Parameters Adaptive Control of Uncertain Chaotic Systems

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Abstract

The bound of static uncertain function of chaotic system is required to be known by using adaptive method in many past research papers. Or many papers only considered the situation of the systems with simple uncertain nonlinear functions. In this paper, a kind of generalized chaotic systems are taken as an example, and an adaptive control algorithm with static uncertain nonlinear functions is proposed to control chaotic systems. The requirements for nonlinear function of system is relaxed by using the proposed algorithm. It only requires that the bound of the nonlinear functions in the system model exists and some part of it is known. At last, detailed numerical simulation is done to testify the rightness of the proposed method. It shows that the stabilization and tracking of chaotic system with static uncertain functions can be realized with a quick response by using the proposed algorithm.

Keywords: Chaotic systems; Nonlinear function; Adaptive control; Uncertainty; High-gain feedback.

1. Introduction

In the chaotic systems adaptive control field, there has been extensive research on parameters adaptive control of uncertain chaotic systems. But static uncertain function in chaotic systems only appeared in some research fruits in recent years\cite{1-4}. The static uncertain function is required to satisfy growth condition and matched condition in some literature. Later research relax the condition, and the static uncertain function is required to be known bound function and bound factors in the literature\cite{1}. And the literature\cite{1} only research single uncertain nonlinear function in the multidimensional chaotic systems. And the above research only research the chaotic systems with same structure, so the research on generalization to general chaotic systems needs further research.

Take generalized chaotic systems as an example in this paper, the adaptive control problem of static uncertain nonlinear function is researched. Relative to literature\cite{1}, the requirements of nonlinear function are further relaxed in this paper. In summary, the bound function and upper bound of nonlinear function should be known in literature\cite{1-3}, and only some of bound function should be known in this paper\cite{5-9}.
2. Problem Description

Considering the following typical chaotic system with static uncertain nonlinear function:
\[
\dot{x} = f(x) + \Delta(x) + bu
\]  
(1)

Where \( x = [x_1, \ldots, x_n]^T \), \( u = [u_1, \ldots, u_n]^T \) is \( n \) dimensional vector. Take a four dimensional system as an example, the system can be written as follows:
\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, \ldots, x_4) + \Delta_1(x_1, \ldots, x_4) + b_1u_1 \\
\dot{x}_2 &= f_2(x_1, \ldots, x_4) + \Delta_2(x_1, \ldots, x_4) + b_2u_2 \\
\dot{x}_3 &= f_3(x_1, \ldots, x_4) + \Delta_3(x_1, \ldots, x_4) + b_3u_3 \\
\dot{x}_4 &= f_4(x_1, \ldots, x_4) + \Delta_4(x_1, \ldots, x_4) + b_4u_4
\end{align*}
\]  
(2)

Where \( f(x) \) is known function and \( \Delta(x) \) is unknown function, assuming it can be described by static nonlinear function, and \( b_i \) is a known constant. In order to analyze this problem more thoroughly, we study this problem from easy to difficult, assuming the unknown part can be described by static nonlinear function [10-14].

The above model has generalized representativeness, the chaotic systems that we know can be described by this kind of form, such as Lorenz system:
\[
\begin{align*}
\dot{x}_1 &= \alpha (y - x_1) \\
\dot{y} &= \gamma (x - x_3) - y \\
\dot{z} &= x_1 - \beta z
\end{align*}
\]  
(6)

The objective of the research of stabilization is to design a control law \( u = u(x, \hat{\theta}, \hat{q}), \hat{\theta} = g(x, \hat{\theta}), \hat{q} = g(x, \hat{q}) \) such that the status can be stabilised, it means \( x \to 0 \).

The objective of the research of track is to design a control law \( u = u(x, \hat{\theta}, \hat{q}), \hat{\theta} = g(x, \hat{\theta}), \hat{q} = g(x, \hat{q}) \) such that the expectation can be tracked by the status, it means \( x \to x^d \).

Assuming the expectation is \( x_i^d \), without loss of generality, assuming the expected value is a constant, there is \( \dot{x}_i^d = 0 \), so \( x_i^d = 0 \) is a special case of tracking problem. Therefore, the problem of stabilization and track can both be described by the following model:

**Definition:** \( z_i = x_i - x_i^d \), there is:
\[
\dot{z}_i = f_i(x_1, \ldots, x_4) + \Delta_i(x_1, \ldots, x_4) + b_i u_i
\]  
(7)

3. Assumption

Four assumptions are built for the above system to simplify the analysis.

Assumption 1: for \( 1 \leq i \leq n \), there exists a unknown positive constant \( q_i^* \leq d_i \),
\[
|\Delta_i(X, t)| \leq \psi_i(X)
\]  
(8)

where \( d_i \) is a known constant, and \( \psi_i(X) \) is a known non-negative smooth function.

Because it is difficult to get \( \psi_i(X) \), we can consider the situation firstly that \( X \) is a single variable.
Assumption 2: $\Delta_i(X,t)$ satisfies the Fourier expansion, so there exists the Fourier series:

$$\Delta_i(X,t) \approx \sum_{j=0}^{\infty} a_{ij} \sin ijx + b_{ij} \cos ijx \quad (9)$$

Assumption 3: $\Delta_i(X,t)$ satisfies interval absolute integrable, so there exists:

$$\lim_{j \to \infty} a_{ij} \to 0, \quad \lim_{j \to \infty} b_{ij} \to 0$$

There exists $\varepsilon_i > 0$ for $N$ to satisfy:

$$\varepsilon_i = \left| \Delta_i(X,t) - \sum_{j=0}^{N} a_{ij} \sin ijx + b_{ij} \cos ijx \right| \quad (10)$$

then:

$$\Delta_i(X,t) < \varepsilon_i + \left| \sum_{j=0}^{N} a_{ij} \sin ijx \right| + \left| \sum_{j=0}^{N} b_{ij} \cos ijx \right| \leq \varepsilon_i + \sum_{j=0}^{N} \left| a_{ij} \sin ijx \right| + \sum_{j=0}^{N} \left| b_{ij} \cos ijx \right| \quad (11)$$

Assumption 4: choose assumption 1 $\psi_i(X) = \sum_{j=0}^{N} \left| \sin ijx \right| + \sum_{j=0}^{N} \left| \cos ijx \right| + 1 \hat{q}_i = \max(a_i, b_i, \frac{\varepsilon_i}{\max(a_i, b_i)})$

is a unknown constant.

Note: when considering the situation that $X$ is multivariate, we can let nonlinear function $\psi_i(X)$ be expressed by multivariate Fourier expansion, the specific expansion can be referred to Math, the principle is same, this is no longer described[15-17].

4. Robust Adaptive Controller Design

For the above $i$ subsystems:

$$\dot{z}_i = f_i(x_i, \ldots, x_n) + \Delta_i(x_i, \ldots, x_n) + b_iu_i \quad (12)$$

The controlled quantity can be designed as:

$$u_i = f_{i1}(x)(-f_i(x_i, \ldots, x_n) - \hat{q}_i \psi_i(x) \text{sign}(z_i)) \quad (13)$$

$$f_{i2}(x) = b_i^{-1}$$

$$f_{i3}(z_i) = k_{i1}z_i + k_{i2} \left[ \frac{z_i}{2} + \varepsilon_i \right] + k_{i3} \left[ \frac{z_i^2}{2} \right] \quad (14)$$

Then:

$$z_i \dot{z}_i = z_i[\Delta_i(x) - \hat{q}_i^* \psi_i(x) \text{sign}(z_i) - f_{i2}(z_i)] \leq -z_i f_{i2}(z_i) + |z_i| \hat{q}_i^* \psi_i(x)|z_i|$$

$$\leq -z_i f_{i2}(z_i) + |z_i| \hat{q}_i^* \psi_i(x)$$

Definition: $\hat{q}_i = \hat{q}_i - \hat{q}_i$, then $\dot{z}_i = -\hat{q}_i = \psi_i(x)|z_i|$.

Note 1: considering discontinuous switching when the sign function is eliminated, the sign function can be replaced by the mollifier, and the control law can be designed as:
\[ u_i = f_{ji}(x)[-f_j(x_i, \ldots, x_n) - \hat{q}_i \psi_i(x)] \]

\[ z_i = z_i[A_j(x) - \hat{q}_i \psi_i(x)\text{sign}(z_i)] - f_{ji}(z_i) \leq -z_i f_{ji}(z_i) + \]

\[ |z_i|g_i'(\psi_i(x) - \hat{q}_i \psi_i(x)) \left| z_i \right| + e_{bi} \]

\[ = -z_i f_{ji}(z_i) + |z_i|g_i'(\psi_i(x)) \]

Definition: \( \hat{q}_i = q_i' - \hat{q}_i \left| z_i \right| + e_{bi} \), when \( |z_i| = e_{bi}, \hat{q}_i = q_i' - \hat{q}_i \), there is \( \hat{q}_i = \hat{q}_i = \psi_i(x)\left| z_i \right| \).

Then, the system can converge to the scope of \( |z_i| \leq e_{bi} \).

Note 2: Considering discontinuous switching when the sign function is eliminated, remove the sign function directly, and the control law can be designed as:

\[ u_i = f_{ji}(x)[-f_{ji}(x_i, \ldots, x_n) - \hat{q}_i \psi_i(x)] \]

Then:

\[ z_i \dot{z}_i = z_i[A_j(x) - \hat{q}_i \psi_i(x)\text{sign}(z_i)] - f_{ji}(z_i) \leq -z_i f_{ji}(z_i) + \]

\[ |z_i|g_i'(\psi_i(x) - \hat{q}_i \psi_i(x)) \left| z_i \right| + e_{bi} \]

\[ = -z_i f_{ji}(z_i) + |z_i|g_i'(\psi_i(x)) \]

Definition:

\[ \hat{q}_i = q_i' - \hat{q}_i \text{sign}(z_i) \]

\[ \hat{q}_i = -\hat{q}_i \text{sign}(z_i) \]

Choose \( \hat{q}_i = \text{sign}(z_i)\psi_i(x)\left| z_i \right| \) according to the above situations, the selection of control law is reasonable [18-20].

5. The Approximate Analysis of Unknown Nonlinear Function

Note 3: Considering nonlinear structural information that is known, then assume:

\[ |A_j(X, t)| \leq \sum_{j=1}^{\delta} c_{ji} \psi_j(x) \]

Where \( c_{ji} \) is an unknown positive parameter, and \( \psi_j(x) \) is known nonlinear system structure information function, and the control law can be designed as:

\[ u_i = f_{ji}(x)[-f_{ji}(x_i, \ldots, x_n) - \sum_{j=1}^{\delta} c_{ji} \psi_j(x)] \]

\[ - f_{ji}(z_i)] \]

\[ \dot{c}_ji = \text{sign}(z_i)\psi_j(x)\left| z_i \right| \]

Note 4: Considering a the simplest assumption, at the interval of \( |X| < a_s \):

\[ |A_j(X, t)| \leq c_i \]

Then the control law can be designed as:
\[ u_i = f_j(x)[\dot{f}_i(x_i, \cdots, x_i) - \dot{c}_i - f_j(z_i)] \] 
\[ \dot{c}_i = z_i \] 
\[ (25) \]

And the control law can be arranged as:
\[ u_i = f_j(x)[\dot{f}_i(x_i, \cdots, x_i) - \int z_i dt - f_j(z_i)] \]
\[ (27) \]

If not through the use of high gain feedback makes:
\[ z_i \dot{z}_i = z_i[\Delta_i(x) - \int z_i dt - f_j(z_i)] > 0 \]
\[ (28) \]

Then \( z_i \) will increase, thus lead to status overflow the interval of \( |X| < a_i \), and if the assumption of \( |\Delta_i(X, t)| \leq c_i \) is invalid, thus lead to the system become diffuse.

Thus two important factors that may affect the stability of the system can be seen:

One is trying to use a high gain feedback within an allowable range for real systems, that is \( f_j(z_i) \) is designed a high gain feedback.

The other one is trying to choose known functions to describe uncertain nonlinearity to make the interval of assumption big enough [21-23].

**Note 5:** Considering a nonlinear function belows:
\[ (34) \]

If choose the assumption:
\[ |\Delta_i(X, t)| = 3 + 4x \]
\[ (29) \]

And the control law can be arranged as:
\[ u_i = f_j(x)[\dot{f}_i(x_i, \cdots, x_i) - \int z_i dt - f_j(z_i)] \]
\[ (31) \]

The assumption is available only at situation of \( x \leq (d_i - 3)/4 \).

If choose the assumption:
\[ |\Delta_i(X, t)| \leq c_i + k_i |x| \]
\[ (32) \]

And the control law can be arranged as:
\[ u_i = f_j(x)[\dot{f}_i(x_i, \cdots, x_i) - \int z_i dt - f_j(z_i)] \]
\[ (33) \]

Then choose \( c_i = 3, k_i = 4 \) so that the system can stable in hypothetical condition, so the control law can make system stable.

It is difficult to predict the uncertain part of real system, for example:
\[ |\Delta_i(X, t)| = 3 + 4x^2 \]
\[ (34) \]

Then the interval for the assumption that \( |\Delta_i(X, t)| \leq d_i \) is available is:
\[ x \leq \sqrt{(d_i - 3)/4} \]
\[ (35) \]

The interval for the assumption that \( |\Delta_i(X, t)| \leq c_i + k_i |x| \) is available is:
\[ x \leq k_i / 4 \]
\[ (36) \]

So it is not difficult to know that using the high order Taylor expansion method can improve approximation accuracy when polynomial are used to approximate the unknown nonlinear function. But the order of polynomial is higher, the gain of feedback is bigger, the real system is easier to enter saturation period, and high gain feedback is not allowed by many real systems. So there are irreconcilable conflicts for the control problem of uncertain systems.
6. Numerical Simulation

Take a four dimension system as an example to do the simulation and the model can be described as follows:

\[
\begin{align*}
\dot{x}_1 &= a(x_2 - x_1) + k_{lb} x_4 \cos x_2 + u_1 \\
\dot{x}_2 &= bx_1 - k_{lb} x_1 x_3 + k_{lb} (1 + \sin(x_2 x_3)) x_2 \\
\dot{x}_3 &= -c x_3 + h x_1^2 + k_{lb} (2 - \cos(x_2 x_3 x_4)) x_1 \\
\dot{x}_4 &= -d x_4 + k_{lb} x_3 (3 + \sin(x_1 x_4)) + u_4
\end{align*}
\]

(37)  (38)  (39)  (40)

Considering unknown nonlinear functions meet the assumptions as follows:

\[
\begin{align*}
|k_{lb} x_4 \cos x_2| &\leq q_1^* |x_4| \\
|k_{lb} (1 + \sin(x_2 x_3)) x_2| &\leq q_2^* |x_2| \\
|k_{lb} (2 - \cos(x_2 x_3 x_4)) x_1| &\leq q_3^* |x_1| \\
|k_{lb} x_3 (3 + \sin(x_1 x_4))| &\leq q_4^* |x_3|
\end{align*}
\]

(41)  (42)  (43)  (44)

When \(a = 10\), \(b = 40\), \(c = 2.5\), \(d = 10.6\), \(k = 1\), \(n = 4\), \(k_{lb} = 0.05\). The system is chaotic. Set \(x_1(0) = 1\), \(x_2(0) = -1\), \(x_3(0) = -2\), \(x_4(0) = 2\). The simulation result is as bellows:

![Figure 1. Trajectory of Uncontrolled Chaotic System (1)](image1)

![Figure 2. Trajectory of Uncontrolled Chaotic System (2)](image2)

![Figure 3. Trajectory of State x](image3)

![Figure 4. Trajectory of State y](image4)
Figure 3 to Figure 6 show trajectory of system state, Figure 7 to Figure 10 show tracking of system state, value of expectation is chosen 2. Use of detailed control parameters from appendix P3-2. It can be deduced from these simulation results that the control method in this article can make chaotic system stabilize and track quickly when there exists a static uncertain functions.
7. Conclusion

In this paper, a kind of generalized chaotic systems are taken as an example, and an adaptive control algorithm with static uncertain nonlinear functions is proposed to control chaotic systems. The requirements for nonlinear function of system is relaxed by using the proposed algorithm. It only requires that the bound of the nonlinear functions in the system model exists and some part of it is known. At last, detailed numerical simulation is done to testify the rightness and robustness of the proposed method. It shows that the stabilization and tracking of chaotic system with static uncertain functions can be realized with a quick response by using the proposed algorithm.

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