Low-carbon Closed-loop Logistics Network Design based on Interval Number Multi-attribute Decision and Queuing Theory

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Abstract

In order to solve closed-loop logistics network design problem in the low-carbon economic environment, in this paper, queuing theory is used to calculate the amount of storage of reverse logistics nodes and interval number is used to describe the uncertainty of the logistics network, a bi-objectives logistics network design model which minimum the total cost and carbon emissions of logistics network is established, the solving idea based on interval-number multiple attribute decision theory is proposed, solving algorithm based on genetic algorithm is designed. The feasibility of the model and solution algorithm is verified through case analysis; this method can meet the low-carbon closed-loop logistics network design requirements under uncertainty conditions.

Keywords: logistics network; low-carbon; interval number; multi-attribute decision; queuing theory.

1. Introduction

Currently, low-carbon economic environment requires attention to environmental and social issues. In fact, if logistics network design is scientific, it would be able to reduce energy and resource consumption, reduce environmental impact [1]. Therefore, we have to look for a better logistics network design method to meet the requirements of low-carbon economy.

Logistics network design problem (LNDP) has been a focus of the logistics research field. According to the direction of goods flow, logistics can be divided into forward logistics and reverse logistics, logistics network can also be divided into forward logistics network, reverse logistics network, and closed-loop logistics network which unify forward and reverse logistics network. In the supply chain environment, if the forward and reverse logistics networks were designed respectively, the result may be suboptimal, so we need to build a closed-loop logistics network [2]. Logistics network design is generally commenced under certain conditions, but in fact there are several uncertainties, and uncertainty is an obvious characteristic and challenge of the logistics network design [3]. This article is in the low-carbon economy and uncertain environment, looking for a closed-loop logistics network design method.

The rest of this paper is organized as follows. Section 2 presents a brief review of the literature on LNDP. Problem definition and formulation are described in Section 3 in detail. Queuing analysis of logistics nodes is described in Section 4. The proposed hybrid solution
methodology is given in Section 5. Case analysis is provided in Section 6. Finally, the paper is concluded in Section 7.

2. Literature review

LNDP has been researched for some time, and includes both determine the number of logistics facilities, location, level of technical ability, and determine the amount of transport and the transport routes between logistics nodes [4].

Lakhal et al. [5] propose a general single cycle supply chain network design model, and this can solve the general problem of distribution network design, but does not involve reverse logistics. Min et al. [6] analysis facility location-allocation problem, propose a nonlinear mixed integer programming model. Srivastava [7] set up a logistics facility location-allocation general conceptual model to minimize the total cost, it can solve the general problem of reverse logistics network design. With the deepening of the study, logistics network design needs to satisfy different aims. Du and Evans [8] establish a reverse logistics network optimization model in order to minimize the total cost and total cycle time. Pishvaae et al. [9] research on forward/reverse logistics network, and construct a bi-objective model to minimum the total cost and maximum the network response. Ramezani et al. [4] propose a multi-objective model which maximum the gross income and customer service level, minimum the raw material defect rate.

Logistics network design often faces an uncertain environment; the theories of robust etc. have been used to solve uncertainty problems in network design. Qin et al. [10] use fuzzy programming theory to establish network design model which is from three aspects which are the lowest expected cost and lowest cost under certain probability and highest reliability. Rosa et al. [11] propose a general closed-loop logistics network design model under uncertainty condition based on robust theory. At the same time, some scholars make deeper study on uncertainty. Li and Savachkin [12] consider the facilities failure, supply delay and fund constraints and other uncertain factors. Baghalian et al. [13] consider the uncertainty of the supply side and demand side, and establish network design model based on reliability theory and robust theory.

Recently queuing theory begin to be used to analysis of the uncertainty in logistics, Ishfaq and Sox [14] consider the queuing problem of logistics operation in the research of multimodal transport hub and spoke logistics network design problem. In reverse logistics stage, queue theory can also be used to analysis the inventory of logistics nodes [15-16]. But the reality may be more complex, logistics network design parameters are not necessarily meet specific probability distributions, but according to statistical data or work experience we can determine the value range of parameters. In this case, we need to use interval number to describe the uncertainty of logistics network.

At the same time, we must consider carbon emissions in low-carbon logistics network design, most existing studies convert the amount of carbon emissions to carbon emissions cost, and the aim of network design is to minimize the total cost [17-19], this is standing in the perspective of enterprise. In fact, government pays more attention to total carbon emissions of logistics network, hope to minimize the emission amount. So we need to find some new logistics network design method to control carbon emissions more directly.

For the deficiencies of the existing low-carbon logistics network design studies, the main innovations in this paper are as follows: (1) network design objectives include minimize both carbon emissions and total cost, this can seek a better balance between costs and carbon emissions; (2) describing the uncertainty of network design parameters with interval number, using queuing theory to analysis the inventory of reverse logistics node, these can enhance the applicability of logistics network design model; (3) integrating forward and reverse logistics to construct a closed-loop logistics network, this makes the network design more reasonable.
3. Problem description and proposed model

3.1. Problem description

Closed-loop logistics network is composed of logistics node and logistics line, logistics node include Manufacturer, Distribution Center (DC), Sales Center (SC), Customer Zone (CZ), Collection Center (CC), Inspection Center (IC), such as shown in Figure 1. In the forward logistics process, the freight volume between manufacturer and DC is large, and the modes of transport are coexistence, a certain percentage of cargo use road transport and other cargo use rail or water transport. In other stage, the freight volume is small and only select road transport. In forward logistics process, inventory is mainly used to meet customer demand during lead time; in the reverse logistics stage, inventory is mainly generated by the recycled products forming queue which is because of limited processing capacity.

![Figure 1. Close-loop Logistics Network](image)

From the business perspective, the target of logistics network design is to reduce the total cost which include transportation cost, fixed cost of opening a logistics node, storage cost, time penalty cost etc. From the government perspective, government hopes to minimize the amount of carbon emissions which include emissions in the logistics lines and nodes. Therefore, low-carbon closed-loop logistics network design should unify the government target and enterprise objectives, and solve the bi-objective programming problem.

3.2. Assumptions

The main assumptions considered in the problem formulation are as follows: (1) locations of logistics nodes and the traffic conditions are predefined; (2) setting a SC and CC in each customer zone; (3) each SC and CC are only responsible for one customer zone; (4) for each SC, only one CC provides service for it, and for each CC, only one IC provides service for it. (5) for each IC and IC, only be served by one manufacturer; (6) the flows entering each CC and IC may wait in a queue to receive necessary recovery service; (7) recycled products returned from CZ to CC and from CC to IC follow Poisson distribution; (8) CC and IC cargo handling time obey negative exponential distribution, CC is a single server system, IC is a multi servers system; (9) manufacturer’s production and processing capacity are unlimited; (10) there is a certain proportion between the quantity of recycled products and the demand product of the CZ.
3.3. Sets and indices

- $M$ number of Manufacturers ($m = 1, 2, \ldots, M$)
- $D$ number of Distribution Centers ($d = 1, 2, \ldots, D$)
- $S$ number of Sales Centers ($s = 1, 2, \ldots, S$)
- $Z$ number of Customer Zones ($z = 1, 2, \ldots, Z$)
- $C$ number of Collection Centers ($c = 1, 2, \ldots, C$)
- $I$ number of Inspection Centers ($i = 1, 2, \ldots, I$)
- $K$ number of transport modes ($k = 1, 2, 3$), $k=1$ means the road transport, $k=2$ means the rail transport, $k=3$ means the water transport.

3.4. Parameters

- $C_{ab}^k$ unit transportation cost from a location $a$ to a location $b$ using the $k$-th transport mode
- $G_{ab}^k$ transport distance from a location $a$ to a location $b$ using the $k$-th transport mode
- $E_{ab}^k$ unit carbon emissions from a location $a$ to a location $b$ using the $k$-th transport mode
- $Q_{ab}$ total quantity of product transported from a location $a$ to a location $b$
- $F_a$ fixed cost of opening a logistics node at location $a$ ($a \in M, D, S, Z, C, I$)
- $H_a$ unit storage cost at logistic node $a$ ($a \in D, S, C, I$)
- $E(N_a)$ expected amount of storage at logistic node $a$ ($a \in D, S, C, I$)
- $LT_{ab}$ lead time of goods sending from a location $a$ to a location $b$ ($a, b \in D, S, Z$)
- $CH_a$ storage capability of logistics node $a$ ($a \in D, S, C, I$)
- $CS_a$ unit processing capability of logistics node $a$ ($a \in C, I$)
- $E_a^k$ unit carbon emissions of storage at logistics node $a$ ($a \in D, S, C, I$)
- $J(t_{ab})$ time penalty cost function from a location $a$ to a location $b$ using the $K$-th transport mode ($a, b \in M, D$)
- $T_d$ allowed arrival time of distribution center $d$
- $\delta_d$ time penalty cost coefficient of distribution center $d$
- $PT^k$ transport prepare time of the $k$-th transport mode
- $V^k$ transport speed of the $k$-th transport mode
- $R_z$ product demand of customer zone $z$
- $R_z'$ quantity of recycled product of customer zone $z$
- $\beta_i$ utilization rate of recycled products in inspection center $i$
- $\gamma_z$ proportion of recycled products and consumer products at customer zone $z$

3.5. Decision variables

- $\alpha_{ab}^k$ ratio of goods transporting from manufacture to distribution center $d$ using the $k$-th transport mode
3.6. Network design model based on interval number

According to the research needs, we should prescribe interval number and its operation rules:

**Definition 1** Let $R$ be a real, closed interval $[\underline{x}, \bar{x}]$ is an interval number which denote by $(x)^{l}$, $\underline{x} < x < \bar{x} \in R$. If $x = \bar{x}$, then $(x)^{l}$ degenerate into a real number, if $\underline{x} = x$, then $(x)^{l}$ degenerate into a real number.

Let $(a)^{l} = [\underline{a}, \bar{a}]$, $(b)^{l} = [\underline{b}, \bar{b}]$, and $k \geq 0$, the following is operation rules:

**Rule 1**

$$(a)^{l} + (b)^{l} = [\underline{a} + \underline{b}, \bar{a} + \bar{b}]$$

**Rule 2**

$$(a)^{l} \times (b)^{l} = [\underline{a} \times \underline{b}, \bar{a} \times \bar{b}]$$

**Rule 3**

$$\frac{(a)^{l}}{(b)^{l}} = \left[ \frac{\underline{a}}{\underline{b}}, \frac{\bar{a}}{\bar{b}} \right], (b > 0)$$

**Rule 4**

$$k(a)^{l} = [k\underline{a}, k\bar{a}], k \geq 0$$, if $k = 0$, then $k(a)^{l} = 0$

**Rule 5**

$$(a)^{l} > (b)^{l}$$ if and only if $\underline{a} > \underline{b}$.

According to the definition and operation rules of interval number, the objective functions of the logistics network design model are equation 1 and 2. Using the lower bound of each interval parameters, we can get the lower bound of the objective function $Z_{\underline{Z}}$, and the upper bound $\bar{Z}$ is the calculation results of the upper bound of each interval parameters.

\[
\begin{align*}
\min(Z_{\underline{Z}}) &= \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} (\delta_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
&\quad + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
&\quad + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
\end{align*}
\]

\[
\begin{align*}
\min(Z_{\bar{Z}}) &= \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} (\delta_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
&\quad + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
&\quad + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{l} (\beta_{ij})^{m} (\gamma_{ij})^{m} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{l} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{l} + \sum_{i=1}^{m} \sum_{j=1}^{h} (\alpha_{ij})^{m} (\beta_{ij})^{m} (\gamma_{ij})^{m} \\
\end{align*}
\]

Subject to:
\[(Q_m)_{i} = (R_e, X_{m}) \quad \forall s \in (1, 2, ..., S), \forall c \in (1, 2, ..., C), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (3)

\[Q_{ee} = R_e X_{ee} \quad \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (4)

\[R_e \leftarrow R_e, \quad \forall \ell \in (1, 2, ..., \ell) \]  \quad (5)

\[(Q_m)_{i} = \sum_{s=1}^{S} \sum_{c=1}^{C} (Q_m)_{i}^{s} X_{m} \quad \forall s \in (1, 2, ..., S), \forall c \in (1, 2, ..., C), \forall \ell \in (1, 2, ..., \ell) \]  \quad (6)

\[(Q_m)_{i} = \sum_{d=1}^{D} \sum_{m=1}^{M} (Q_m)_{i}^{d} X_{m} \quad \forall d \in (1, 2, ..., D), \forall m \in (1, 2, ..., M) \]  \quad (7)

\[Q_{ee} = \sum_{s=1}^{S} \sum_{c=1}^{C} Q_{ee}^{s} X_{ee} \quad \forall s \in (1, 2, ..., S), \forall c \in (1, 2, ..., C), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (8)

\[Q_{ee} = \sum_{d=1}^{D} \sum_{m=1}^{M} Q_{ee}^{d} X_{ee} \quad \forall d \in (1, 2, ..., D), \forall m \in (1, 2, ..., M) \]  \quad (9)

\[(E(N_e))^\ell = (Q_m)_l \cdot (T_{l})^\ell \quad \forall s \in (1, 2, ..., S), \forall c \in (1, 2, ..., C), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (10)

\[(E(N_e))^\ell = (Q_m)_l \cdot (T_{l})^\ell \quad \forall d \in (1, 2, ..., D), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (11)

\[(E(N_e))^\ell = (G_m)_l \cdot (V_l)^\ell \quad \forall d \in (1, 2, ..., D), \forall m \in (1, 2, ..., M), \forall \ell \in (1, 2, ..., \ell) \]  \quad (12)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall d \in (1, 2, ..., D), \forall m \in (1, 2, ..., M), \forall \ell \in (1, 2, ..., \ell) \]  \quad (13)

\[E(N_e) \subseteq CH, \quad \forall s \in (1, 2, ..., S) \]  \quad (14)

\[E(N_e) \subseteq CH, \quad \forall c \in (1, 2, ..., C) \]  \quad (15)

\[E(N_e) \subseteq CH, \quad \forall e \in (1, 2, ..., E) \]  \quad (16)

\[E(N_e) \subseteq CH, \quad \forall \ell \in (1, 2, ..., \ell) \]  \quad (17)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall d \in (1, 2, ..., D), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (18)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall s \in (1, 2, ..., S), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (19)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall c \in (1, 2, ..., C), \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (20)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall e \in (1, 2, ..., E), \forall \ell \in (1, 2, ..., \ell) \]  \quad (21)

\[
\sum_{i=1}^{\ell} \left( \sum_{\ell=1}^{\ell} \left( \sum_{t=1}^{T_l} \right) \right) \quad \forall \ell \in (1, 2, ..., \ell) \]  \quad (22)
\[ \sum_{m=1}^{n} X_{md} = 1 \quad \forall d \in \{1, 2, \ldots, D\}, \forall m \in \{1, M\} \tag{23} \]

\[ \sum_{i=1}^{l} X_{mi} = 1 \quad \forall i \in \{1, 2, \ldots, l\}, \forall i \in \{1, M\} \tag{24} \]

\[ \sum_{m=1}^{n} X_{mi} = 1 \quad \forall i \in \{1, 2, \ldots, I\}, \forall m \in \{1, M\} \tag{25} \]

\[ \sum_{d=1}^{D} a_{ij} = 1 \quad \forall d \in \{1, 2, \ldots, D\}, \forall k \in \{1, K\} \tag{26} \]

\[ Q_{cm} \geq Q_{i} \quad \forall c \in \{1, 2, \ldots, C\}, \forall m \in \{1, M\}, \forall i \in \{1, 2, \ldots, I\} \tag{27} \]

\[ E \left( N_{j} \right) = \Lambda \quad \forall j \in \{1, 2, \ldots, S\} \tag{28} \]

In the model, objective function (1) means minimizing the total cost which include the transportation cost, fixed cost of opening a logistics node, storage cost and time penalty cost; objective function (2) means minimizing the carbon emissions which are generated during the transportation process and storage procedure.

Constraint (3) represents the interval traffic volume between SC and CZ is equal to the interval demand of CZ. Constraint (4) represents the traffic volume between CC and CZ is equal to the amount of recycled products of CZ. Constraint (5) represents that for each CZ, the amount of recycled products is equal to the upper bound of demand by the upper bound of the proportion of recycled products. Constraint (6) makes sure that the interval traffic volume between DC and SC is equal to the interval inventory of SC plus the interval traffic volume between SC and CZ. Constraint (7) makes sure that the interval traffic volume between manufacturer and DC is equal to the interval inventory of DC plus the interval traffic volume between DC and SC. Constraint (8) makes sure that the traffic volume between IC and CC is equal to the inventory of IC plus the traffic volume between CC and IC. Constraint (9) makes sure that the traffic volume between manufacturer and IC is equal to the inventory of IC plus the traffic volume between IC and CC. Constraint (10) requires that the interval inventory of SC is equal to the total interval demand of CZ during the interval lead time. Constraint (11) requires that the interval inventory of DC is equal to the total interval demand of SC during the interval lead time. Constraints (12) and (13) calculate the interval transportation time and time penalty cost of each mode between manufacturer and DC. Constraints (14)-(17) enforce the capacity restrictions at SC, DC, CC and IC respectively. Constraints (18) and (19) ensure that SC and CZ is one one corresponding. Constraints (20) and (21) ensure that CC and CZ is one one corresponding. Constraint (22) assures that for each SC, only one DC serves it. Constraint (23) assures that for each DC, only one manufacturer serves it. Constraint (24) assures that each CC only sends recycled products to one IC. Constraint (25) assures that each IC only sends recycled products to one manufacturer. Constraint (26) makes sure that the value of freight proportion is in a reasonable range. Constraints (27) and (28) represent that the traffic volume and inventory are non-negative.

4. Queuing analysis of logistics nodes

According to the hypothesis, recycled products will form a queue in CC and IC, each CC only serves one CZ, and it is \([M / M / 1 / N]\) system. The basic features of CC are as follows:
Arrival rate $\lambda_i = \sum_{c=1}^{z} R_i x_{ci}$, service rate $u_{cS_i} = CS_i$, service intensity $\rho_{ic} = \lambda_i / u_{cS_i}$, service station idle rate $\rho_{ic} = \sum_{c=1}^{z} R_i x_{ci}$, expected value of queue length $L_{ic} = (\rho_{ic}^x - 1) / (1 - \rho_{ic})$, expected inventory of CC $E(N_i) = L_{ic}$. 

Every IC serves many CCs, and IC will set a service station for each CC, so IC is $[M / M / S / N]$ system. The basic features of IC are as follows:

Arrival rate $\lambda_i = \sum_{c=1}^{z} Q_i x_c$, service rate $u_{cS_i} = CS_i$, service station number $s_i = \sum_{c=1}^{z} x_c$, service intensity $\rho_{ic} = \lambda_i / (u_{cS_i} s_i)$, service station idle rate $\rho_{ic} = \sum_{c=1}^{z} Q_i x_c s_i / (u_{cS_i} (1 - \rho_{ic}))$, expected value of queue length $L_{ic} = \rho_{ic}^x / s_i (1 - \rho_{ic})$, expected inventory of IC $E(N_i) = L_{ic}$. 

5. Proposed solution approach

5.1. Solving method

The low carbon closed-loop logistics network design model has two objectives: minimize the total cost and the carbon emissions, these two objectives can be seen as two attributes of network design decisions. Using the TOPSIS decision method mentioned in the literature [20], we can transfer the multi-objectives into multi-attribute to make decision. The basic steps are as follows:

Step 1: Generating alternative solutions sets, $X = \{x_1, x_2, \ldots, x_m\}$. 

Step 2: Establishing bi-attribute decision evaluation matrix, the decision attributes set is $Z = \{Z_1, Z_2\}$, the weight vector set is $W = \{w_1, w_2\}$, and $w_1 + w_2 = 1$. Measure every scheme $x_i$ according to attribute $Z_k$, we can get the evaluation value $(a_{ik})^L$, and the interval evaluation matrix is $(A)^L = (a_{ik})^L$. 

Step 3: Standardizing the interval evaluation matrix. The network’s total cost and carbon emissions are belong to cost index, the standardize method is as equation (30), and we can get the standard interval evaluation matrix $((E_{ik})^L = ((e_{ik})^L)$. 

$$e_{ik} = (1 / a_{ik}) / \sum_{i=1}^{n} (1 / a_{ik})$$

Step 4: Calculating the weighted standard decision matrix $(v_{ik})^L = w_1 \cdot (e_{ik})^L$, determining the positive ideal solution and negative ideal solution through equation (31), using equation (32) to compute the distance between the evaluation object and the positive and negative ideal solutions respectively.
\[
\begin{align*}
\{ v^+ \} &= \{ v^*_1, v^*_2, \ldots, v^*_n \} = \left\{ \max_{e_{ij}} \{ e_{ij} \} : i = 1, 2, \ldots, m ; k = 1, 2, \ldots, n \right\} \\
\{ v^- \} &= \{ v^-_1, v^-_2, \ldots, v^-_n \} = \left\{ \min_{e_{ij}} \{ e_{ij} \} : i = 1, 2, \ldots, m ; k = 1, 2, \ldots, n \right\}
\end{align*}
\]

(31)

\[
\begin{align*}
D_i^+ &= \frac{1}{\sqrt{2}} \left[ \sum_{i,j} \left( |L_{ij} - v^+_i| + |U_{ij} - v^+_i| \right) \right]^{1/\rho}, \quad p \geq 1 \\
D_i^- &= \frac{1}{\sqrt{2}} \left[ \sum_{i,j} \left( |L_{ij} - v^-_i| + |U_{ij} - v^-_i| \right) \right]^{1/\rho}, \quad p \geq 1
\end{align*}
\]

(32)

Step 5: Computing the relative degree of approximation \( C_i \) of each scheme through equation (33), sorting \( C_i \) and larger \( C_i \) indicates closer with positive ideal solution, so the ranking position is more front. We select the maximum \( C_i \) program as the optimal solution.

\[
C_i = \frac{D_i^+}{D_i^+ + D_i^-}, \quad \forall i \in \{1, 2, \ldots, m\}.
\]

(33)

5.2 Solving algorithm

According to the characteristics of the model, based on genetic algorithm, the solving algorithm is as follows:

5.2.1. Chromosome coding method: Chromosome is three-dimensional structure as Fig. 2. The Z axis is 1 represents the first chromosome, the X axis are 1, 2, 3 represent the first, second and third transport mode which are road, rail, and water transport. The Y axis from 1 to \( D \) indicates \( D \) DCs, three values corresponding to the row represent the \( a_{ij} \); from \((D+1)\) to \((D+D)\) indicates \( D \) DCs, every \( X=2 \) and \( X=3 \) elements of a row are 0, each \( X=1 \) element of a row is \( p \), \( p \) is an integer from 1 to \( M \), this indicates that a DC select manufacturer \( p \); from \((D+D+1)\) to \((D+D+D+I+S+C)\) indicate the logistics service provider of IC, SC, and CC respectively, and the encode mode is the same as \((D+1)\) to \((D+D)\) segment.

![Figure 2. Chromosome Coding Structure](image)

5.2.2. Genetic solving process: First, generating the initial population based on the chromosome coding structure. Second, decoding every chromosome and bringing it into the objective functions, and we can get the value of objective functions. Each chromosome corresponds to an alternative solution, according to the interval multi-attribute decision making method which has been mentioned above, we can compute the \( C_i \), and take \( C_i \) as the fitness of chromosomes. After that, using fixed proportion (gap) method, choosing chromosomes which

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have large fitness value to perform crossover, mutation, regenerating populations, and other operations. When iterations reach the given max evolution times, stop running and output the results.

5.2.3. Judging iterative effect: The fitness is a relative degree of approximation, and each genetic operation will produce new species, the relative degree of approximation within a population will change, iterations can not guarantee the fitness increasing, so stable fitness can not be used as the iterative effect judgment standard. But if the algorithm was convergent, the best chromosome should be stable, so observing the optimal chromosome’s change can determine whether the algorithm has converged.

6. Case study
An enterprise has two production plants in Hunan Province, China, 14 SCs and 14 CCs at the province’s 14 cities. Among the 14 cities, three places can build DC and IC. The unit carbon emissions during transportation use data from literature [21], and without considering the logistics nodes’ carbon emissions. The parameters of logistics network design model are as shown in Table 1~3, the demand of CZ is interval number, and other parameters are determinate number.

<table>
<thead>
<tr>
<th>Table 1. Distance between Manufacturer and DC,IC</th>
<th>Unit: km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturer 1</td>
</tr>
<tr>
<td></td>
<td>Road</td>
</tr>
<tr>
<td>DC1/IC1</td>
<td>70</td>
</tr>
<tr>
<td>DC2/IC2</td>
<td>171</td>
</tr>
<tr>
<td>DC3/IC3</td>
<td>139</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Distance and Demand</th>
<th>Unit: km,ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC, CC</td>
<td>DC1, DC2, DC3, Demand</td>
</tr>
<tr>
<td>1</td>
<td>59</td>
</tr>
<tr>
<td>2</td>
<td>172</td>
</tr>
<tr>
<td>3</td>
<td>222</td>
</tr>
<tr>
<td>4</td>
<td>182</td>
</tr>
<tr>
<td>5</td>
<td>182</td>
</tr>
<tr>
<td>6</td>
<td>182</td>
</tr>
<tr>
<td>7</td>
<td>182</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Parameters of Logistics Network Design Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Name</td>
</tr>
<tr>
<td>Unit cost of road, rail, water</td>
</tr>
<tr>
<td>Speed of road, rail, water</td>
</tr>
<tr>
<td>Prepare time of road, rail, water</td>
</tr>
<tr>
<td>Unit storage cost of DC, SC, IC, CC</td>
</tr>
<tr>
<td>Storage and process capability of CC</td>
</tr>
<tr>
<td>Storage and process capability of IC</td>
</tr>
</tbody>
</table>
Using MATLAB software, setting the initial population be composed of 100 chromosomes, the generation gap is 0.9, crossover rate is 0.85, mutation rate is 0.2, maximum number of iterations is 500, calculating results are shown as Table 4. As can be seen from the table, the network structures are consistent under the first two scenarios, but the transportation modes between manufacturer and DC have some differences. When only consider logistics cost (Scenario 1), we will choose only road transport, and this is the same with current enterprise practice. When consider the balance of logistics cost and carbon emissions (Scenario 2), we will select the coexistence of road and rail transport. When only consider carbon emissions (Scenario 3), network structure will change, and we select only rail transport between manufacturer and DC. Thus, if the government implements carbon emissions constraint, enterprises had to consider cost and carbon emissions, so the overall structure of logistics network will not basically change, but the transport mode will tend to low carbon.

The rail and water transport have certain advantages from the perspective of transportation cost, but because the transport prepare time is too long, the delivery time is too long, this create a great time penalty cost and the total transportation cost being too high. Currently Chinese government does not implement restrictions on carbon emissions, so enterprises always choose road transport.

Under optimal network structure, if only consider cost (Scenario 1), and change the Time penalty cost coefficient, we can get some results as Table 5. As can be seen, with the reduction of the coefficient, transportation mode gradually shift from road to rail or water transport. So in the transportation market, for the goods whose time request are not high, enterprises will choose the cheaper and more environment friendly transport modes.

At the same time, under different scenarios, the upper bound and lower bound of the design objectives constitute Pareto optimality respectively. The area between the upper and lower bounds of the Pareto optimal value form a Pareto optimal interval as be showed on Fig 3 and the optimal values of logistics network design will fall within this range.

### Table 4. Logistics Network Design Result Under Different Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>TOPSIS attribute calculation weight</th>
<th>Carbon emissions (10^2 ton)</th>
<th>Total logistics cost (10^6 Yuan)</th>
<th>Ratio of the transport mode of DC1</th>
<th>Ratio of the transport mode of DC2</th>
<th>Ratio of the transport mode of DC3</th>
<th>SC be served by DC1</th>
<th>SC be served by DC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.1]</td>
<td>[3.357,3.671]</td>
<td>[5.153,5.591]</td>
<td>[1,0,0]</td>
<td>[1,0,0]</td>
<td>[1,0,0]</td>
<td>1,4,5,6,7,8,12</td>
<td>3,10,13,14</td>
</tr>
<tr>
<td>2</td>
<td>[0.5,0.5]</td>
<td>[2.959,3.241]</td>
<td>[5.590,6.062]</td>
<td>[0,1,0]</td>
<td>[0,1,0]</td>
<td>[0,1,0]</td>
<td>1,4,5,6,7,8,12</td>
<td>3,10,13,14</td>
</tr>
<tr>
<td>3</td>
<td>[1.0]</td>
<td>[2.027,2.062]</td>
<td>[5.902,6.398]</td>
<td>[0,1,0]</td>
<td>[0,1,0]</td>
<td>[0,1,0]</td>
<td>1,4,5,6,7,8,12</td>
<td>3,10,13,14</td>
</tr>
</tbody>
</table>
Table 5. Logistics Network Design Result under Different Time Penalty

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1-1</th>
<th>1-2</th>
<th>1-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time penalty cost coefficient of DC</td>
<td>1.07</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
<td>Ratio of the transport mode of DC1</td>
<td>[1.0,0.1]</td>
<td>[0.9,0.1]</td>
<td>[0.8,0.1]</td>
</tr>
<tr>
<td>Ratio of the transport mode of DC2</td>
<td>[1.0,0.1]</td>
<td>[0.9,0.1]</td>
<td>[0.8,0.1]</td>
</tr>
<tr>
<td>Ratio of the transport mode of DC3</td>
<td>[1.0,0.1]</td>
<td>[0.9,0.1]</td>
<td>[0.8,0.1]</td>
</tr>
</tbody>
</table>

7. Conclusions

In the low-carbon economic environment, when design a logistics network, we not only need to focus on costs, benefits and other economics indicators, but also meet the energy saving and environment protection requirements. In this context, this study expand from one-way logistics network to closed-loop logistics network, use interval number theory to analyze network’s uncertainty, use queue theory to calculate the amount of reverse logistics. With the lowest emissions and minimum cost as the objectives, a bi-objectives closed-loop logistics network design model has been constructed based on interval number theory, solving algorithm has been proposed, and the model has been tested through case analysis. Compared with existing research, the logistics network design method presented in this paper can better adapt to uncertain environment, balance the logistics cost and carbon emissions more directly, and improve the low carbon logistics network design theory.
Acknowledgments

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References
