Low-Complexity Signal Vector Based Detection Algorithm for Spatial Modulation

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Abstract

In this paper, signal vector based detection (SVD) algorithm for spatial modulation (SM) is modified to achieve a near maximum-likelihood (ML) performance and reduces the complexity compared to ML. First, the proposed low-complexity SVD (LC-SVD) algorithm orders the antenna index list based on the angle between the received vector \(y\) and the channel vector \(h_j\), and then it estimates symbol by compensating the channel attenuation with transmitting antenna index list. We can trade-off between the performance and the complexity by changing the number of the candidate transmitting antennas. The theoretical analysis and simulation results show that the LC-SVD algorithm can achieve a near-ML performance with lower complexity.

Keywords: Spatial modulation, Complexity, SVD, Multiple-Input-Multiple-Output

1. Introduction

Spatial modulation (SM) \([1-3]\) is a recent low-complexity method for multiple-input-multiple-output (MIMO) system. MIMO system is proposed to solve the limited spectrum resources used in communications system and the reliability in high speed wireless communications, and V-BLAST \([4]\) is one of space time coding schemes for MIMO system, which is known for its high spectral efficiency. There are several main problems for V-BLAST system:

1) the high inter-channel interference (ICI) at the receiver due to simultaneous transmissions on the same frequency from multiple antennas;
2) the high bit error rate (BER) when deep fading happens in some sub-channels;
3) the number of receiving antennas \(N_r\) must be greater or equal to the number of the transmitting antennas \(N_t\).

In SM system, different from the traditional MIMO system, such as V-BLAST, only one transmitting antenna is active and modulated symbols from the constellation at one time slot by spatial multiplexing technology. In this way, SM can effectively avoid the inter-channel interference at the receiver. What is more, SM system can be applied to the MIMO systems in which the number of receiving antennas \(N_r\) is less than the number of transmitting antennas \(N_t\), and there can be only one receiving antenna.

There are several detection algorithms been proposed for SM systems. Maximum-likelihood (ML) algorithm which searches all the transmitting antennas and symbols from the constellation was proposed in \([5]\). It has optimal performance but with high computational complexity. ML’s computational complexity will linearly increase with the increase of the number of transmitting antennas \((N_t)\), the number of receiving antennas \((N_r)\) and the size of the modulation scheme \((M)\). Maximum ratio combing
(MRC) algorithm was proposed in [6], which estimates the transmitting antenna first and then demodulates constellations symbols. The estimation of antenna will greatly influence the demodulation results. MRC algorithm is proposed to reduce the computational complexity significantly but with rapid performance degradation. SM sphere decoding (SD) algorithm [7] is a modified algorithm of the ML. It provides a near-ML performance and reduces the complexity in the case of a large number of receiving antennas. However, the advantage of the SM system is the less using of receiving antennas. In this case, the complexity of SD is still considerably high. Signal vector based detection (SVD) method has been proposed in [8]. This new method does the detection of the best candidate transmitting antenna first, and then the detection of the constellation symbol. SVD detection performs better than MRC with a lower complexity compared to ML. However, a comment on SVD algorithm in [9] is proposed to prove that the SVD scheme performs very poorly compared to the optimal detection.

In this paper, an improved SVD algorithm called low-complexity signal vector based detection (LC-SVD), is proposed to achieve a near optimal performance and reduce its complexity compared with ML. With the modification in the proposed LC-SVD, first of all, a list of best candidate transmitting antenna index detection is computed to improve its performance. Then, to reduce its complexity, the proposed LC-SVD estimates the constellation symbol by compensating the channel attenuation of each transmitting antenna in the index list. At last, the optimal combination of transmitting antenna and constellation symbol is calculated. The simulation results show that the LC-SVD has a near-ML performance but with a much lower complexity.

The rest of this paper is organized as follows: Section II presents the system model and the MRC and ML optimum detector are also discussed. In Section III, the SVD algorithm and proposed LC-SVD algorithm for SM system are described, and complexity theoretical analysis is made. Section IV presents the simulation of BER performance comparison and complexity comparison between LC-SVD and ML detector. Finally, Section V concludes the paper.

2. System Model

2.1. SM Modulator

The system block diagram of spatial modulation is shown in Figure 1. In SM system, the bitstream emitted by a binary source is divided into blocks containing $m = \log_2(N_t) + \log_2(M)$ bits, where $m$ is the spectra efficiency at one time slot. The first $\log_2(N_t)$ bits are used to select the antenna which is active for data transmission, and the other transmit antennas are kept silent. The second $\log_2(M)$ bits are used to choose a symbol in the constellation diagram.

For SM system, only one antenna is active to carry constellation symbols at one time slot, so only one element of $x$, the matrix of transmitting symbols, is nonzero. Thus the SM mapped output at one time slot can be expressed as

$$x_{jq} = [0...x_q...0]^T$$

where $x_{jq}$ is the $j_{th}$ column vector of $x$, and has dimension of $N_t \times 1$. $x_q$ denotes the symbol carried by the active antenna from an M-ary constellation with $q \in [1 : M]$. The mapping table with 2 transmitting antenna and 4QAM modulation is depicted in TABLE 1.
Then, the $N_r \times 1$ received vector at one time slot can be written as follow
\[ y = Hx + n \] (2)

where $H$ is complex channel matrix of MIMO fading channel with dimension of $N_x \times N_r$, $n$ is the $N_r$ dimension Additive White Gaussian Noise (AWGN) with zero-mean and variance $\sigma^2$ per dimension at the receiver input.

From (1), (2) can be simplified as follow
\[ y = h_j x_q + n \] (3)

where $h_j$ is the $j_{th}$ column of $H$.

2.2. MRC detector

Antenna index estimation based maximum ratio combining (MRC) estimates the transmitting antenna first and then demodulates constellations symbols, which can be described as follows

\[ g_j = h_j^H y \] (4)

\[ j_{MRC} = \arg \max_{j \in [1...N]} |g_j| \] (5)

\[ q_{MRC} = Q(g) \] (6)

where $Q(\cdot)$ is constellation quantization function. Based on the estimated $j_{MRC}$ and $q_{MRC}$, the demodulated data can be obtained through demapping.

### Table 1. The Mapping Block Diagram of Spatial Modulation

<table>
<thead>
<tr>
<th>bitstream</th>
<th>Space bit</th>
<th>Antenna index</th>
<th>Modulation bit</th>
<th>Symbol</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0</td>
<td>1</td>
<td>00</td>
<td>+1+j</td>
<td>[+1+j, 0]</td>
</tr>
<tr>
<td>001</td>
<td>0</td>
<td>1</td>
<td>01</td>
<td>+1- j</td>
<td>[+1- j, 0]</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>-1+j</td>
<td>[-1+j, 0]</td>
</tr>
<tr>
<td>011</td>
<td>0</td>
<td>1</td>
<td>11</td>
<td>-1- j</td>
<td>[-1- j, 0]</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>2</td>
<td>00</td>
<td>+1+j</td>
<td>[0, +1+j]</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
<td>2</td>
<td>01</td>
<td>+1- j</td>
<td>[0, +1- j]</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
<td>2</td>
<td>10</td>
<td>-1+j</td>
<td>[0, -1+j]</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
<td>2</td>
<td>11</td>
<td>-1- j</td>
<td>[0, -1- j]</td>
</tr>
</tbody>
</table>
The MRC algorithm can only work efficiently under certain conditions. Without considering the noise, we can get

\[ g_j = h_j^H h_j x_q \]  

(7)

We can see that if we want to get the correct antenna index, \( |g_j| \) must be maximum when \( j = j_{MRC} \). We require

\[ |h_{j_{MRC}}^H h_{j_{MRC}}| \geq |h_j^H h_{j_{MRC}}|, (j = 1, 2, \ldots, N) \]  

(8)

Therefore, MRC detector is only applicable to some certain channels.

2.3. ML-Optimum Detector

The optimal ML-Optimum detector is described in [10], which searches all the transmitting antennas and constellation symbols, then takes the group with minimum Euclidean distance from the received vector as output. The optimal ML joint detection can be described as

\[ \{ j_{ML}, q_{ML} \} = \arg \max_{j \in \{1, \ldots, N\} \atop q \in \{1, \ldots, M\}} p(y \mid h_j, x_q) \]

\[ = \arg \min_{j \in \{1, \ldots, N\} \atop q \in \{1, \ldots, M\}} \|y - h_j x_q\|^2 \]  

(9)

\[ = \arg \min_{j \in \{1, \ldots, N\} \atop q \in \{1, \ldots, M\}} \left\{ \sum_{r=1}^{N} |y_r - h_{j_r} x_q|^2 \right\} \]

where \( \|p\|^2_f \) denotes the Frobenius norm, \( y_r \) and \( h_{j_r} \) are the \( r \)-th entries of \( y \) and \( h_j \) respectively. ML detection searches all the transmitting antennas, receiving antennas and constellation symbols so that it can achieve optimum performance but the complexity is very high, which can be shown from the complexity analysis in a later section.

3. Low-Complexity Signal Vector Based Detection

3.1. SVD

The SVD is based on the observation that, without the consideration of the noise, the received vector \( h_j x_q \) is with the same direction of the channel vector \( h_j \). Figure 2 shows the Hermitian angle between the channel vector \( h_j \) and the received vector \( y \).

![Figure 2. Hermitian Angle Between h_j And y](image)

Let \( \theta_j \) denote the Hermitian angle between \( h_j \) and \( y \). So \( \theta_j \) can be expressed as follow
\[ \theta_j = \arccos \rho_j \text{ with } \alpha_j = \frac{\langle h_j, y \rangle}{\|h_j\|\|y\|} \text{ and } \rho_j = \|\alpha_j\| \] (10)

where \( \langle \cdot, \cdot \rangle \) denotes the inner product in the Hilbert space. Then, the antenna can be estimated by

\[ j_{SVD} = \arg \min_{j \in \{1,...,N_t\}} \theta_j \] (11)

For the symbol detection, the traditional demodulation is performed to recover the constellation symbol, assuming the \( j_{SVD} \)-th transmit antenna being activated. The symbol can be estimated by

\[ q_{SVD} = \arg \min_{q \in \{1,...,M_f\}} \|y - h_{j_{SVD}}x_q\|^2 \] (12)

### 3.2. Proposed LC-SVD

The performance of SVD is unsatisfied compared to ML. To improve performance, let \( P \) denote the list of candidate antennas which have the smallest angles among the total angles \( \{\theta_1,...,\theta_{N_t}\} \) calculated by (10). Without loss of generality, assume \( P = \{j_1,...,j_p\} \) with \( 1 \leq p \leq N_t \). \( j_1 \) and \( j_p \) denote the antenna index with the minimal and maximal angles, respectively.

However, the complexity will increase rapidly if it searches all the constellation symbols for each antenna in List \( P \). To reduce the complexity, the proposed LC-SVD algorithm obtains the constellation symbol of each candidate antenna in the list \( P \) instead of searching all the constellation symbols. For compensating the channel attenuation, as mentioned in [12], every column of \( H \) should be normalized by its norm in order to improve the accuracy of demodulation. For the symbol demodulation, equation (2) is required to be left multiplied by \( h_j^H \) to get the estimation of \( x_q \). Then the received vector \( y \) is required to be multiplied by \( h_j^H \) as

\[ x_{q_j^1} = h_j^H y \quad (j \in P) \]
\[ h_j^H = (h_j^H h_j)^{-1} h_j^H = h_j^H / \|h_j\|^2 \] (13)

where \( (\bullet)^+ \) is Moore-Penrose inverse of a matrix, \( P = \{j_1,...,j_p\} \) with \( 1 \leq p \leq N_t \). Then the estimated symbol is obtained by the demodulator

\[ x_{q_j^1} = Q(x_{q_j^1}) \] (14)

where \( Q(\bullet) \) is constellation quantization function. Then, the final antenna index and constellation symbol can formally be written as follows

\[ [j,q] = \arg \min_{j \in P} \|y - h_j x_{q_j^1}\| \] (15)
3.3. Complexity Analysis

We use the number of real multiplications of the detectors to describe the complexity of the algorithms. It’s worth mentioning that the complexity of $(\bullet)^H$ and $Q(\bullet)$ can be neglected because they do not require additional operation. The ML detection needs $6MN_rN_r$ real multiplications [13] from (9). For SVD detection, the calculation of $N_r$ angles in (10) needs $8N_rN_r + 2N_r$ and the symbol estimation in (12) needs $6MN_r$ real multiplication, respectively. Therefore, the total complexity of SVD is $8N_rN_r + 2N_r + 6MN_r$. For LC-SVD, the angle calculation needs $8N_rN_r + 2N_r$ and Symbol demodulation from (13) needs a total of $2pN_r + p(N_r - 1)$ [14] real multiplications. Hence, the total complexity of all algorithms mentioned can be expressed as

$$C_{ML} = 6MN_rN_r$$

$$C_{SVD} = 8N_rN_r + 2N_r + 6MN_r$$

$$C_{LC-SVD} = 8N_rN_r + 2N_r + 8pN_r + p(N_r - 1)$$

(16)

where $C$ denotes the complexity of the algorithms. When $C_{LC-SVD} \leq C_{ML}$, we can get the inequalities which can be written as follows

$$p \leq \frac{6M - 8}{9 - \frac{1}{N_r}}N_r - \frac{2}{9 - \frac{1}{N_r}}$$

(17)

It is easy to find from (17) that the LC-SVD has less complexity than the ML detection when $6M - 8 \geq 9$, $M > 17/6 \approx 3$, which is suitable for most modulation systems. The detailed comparison will be discussed in the next section.

4. Simulation Results

4.1. BER Performance

MATLAB is used as the simulation platform. The parameters in the simulation experiment are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel receive antennas</td>
<td>Rayleigh fading channel</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

Simulations are performed for uncoded SM system with spectral efficiency of 6 bits per time slot and 8 bits per time slot under Rayleigh fading channel [15], respectively. For the case of 6 bits per time slot, two configurations of SM system are considered: one is 8 transmitting antennas with 8-QAM modulation, and the other is 4 transmitting antennas with 16-QAM modulation. For the case of 8 bits per time slot, two configurations are also considered: one is 16 transmitting antennas with 16-QAM modulation, and the other is 32 transmitting antennas with 8-QAM modulation. There are two kinds of configurations, one is the higher order modulation with less transmitting antennas, and the other is the lower order modulation with more transmitting antennas. For convenience, we call them high order system and low order system respectively. The BER performance comparisons
between ML, SVD, LC-SCD with \( p = 2 \), \( p = 3 \) and \( p = 4 \) are simulated for high order system and low order system, respectively.

The simulation results of 6 bits per time slot can be found in Figure 3 and Figure 4, which shows the comparison of the BER Performance for ML, SVD, LC-SCD with \( p = 2 \), \( p = 3 \) and \( p = 4 \). While the simulation results of 8 bits per time slot can be found in Figure 5 and Figure 6. We can roughly see that LC-SVD performs much better than SVD and can achieve a near-ML performance.

As 8-QAM and 8 transmit antennas are employed in Figure 3, the spectra efficiency is 6 bits per time slot. When \( N_t = 4 \) and \( BER = 1 \times 10^{-4} \), the LC-SVD with \( p = 2 \) has about 1.5 dB performance gain compared to the SVD, while about 0.5 dB performance loss compared to the ML detection. When \( p = 3 \), the performance of LC-SVD is much closer to ML. And when \( p = 4 \), the LC-SVD almost has the same performance as ML.
The similar result can be found in Figure 4, where the normalized 16-QAM is employed. To keep the same spectral efficiency (6 bits per time slot), the number of the transmit antennas is 4. The LC-SVD also performs much better than SVD in the high order system. When $BER = 1 \times 10^{-4}$, the LC-SVD with $p = 2$ has about 3 dB performance gain compared to the SVD, while about 0.3 dB performance loss compared to the ML detection. And when $p = 3$ and $p = 4$, the LC-SVD almost has the same performance as ML.

The simulation results of 8 bits per time slot can be found in Figure 5 and Figure 6. The high order system shows the BER performance comparisons in Figure 5, where the 16-QAM and 6 transmit antennas are employed. The low order modulation and a larger number of antenna bits are simulated in Figure 6. As 8-QAM is employed, the number of transmit antennas is set to 32.

The spectra efficiency is 8 bits per time slot as 16-QAM and 16 transmit antennas are used in Figure 5. In this case, the advantage of LC-SVD is more obvious. When $BER = 2 \times 10^{-3}$, the LC-SVD with $p = 2$ has about 2 dB performance gain compared to the SVD, while about 1 dB performance loss compared to ML detection. And when $p = 3$ and $p = 4$, the LC-SVD almost has the same performance as ML.

![Figure 5. BER Performance Comparison in a Nt=16,16QAM Uncoded SM System (8 bits per time slot)](image-url)
Similar result is shown in Figure 6, where 8-QAM and 32 transmit antennas are employed. The LC-SVD detector also keeps its advantage compared to the SVD detector. When $BER = 2 \times 10^{-3}$, the LC-SVD with $p = 2$ has about 3 dB performance gain compared to the SVD, while less than 1 dB performance loss compared to ML detection. The LC-SVD with $p = 3$ also has about 0.5 dB performance loss compared to ML detection. However, the LC-SVD with $p = 4$ almost has the same performance as ML.

From Figure 3, Figure 4, Figure 5, and Figure 6, it is obvious that the proposed LC-SVD has better performance than SVD and can achieve a near-ML performance in both high order system and low order system. In addition, the LC-SVD can make an excellent trade-off between performance and complexity by changing the value of $p$.

### 4.2. Comparison on Complexity

Figure 7 and Figure 8 show the complexity comparison between ML and LC-SVD with $p = 2$, $p = 3$ and $p = 4$. From Figure 3, Figure 4, Figure 5, and Figure 6, we can see that the LC-SVD can achieve a near-ML performance. While Figure 7 and Figure 8 show the advantage of complexity of the proposed LC-SVD compared to ML.

As all we know, for SM system, the number of the transmitting antennas and the size of the modulation scheme are two main factors which will greatly influence the complexity of the detector. In Figure 7 and Figure 8, the x axis $\log_2(M), \log_2(N_t)$ denotes the bits determining the constellation symbol and choosing the transmitting antenna respectively.

The number of real multiplication comparison between ML and LC-SVD with $p = 2$, $p = 3$ and $p = 4$ are shown in Figure 7. Figure 7 denotes that the complexity of ML and LC-SVD will increase with the raise of the modulation order or the number of transmitting antennas in the SM system, whereas the complexity of LC-SVD grows slower than the ML. And the complexity of LC-SVD is lower than ML all the time. When $\log_2(M) = \log_2(N_t) = 6$, the number of real multiplications of ML detector is $C_{ML} \approx 8 \times 10^3$, while the number of real multiplications of the LC-SVD detector is $C_{LC-SVD} \approx 10^2$, the complexity of LC-SVD is much lower than the ML.
detector in the case of high modulation order and a large number of transmitting antennas. The relative computational complexity is more apparently shown in Figure 8.

![Figure 7. Complexity Comparison on the Number of Real Multiplication](image)

Figure 7. Complexity Comparison on the Number of Real Multiplication

![Figure 8. Relative Complexity Comparison](image)

Figure 8. Relative Complexity Comparison

Figure 8 shows the relative computational complexity of the LC-SVD with respect to the ML algorithm, which can be written by

\[ C_{rel} = \log_{10}( \frac{C_{LC-SVD}}{C_{ML}} ) \]  \hspace{1cm} (18)

Figure 8 denotes that the relative computational complexity \( C_{rel} \) decreases with the raise of the modulation order or the number of transmitting antennas, \( \frac{C_{LC-SVD}}{C_{ML}} \) decreases with the raise of the modulation order or the number of antenna bits. When \( \log_2(M) = \log_2(N_t) = 4 \), the relative computational complexity \( C_{rel} \approx 10^{-1} \), the complexity of \( C_{LC-SVD} \) is ten times as large as \( C_{ML} \).
From Figure 7 and 8, we can see that the complexity of LC-SVD is lower than the ML detector in SM system. In the case of high modulation order and large number of transmitting antennas, the superiority of the LC-SVD detector is more significant than that of ML detector in complexity.

5. Conclusions

As a new transmitting method for MIMO system, SM has excellent prospects for development. In this paper, a low-complexity signal vector based detection algorithm has been proposed for SM systems. It has been proved that the proposed LC-SVD retains a near-ML performance with much lower complexity, especially in the case of high order modulation and the large number of transmitting antennas. In addition, we can make trade-off between the performance and the complexity by changing the number of the candidate transmitting antennas. According to this, a higher value of p can be choose for systems requiring higher performance, whereas a smaller one will be better.

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References

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