & \cdots & x_m(k-\omega+1) \\ x_1(k-\omega) & x_2(k-\omega) & \cdots & x_m(k-\omega) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(k) & x_2(k) & \cdots & x_m(k) \end{bmatrix} \quad (1)$$

Therefore, the first and second order statistics such as average $\mu_i$, variance $\upsilon_i$, correlation coefficients $\text{corr}_{ij}$, auto-correlation coefficients $\text{autocor}^d_l$ and cross correlation coefficient $\text{crosscor}^d_l$ can be represented as follows, of which $d$ is the time delay of variables.

$$\mu_i = \frac{1}{\omega} \sum_{i=0}^{\omega-1} x_i(k-1) \quad (2)$$

$$\upsilon_i = \frac{1}{\omega} \sum_{i=0}^{\omega-1} \left[ x_i(k-1) - \mu_i \right]^2 \quad (3)$$

$$\text{corr}_{ij} = \frac{1}{\omega} \sum_{i=0}^{\omega-1} \frac{[x_i(k-1)-\mu_i][x_j(k-1)-\mu_j]}{\sqrt{\upsilon_i \upsilon_j}} \quad (4)$$

$$\text{autocor}^d_l = \frac{1}{\omega-d} \sum_{i=0}^{\omega-d-1} \frac{[x_i(k-1)-\mu_i][x_i(k+d-l)-\mu_i]}{\upsilon_i} \quad (5)$$

$$\text{crosscor}^d_l = \frac{1}{\omega-d} \sum_{i=0}^{\omega-d-1} \frac{[x_i(k-1)-\mu_i][x_j(k+d-l)-\mu_j]}{\sqrt{\upsilon_i \upsilon_j}} \quad (6)$$

Besides, high order statistics such as skewness and kurtosis may also be expressed as follows:

$$\text{skewness}_i = \frac{1}{\omega} \sum_{i=0}^{\omega-1} \frac{[x_i(k-1)-\mu_i]^3}{\left( \frac{1}{\omega} \sum_{i=0}^{\omega-1} [x_i(k-1)-\mu_i]^2 \right)^{3/2}} \quad (7)$$

$$\text{kurtosis}_i = \frac{1}{\omega} \sum_{i=0}^{\omega-1} \frac{[x_i(k-1)-\mu_i]^4}{\left( \frac{1}{\omega} \sum_{i=0}^{\omega-1} [x_i(k-1)-\mu_i]^2 \right)^2} - 3 \quad (8)$$

If we line up these statistics continuously, a row vector $S_k$ called sample statistics can be obtained. Then, the sliding window of $X_k$ will be moved forward one sample at a time. This way, a new sample will replace the old one as the sample statistics of this new window. As sample statistics $S_k$ contain statistics information of different orders in current sample $X_k$ and $\omega$ samples before $X_k$, it is much easier to do the post-event analysis by
using $S_k$ instead of $X_k$. Besides, it can be observed from calculation formulas of these statistics that there is a strong and complicated nonlinear relation between different order statistics. For example, according to Eq. (3), Eq. (7) and Eq. (8), the nonlinear relationship between variance $\nu_i$ and high order statistics are as follows:

\[
\text{skewness}_i = \frac{1}{\omega} \sum_{k=0}^{n-1} \left[ x_i (k-1) - \mu_i \right]^3 / \left( \nu_i \right)^{7/2} 
\]

\[
\text{kurtosis}_i = \frac{1}{\omega} \sum_{k=0}^{n-1} \left[ x_i (k-1) - \mu_i \right]^4 / \left( \nu_i \right)^2 - 3 
\]

As such, only feature extracting of PCA is far from enough. Therefore, it makes the nonlinear kernel transformation of the sample statistics become more of a necessity.

### 3. Improved Kernel Principal Component Analysis

Most data, which is produced by devices in military barracks, contains nonlinear characters. Furthermore, to analyze the feature of samples, the high order information is also very important. After mapping the original data space into statistical sample space, the data we get and are prepare to do the transformation using kernel function successfully contain the high order statistics information. And specifically, skewness describes the asymmetry of probability density function (PDF); and kurtosis describes the non-Gaussianity of PDF, which is also effective to reduce the impacts of Gaussian noise as well. Nevertheless, the nonlinear part of original data in RM-FD system of military barracks has not been solved yet. Meanwhile, the nonlinear relationship between the new added statistics still needs to be solved. By analyzing the sample statistics using KPCA, they can be projected into high dimensional kernel space so that the nonlinear problem can be solved.

In order to observe the validity and practicability of this fault diagnosis method, $\mu, \nu, \text{skewness}, \text{kurtosis}$ are chosen instead of the correlation coefficients which describe the sequential relationship in MSKPCA. Then, the corresponding sample of statistics can be obtained from Eq. (1):

\[
S_k = [\mu \ \nu \ \text{skewness} \ \text{kurtosis}] 
\]

As time goes on, the sliding window is required to move forward to ensure the instantaneity of MSKPCA so that the latest sample of statistics could be tracked down, then the sample matrix of statistics is gained.

MSKPCA method mainly comprises the steps listed below:

1. The matrix of training dataset $X$ is processed to calculate the order statistics according to moving time window method. Permute the order statistics as a row vector, the sample of statistics $S_k$ will be obtained in order to get the matrix of sample of statistics $S \in R^{n-\omega+1 \times 4m}$. At this point, the first transformation of data space is completed;

2. Firstly, standardize matrix of sample statistics $S$ using average $\mu$ and variance $\nu$. In order to project matrix $S$ into kernel space, nonlinear kernel function mapping is used. Then, kernel matrix $K \in R^{(n-\omega+1)(n-\omega+1)}$ is given and the second transformation of data space is completed. The kernel function in this transformation is as follows:

\[
k(x,y) = \exp\left(-\frac{||x-y||^2}{c}\right) 
\]

In Eq. (12), $c$ is the width of Gaussian kernel function, which has a range of choice between $5m$~$10m$, $m$ represents the number of variables.

3. Centered matrix $K_l$ can be obtained by normalizing kernel matrix $K$ as follows:

\[
K_l = K - IK - KL - IK 
\]

Where $l = \frac{1}{n-\omega+1} \cdot \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$.

4. By extracting $P_c$ primary component, the corresponding eigenvectors are considered as load vectors. Then, the statistics of $T^2$ and SPE can be calculated by load
matrix \( P \in \mathbb{R}^{(n-\omega+1) \times P_c} \).

\[
T^2 = K_l^T P A^{-1} P^T K_l \quad (14)
\]

Where control limit of \( T^2 \) is:

\[
T^2_a = \frac{P_c (n-1)}{n-P_c} F_{\alpha} (P_c, n-P_c) \quad (15)
\]

SPE = \( r^T r, r = (I - PP^T) K_l \) \quad (16)

Where control limit of SPE is:

\[
SPE_a \sim \chi^2_0 \quad (17)
\]

In Eq. (15), \( F_{\alpha} (P_c, n-P_c) \) represents an F distribution which \( P_c \) and \( n-P_c \) are degrees of freedom, and \( \alpha \) is confidence limit. In Eq. (17), \( g = b/2a, h = 2a^2/b \), of which \( a, b \) represent the average and variance of SPE statistics of normal parts. In both statistics of \( T^2 \) and SPE, the confidence limits are set to 99%.

(5) On-line monitoring using MSKPCA. First of all, \( \omega-1 \) normal parts and new samples combine to form the first window to calculate the samples of statistics; Then, move forward every sample once time to form a new samples of statistics; Calculate \( T^2 \) and \( SPE \) of the new samples of statistics, and compare them to their confidence limits respectively to find out if there is any fault.

4. Simulation Experiment

In order to verify the validity and utility of KPCA、SPA and MSKPCA, a typical nonlinear numerical example [15]-[17] is conducted. This example includes three variables \( x_1, x_2, x_3 \), which are all generated by one factor \( t \).

\[
x_1 = t + e_1 \quad (18)
\]

\[
x_2 = t^2 - 3t + e_2 \quad (19)
\]

\[
x_3 = -t^3 + 3t^2 + e_3 \quad (20)
\]

Where \( t \in [0.01,2] \), and \( e_1, e_2, e_3 \) are independent noise variables \( \mathcal{N}(0,0.01) \). Training set which comprising 500 normal data were generated according to these formulas. Besides, test set contains 1,000 data in which series faults were artificially introduced. In this example, four faults were listed as follows:

Fault 1: A step change of \( x_2 \) by -0.4 was introduced;

Fault 2: \( x_1 \) drifted slowly by adding 0.001(k-500) to the \( x_1 \) value of each sample, where \( k \) is the sample number;

Fault 3: Parameters of \( x_3 \) were changed slightly, \( x_3 = -1.1t^3 + 3.2t^2 + e_3 \);

Fault 4: hidden variable \( t \) reduced 0.5.

In PCA, KPCA and MSKPCA, respective key parameters of these three fault diagnosis methods are provided. Based on methods of average eigenvalues, one principal component has been chosen for PCA, while three instead have been chosen for KPCA. In MSKPCA, because of the using of transformation of kernel function, the number of principal components (PCs) is larger than the number of PCs before nonlinear transformation. In this paper, twenty of PCs have been chosen for MSKPCA method. Specific key parameter settings are as indicated (see table 1). From this table, the control limits of statistics of every method are set to 99%. And kernel parameter is set to 5m, which m represents to the number of variance before kernel transformation. Besides, after repeated experiment, 12 is selected as the window width of MSKPCA method.

| Table 1. Parameter Settings of Three Methods in Nonlinear Numerical Example |
|------------------|------------------|------------------|
| Method           | PCs | Kernel Parameter | Window Width |
| PCA              | 1   | ----------       | ----------   |

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Compared with other two fault diagnosis methods, MSKPCA is of good fault detection performance. False alarm rates and missed alarm rates of respective methods can be seen from Table 2 and Table 3. As Table 2 shows, the false alarm rates of three methods are within acceptable limits which T2 statistics in MSKPCA is a little higher than others. However, from Table 3, MSKPCA is of the best performance in all four faults. In the same case, the missed alarm rate of MSKPCA is significantly less than those of PCA and KPCA. It illustrates that the addition of higher statistics not only makes the useful information been adequately digged out, but also has better efficiency to fault diagnosis.

**Table 2. False Alarm Rates of Three Methods in Different Faults in Nonlinear Numerical Example**

<table>
<thead>
<tr>
<th>Fault</th>
<th>PCA</th>
<th>KPCA</th>
<th>MSKPCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T2</td>
<td>SPE</td>
<td>T2</td>
</tr>
<tr>
<td>1</td>
<td>1.2</td>
<td>10.6</td>
<td>1.8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>8.8</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>1.4</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>12.8</td>
<td>7.2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 3. Missed Alarm Rates of Three Methods in Different Faults in Nonlinear Numerical Example**

<table>
<thead>
<tr>
<th>Fault</th>
<th>PCA</th>
<th>KPCA</th>
<th>MSKPCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T2</td>
<td>SPE</td>
<td>T2</td>
</tr>
<tr>
<td>1</td>
<td>97.4</td>
<td>4.6</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>99.8</td>
<td>48.6</td>
</tr>
<tr>
<td>3</td>
<td>17.8</td>
<td>22.3</td>
<td>41.4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3.4</td>
<td>73.8</td>
</tr>
</tbody>
</table>

Fault 1 as described earlier is a typical step fault. When PCA method is used, it is almost impossible to identify this kind of fault as shown in Figure 2. Similarly, the detection performance of KPCA method is not very well, either. A high missed alarm rate of KPCA method is demonstrated in Table 3 and Figure 3 visually. However, the proposed method, which contains the higher order statistics and goes through twice transformations, has a much better performance than the previous two methods in diagnosing fault 1. The monitoring results of MSKPCA method for fault 1 are shown in Figure 4.
Figure 2. Monitoring Results of PCA for Fault 1 in the Numerical Example
Fault 2 as described earlier is a slowly drift fault. It is not detectable in PCA method. Furthermore, this kind of fault can only be detected in KPCA method after 850 points, which is of bad real-time performance. However, using MSKPCA method, fault can be
detected after 5 points at the very beginning of fault occurring. Figure 5, Figure 6 and Figure 7 gives the monitoring results of different methods for fault 2.

![PCA (T2)](image)

![PCA (SPE)](image)

**Figure 5. Monitoring Results of PCA for Fault 2 in the Numerical Example**

![KPCA (T2)](image)

![KPCA (SPE)](image)
5. Conclusions

Fault diagnosis of devices attracts an increasing attention in recent years, as a result of growing demand for higher performance, efficient, reliability and better real-time in RM-FD system of military barracks.

In order to solve the problem of not taking advantage of higher statistics in feature
extracting while using traditional KPCA, MSKPCA method is proposed by combining statistical analysis with kernel function method. According to moving time window method, MSKPCA firstly calculates the corresponding statistics that includes higher order ones such as skewness and kurtosis every sample at a time. Then the transformation of nonlinear kernel function is used to extract features of sample statistics. By using space transformation twice, not only the higher statistics of original data have been well used, but also the nonlinear features of original data have been effectively processed.

MSKPCA is a common fault diagnosis approach of every intelligent device in military barracks as it only needs the process data in RM-FD system. Common devices that will be used a lot in military barracks such as smart power meters, water meters, calorimeters and transducers can be detected and found if faults especially nonlinear faults occur in them. By using MSKPCA in RM-FD system of military barracks, the efficiency and reliability will be improved greatly.

Finally, to validate proposed method, a nonlinear numerical example was conducted. The experiment results showed that compared with PCA and traditional KPCA method, MSKPCA has higher efficiency and better real-time character in nonlinear process fault diagnosis. Although, how to choose a proper window width and the influences caused by different window widths need to be further discussed.

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References


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