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Analysis Of the Total Electrical Length Extremum Of TSSIR Using Lagrange Multipliers Method

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Abstract

A systematic approach to determine the extreme value of total electrical length for tri-section stepped impedance resonator (TSSIR) is proposed in this paper. It utilizes the Lagrange multipliers method to search the extremum condition. Result demonstrates that the maximum resonator length of TSSIR is limited to triple that of the corresponding uniform impedance resonator (UIR). Spurious resonant frequency of λ/2 TSSIR is mainly analysized under the extremum condition ($\theta_1 \neq \theta_2 \neq \theta_3$). Through comparison between the proposed approach and previously assumed condition ($\theta_1 = \theta_2 = \theta_3$), the staircase discontinuity of transmission line can be reduced using the paper theory to achieve same harmonic ratio. Last, a simulated example is presented to verify correctness of the method. This paper aims to provide a theoretical basis and general guideline for the filter design using TSSIR.

Keywords: *Tri-section stepped-impedance resonator (TSSIR); Lagrange multipliers method; total electrical length; extreme value condition; staircase effect; harmonic ratio*

1. Introduction

Uniform impedance resonator (UIR) was widely used in the design of microwave filters [1]. In order to improve many shortcomings of UIR, step impedance resonator (SIR) structure changes out on this basis and applies to the circuit design [2-3]. The conventional SIR originally is proposed in [4]. Two-section SIR, which was first proposed by Makimoto and Yamashita in 1980.

To achieve better filter characteristics and add design flexibility, tri-section stepped impedance resonator (TSSIR) is used to design filter [2]-[4]. The dual-band trisection SIR can provide the desired dual-band response, and the lowpass fillter can improve the out-of-band performance by suppressing the armonics and spurious responses[5].

It can It is well known that the maximum or minimum total electrical length of SIR is obtained in the case of the same electric length ($\theta_1=\theta_2$) [6]. However, to TSSIR, most of designs only assume that each of the electrical length was equal for facilitating calculation without concrete proof about the extreme condition. For instance, a tri-band microstrip bandpass filter using spur-line-loaded TSSIR was proposed with assuming all three sections of the TSSIR have the same electric length ($\theta_1=\theta_2=\theta_3$) [7]; Whereas in[8], the electrical length of two sections for TSSIR were assumed to be same but different with the third section ($\theta_1=\theta_2\neq\theta_3$) for gaining relationship of the total electrical length with θ_1. Thus, the minimum value of total electrical length can be determined. A dual-band bandstop filter design using TSSIR with three different characteristic impedances and electric lengths ($Z_1>Z_3>Z_2$, $\theta_1\neq\theta_2\neq\theta_3$) was proposed in [9] for reducing circuit size. In [10], the fundamental resonance and spurious responses of a symmetrical quad-section stepped impedance resonator (QSIR) could be derived in the equal section electric length condition.

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In section 2, the Lagrange multiplier method extremum principle is applied to determine the extreme value of TSSIR total electrical length when is resonating. In section 3, harmonic characteristic about λ/2 TSSIR is analysized with unequal electrical lengths and equal electrical lengths [11]. Under the extremum condition, the first and second harmonic ratio can be smaller. Finally, to verify the design formulas one experimental bandpass filters is designed and fabricated. The experimental performance data are shown to be in close agreement with the design data.

2. Analysis of Total Electrical Length Extremum of Tssir

In this section, Lagrange multipliers method is used to analyze and calculate the extreme value about total electrical length of TSSIR that satisfied the resonance condition.

Without loss of generality, the geometry of a unit TSSIR considered in this letter is depicted in Figure 1.

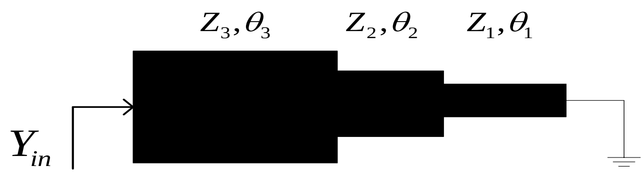


Figure 1. The General Geometry of the Unit TSSIR

The structure consists of three different characteristic impedance lines Z_1, Z_2 and Z_3, with the corresponding electrical length given by θ_1, θ_2 and θ_3, respectively. In Figure 1, the admittance, Y_{in}, can be written as (1).

$$Y_{in} = \frac{Z_2Z_3 - Z_1Z_3\tan\theta_1\tan\theta_2 - Z_1Z_2\tan\theta_1\tan\theta_3 - Z_2^2\tan\theta_2\tan\theta_3}{jZ_3\left(Z_1Z_2\tan\theta_1 + Z_2^2\tan\theta_2 + Z_2Z_3\tan\theta_3 - Z_1Z_3\tan\theta_1\tan\theta_2\tan\theta_3\right)} \qquad (1)$$

At resonance, the input admittance becomes zero ($Y_{in} = 0$). To facilitate analysis, two impedance ratios, $K_1=Z_2/Z_1$ and $K_2=Z_3/Z_2$ are defined. As the TSSIR will degenerate to a UIR in the case of both K_1=1 and K_2=1 and degenerate to a two-section SIR with K_1=1 or K_2=1, and both structures have been extensively studied and reported, they will not be discussed in this paper. Thus, $K_1 \neq 1$ and $K_2 \neq 1$ are assumed throughout this work. Y_{in}, can be written as

$$Y_{in} = \frac{1 - \frac{1}{K_1}\tan\theta_1\tan\theta_2 - \frac{1}{K_1K_2}\tan\theta_1\tan\theta_3 - \frac{1}{K_2}\tan\theta_2\tan\theta_3}{jZ_3\left(\frac{1}{K_1K_2}\tan\theta_1 + \frac{1}{K_2}\tan\theta_2 + \tan\theta_3 - \frac{1}{K_1}\tan\theta_1\tan\theta_2\tan\theta_3\right)} \qquad (2)$$

The fundamental resonance condition can be expressed as

$$1 - \frac{1}{K_1}\tan\theta_1\tan\theta_2 - \frac{1}{K_1K_2}\tan\theta_1\tan\theta_3 - \frac{1}{K_2}\tan\theta_2\tan\theta_3 = 0 \qquad (3)$$

It is not clair to search for the extreme conditions of total electrical length for TSSIR from one equation with two variables.

Total electrical length between the ends of TSSIR can be expressed as

$$f\left(\theta_1, \theta_2, \theta_3\right) = \theta_1 + \theta_2 + \theta_3$$

(4)

Set

$$L(\theta_1,\theta_2,\theta_3) = f(\theta_1,\theta_2,\theta_3) + \lambda\varphi(\theta_1,\theta_2,\theta_3)$$
(5)

Which,

$$\varphi(\theta_1,\theta_2,\theta_3)=1-\frac{1}{K_1}\tan\theta_1\tan\theta_2-\frac{1}{K_1K_2}\tan\theta_1\tan\theta_3-\frac{1}{K_2}\tan\theta_2\tan\theta_3 \quad (6)$$

According to the Lagrange multipliers principle, the following equation can be obtained. The main proof process is described at the **Appendix 1**.

$$\begin{cases} \tan\theta_1 = \pm\sqrt{K_1} \\ \tan\theta_2 = \pm\dfrac{\sqrt{K_1K_2}\pm 1}{\sqrt{K_2}\mp\sqrt{K_1}} \\ \tan\theta_3 = \pm\sqrt{K_2} \end{cases} \quad (7)$$

To simplify analysis, we only consider θ_1, θ_2, θ_3 Є (0, π/2) here.

In terms of the above analysis, TSSIR will have extremum value in the case of K_1* K_2>1. At this moment, three sections of TSSIR have different electric length ($\theta_1 \neq \theta_2 \neq \theta_3$). The extremum value is written as

$$\begin{aligned} \theta_{TE} &= \theta_1+\theta_2+\theta_3 \\ &= \tan^{-1}\sqrt{K_1}+\tan^{-1}\left(\frac{\sqrt{K_1K_2}-1}{\sqrt{K_1}+\sqrt{K_2}}\right)+\tan^{-1}\sqrt{K_2} \end{aligned} \quad (8)$$

Taking into account the extreme conditions, normalized length of the resonator can be expressed as,

$$L_n = \frac{2\theta_T}{\pi} = \frac{2\left(\tan^{-1}\sqrt{K_1}+\tan^{-1}\left(\dfrac{\sqrt{K_1K_2}-1}{\sqrt{K_1}+\sqrt{K_2}}\right)+\tan^{-1}\sqrt{K_2}\right)}{\pi} \quad (9)$$

Figure 2 shows the relationship between impedance ratios and normalized length of the resonator,

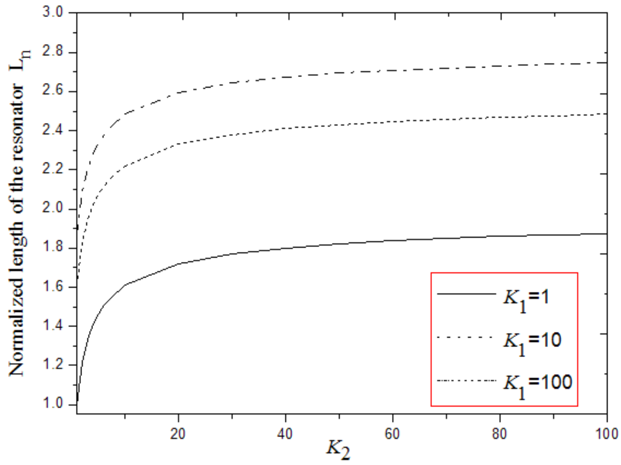


Figure 2. Relationship between Impedance Ratios and Normalized Length of the Resonator

From the Figure 2 we can see that the maximum resonator length of TSSIR is limited to triple that of the corresponding UIR.

3. Theoretical Analysis of Spurious Frequency

On the basis of previous discussion, spurious resonant frequency about λ/2 TSSIR is analysed in this section.

The geometry of a λ/2 TSSIR is depicted in Figure 3, where the dashed line represents the symmetry plane.

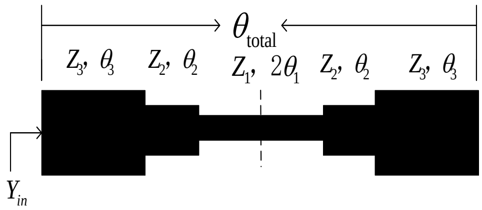


Figure 3. The General geometry of the λ/2 TSSIR

At resonance, the input admittance becomes zero (Y_{in} = 0). The resonance condition can be written as

$$\left(K_1\tan\theta_1+\tan\theta_2+\frac{1}{K_2}\tan\theta_3-\frac{K_1}{K_2}\tan\theta_1\tan\theta_2\tan\theta_3\right)*$$
$$\left(1-\frac{1}{K_1}\tan\theta_1\tan\theta_2-\frac{1}{K_1K_2}\tan\theta_1\tan\theta_3-\frac{1}{K_2}\tan\theta_2\tan\theta_3\right)=0 \tag{10}$$

Suppose that fundamental frequency is f_0, the corresponding electrical length of TSSIR is θ_{10}, θ_{20}, θ_{30}, respectively. θ_{10}, θ_{20} and θ_{30} should satisfy the relations,

$$1-\frac{1}{K_1}\tan\theta_{10}\tan\theta_{20}-\frac{1}{K_1K_2}\tan\theta_{10}\tan\theta_{30}-\frac{1}{K_2}\tan\theta_{20}\tan\theta_{30}=0 \tag{11}$$

The equation (11) can be simplified to,

$$\tan^{-1}\left(\frac{1}{K_1}\tan\left(\theta_{10}\right)\right)+\theta_{20}+\tan^{-1}\left(\frac{1}{K_2}\tan\left(\theta_{30}\right)\right)=\frac{\pi}{2} \tag{12}$$

Taking into account the extreme conditions of total electrical length, the value of θ_{10}, θ_{20} and θ_{30} is shown.

$$\theta_{10}=\tan^{-1}\sqrt{K_1}\quad,\quad \theta_{20}=\tan^{-1}\frac{\sqrt{K_1K_2}-1}{\sqrt{K_1}+\sqrt{K_2}}\quad,\quad \theta_{30}=\tan^{-1}\sqrt{K_2} \tag{13}$$

Set the first and second harmonic ratio as $F_{s1} = f_{s1} / f_0$, $F_{s2} = f_{s2} / f_0$. f_{s1}, f_{s2} are the first and second spurious frequency respectively. According to the resonance condition, the following equation should be contented.

$$\tan^{-1}\left(K_1\tan\left(\theta_{10}\cdot F_{s1}\right)\right)+\theta_{20}\cdot F_{s1}+\tan^{-1}\left(\frac{1}{K_2}\tan\left(\theta_{30}\cdot F_{s1}\right)\right)=\pi \tag{14}$$

$$\tan^{-1}\left(\frac{1}{K_1}\tan\left(\theta_{10}\cdot F_{s2}\right)\right)+\theta_{20}\cdot F_{s2}+\tan^{-1}\left(\frac{1}{K_2}\tan\left(\theta_{30}\cdot F_{s2}\right)\right)=\frac{3\pi}{2} \tag{15}$$

To better understand the harmonic characteristics of TSSIR with unequal electrical lengths, a characteristics comparison will be drawn between TSSIR with unequal electrical lengths and equal electrical lengths. With all three sections of the TSSIR assumed to have the same electric length (*i.e.*, $\theta_1=\theta_2=\theta_3=\theta_0'$), the corresponding electric lengths for the first second resonant modes of the TSSIR can be derived as (16) and (17) from [11].

$$\theta_{01}' = \tan^{-1}\sqrt{\frac{K_1K_2}{1+K_1+K_2}} \tag{16}$$

$$\theta_{02}' = \tan^{-1}\sqrt{\frac{1+K_2+K_1K_2}{K_1}}$$

(17)

Table I shows the comparison of total electrical length of a λ/2 TSSIR when $\theta_1\neq\theta_2\neq\theta_3$ and $\theta_1=\theta_2=\theta_3$ for various sets of K_1 and K_2 values. When $K_1*K_2<1$, the total electrical length of TSSIR with $\theta_1\neq\theta_2\neq\theta_3$ is smaller than with $\theta_1=\theta_2=\theta_3$, whereas in the case of $K_1*K_2>1$, the total electrical length of TSSIR with $\theta_1\neq\theta_2\neq\theta_3$ is larger than with $\theta_1=\theta_2=\theta_3$.

For $K_1=1$ and $K_2\neq1$ or $K_2=1$ and $K_1\neq1$, the same result was observed. Due to limited space, we only discuss the condition of $K_1=1$ and $K_2\neq1$ here. According to (8), we have:

$$\theta_{T0} = 2\left(\frac{\pi}{4}+\tan^{-1}\frac{\sqrt{K_2}-1}{1+\sqrt{K_2}}+\tan^{-1}\sqrt{K_2}\right)=4\tan^{-1}\sqrt{K_2} \quad (K_2 \text{ takes any value}) \tag{18}$$

Whereas by using (16) with assumption of $\theta_1=\theta_2=\theta_3$ and $K_1=1$ following equation is obtained:

$$\theta_{T0} = 6\left(\tan^{-1}\sqrt{\frac{K_2}{1+1+K_2}}\right)=4\tan^{-1}\sqrt{K_2} \quad (K_2=1)$$

(19)

That is, the former conclusion is not suitable for all the values of impedance ratio in (18), while the paper conclusion is suitable in (19). TSSIR degenerates to a SIR.

When K1=1 and K2=1, the two conclusion agree with each other, TSSIR will degenerates to a UIR. These specific numerical values are shown in bold in Table I.

Table 1. Comparison of Fundamental Characteristics of TSSIR

	The text result($\theta_1\neq\theta_2\neq\theta_3$)		$\theta_1=\theta_2=\theta_3$ result	
K_1 and K_2 values	θ_T (rad)	f_{s1}/f_{s0}	θ_T (rad)	f_{s1}/f_{s0}
K_1=0.6, K_2=0.4	1.75	3.37	2.00	3.08
K_1=0.8, K_2=0.6	2.42	2.49	2.52	2.42
K_1=1.0, K_2=0.8 (SIR)	**2.92**	2.08	**2.95**	2.07
K_1=1.0, K_2=1.0 (UIR)	**3.14**	2.00	**3.14**	2.00
K_1=1.5, K_2=1.0 (SIR)	**3.54**	1.66	**3.48**	1.71
K_1=2.0, K_2=1.5	4.32	1.52	4.15	1.65
K_1=2.5, K_2=2.0	4.71	1.29	4.57	1.39

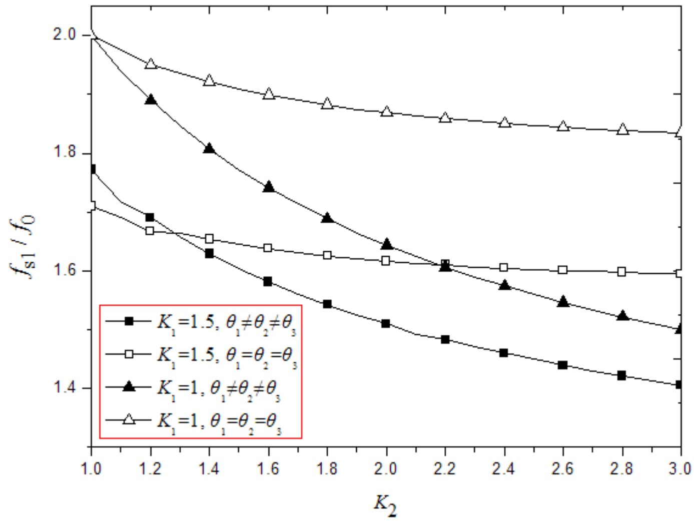


Figure 4. Comparison of Frequency Ratio f_{s1} / f_0 Versus K_1 with Various K_2

Figure 4 compares $f_{s1} / f_0 = F_{s1}$ with $f_{s1}' / f_0 = \theta_{02}' / \theta_{01}'$ for the same impedance ratios. It is observed that the F_{s1} is smaller than f_{s1}' / f_0 ($\theta_1=\theta_2=\theta_3$) in the condition of $K_1 * K_2 > 1$. With the continuous increasement of values of K_1 and K_2, the ratio will become smaller. Hence, it is appropriate to choose $\theta_1 \neq \theta_2 \neq \theta_3$ if low loss characteristics are important. And it also can widen the bandwidth of filter. Besides, find that the value of K_2 under the different electric length is smaller than the value under the same electric length for identical first harmonic ratio and K_1. That is, to achieve same harmonic ratios, the staircase discontinuity of transmission line can be reduced when choosing $\theta_1 \neq \theta_2 \neq \theta_3$. Thus, the measured result may be more accurate.

4. Numerical Experiment

To reinforce the statement about the above section, a filter has been designed using λ/2 TSSIR for performance demonstration with the following specifications on a substrate with ε_r=4.5 and thickness h=0.8 mm. Comparison of frequency response between using the peper method and the previous method have been made. Center frequency f_0 is 2.25GHz. The structure of TSSIR filter is shown as Figure 5. The impedance ratios K_1 and K_2 have been chosen to be 1.5, 2, respectively. The impedance of each section is designed to be Z_1=33.3Ω, Z_2=50Ω, and Z_3=100Ω. The electric dimensions of the filter are as follows:

(1) Different electric length: θ_1=0.8861rad, θ_2=0.2706rad, θ_3=0.9553rad.

(2) Same electric length: $\theta_1=\theta_2=\theta_3$=0.6847rad.

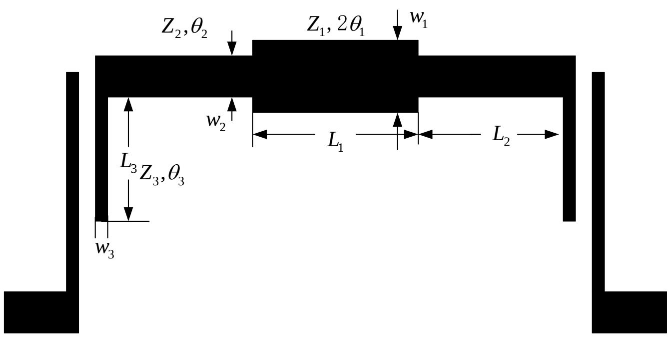
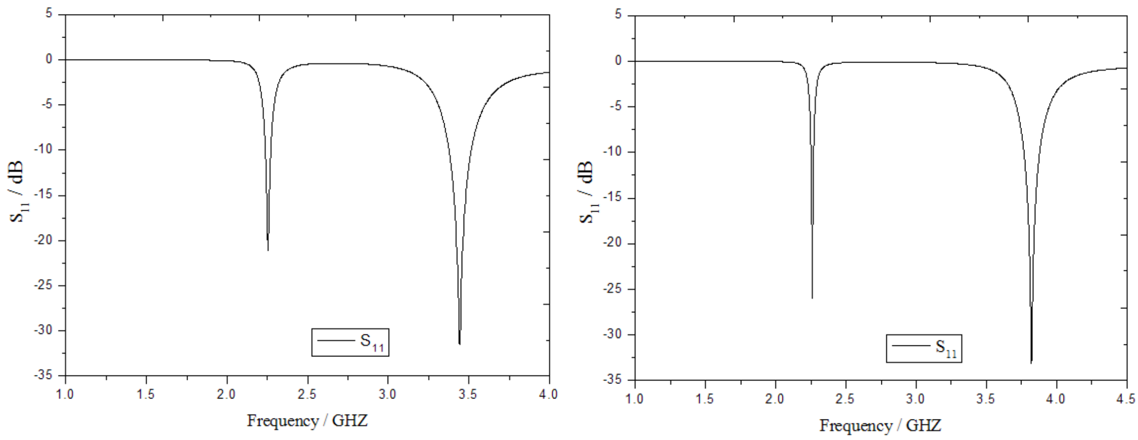


Figure 5. The Structure of TSSIR Filter

Total electrical length is 2.112rad and 2.0541rad, respectively. In Figure 6, simulated response curve of TSSIR filter using HFSS is displayed. Figure 6 (a) is the frequency response using the paper method, f_0=2.25GHz, f_{s1}=3.43GHz, f_{s1} / f_0 =1.5244. Figure 6 (b) is the frequency response using the same electric length of TSSIR with the previous method, f_0=2.25GHz, f_{s1}=3.82GHz, f_{s1} / f_0 =1.6978. From this figure we can find that the second resonant mode (f_{s1}) is closer to the first resonant frequency in the case of same impedance ratios using the paper method. The impedance ratios required in the design of the TSSIR can be determined easily using a set of analytic formulas presented. On the other hand, the impedance ratios can be smaller in the case of same harmonic ratios. It can make easy for the filter design and improve the design accuracy. By comparison bettwen Figure 4 and 6, find that experimental result show a good agreement between simulation and theoretical calculation.



(a) The Frequency Response Using the Paper Method

(b) The Frequency Response Using the Previous Method

Figure 6. Simulated S11 Response Curve of TSSIR Filter

5. Conclusions

In this paper, a proof method for total electrical length extreme value of TSSIR is presented. According to the above explanation, the condition of extreme value is that all three sections of TSSIR have different electric length ($\theta_1 \neq \theta_2 \neq \theta_3$). The maximum resonator length of TSSIR is limited to triple that of the corresponding UIR. Analysis of spurious resonant frequency about λ/2 TSSIR indicates that the staircase effect of transmission line isn't obvious to achieve same harmonic ratio using different electric length. Besides, using non-equal electrical length which did not bring much trouble to design, and increases the design flexibility simultaneously. Besides, a simulated example is presented to verify correctness of the method. The proposed method provides theoretical basis for the filter design using TSSIR.

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References

[1] G. Matthaei, L. Young, and E.M.T. Jones, "Microwave filters, impedance matching networks and coupling structures", Reprint Edition, Artech House, Norwood, MA, (**1985**).

[2] Y-C Chen, Y-H Hsieh, C-H Lee, and Chung-I G. Hsu. Tri-band microstrip "BPF design using tri-section SIRs", IEEE Antennas and Propagation Society International Symposium, vol. 6 (**2007**).

[3] I-Chin Wang, Ching-Her Lee, L-Y Chen, and Chung-I G. Hsu, "Band-Notched UWB bandpass filter design using tri-section SIR", IEEE Microwave Conference, (**2008**).
[4] C.-H. Lee, C.-I.G. Hsu, and L.-Y. Chen, "Band-notched Ultra-wideband bandpass filter design using combined modified quarter-wavelength tri-section stepped-impedance resonator", IET Microw. Antennas Propag, vol. 3, no. 8, (**2009**).
[5] C-H Lee, Chung-I G. Hsu, and Yu-Cheng Chen, "Compact tri-band BPF design using spur-line-loaded tri-section SIRS", Microwave and Optical Tech Lett, vol. 51, no. 4, (**2009**).
[6] Y Wang, Bing-Zhong Wang, Jianpeng Wang, Shanshan Gao, and Shaoqiu Xiao, "Tri-section stepped-impendance bandpass filter with wide stopband performance", Microwave and Optical Tech Lett, vol. 49, no. 4, **(2007**).
[7] C-K Lung, Kuo-Sheng Chin, and Jeffrey S. Fu, "Tri-Section stepped-impedance resonators for design of dual-band bandstop filter", European Microwave Conference, (**2009**).
[8] D. Packiaraj, M. Ramesh, and A. T. Kalghatgi, "Analysis of a quad-section SIR", Microwave Opt Technol Lett, vol. 50, no. 3, (**2008**).
[9] C-I G. Hsu, C-H Lee, and Y-H Hsieh, "Tri-Band bandpass filter with sharp passband skirts designed using tri-section SIRs", IEEE Microw. Wireless Compon. Lett, vol. 18, no. 1, (**2008**).

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