Reduction of Phase Noise Influence in MISO SFBC SC-FDMA System

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Abstract
It is very important to achieve high performance in the SC-FDMA (single carrier frequency division multiple access) system that is used for LTE (long term evolution) and LTE-advanced system. To simultaneously resolve ICI (inter-subcarrier interference) and PAPR (peak to average power ratio) problem, focused on the two transmit (Tx) / one receive (Rx) antenna configuration, phase noise influence on the 2×1 SFBC (space frequency block coding) SC-FDMA system is first analyzed, then compared with MISO (multiple input single output) SC-FDMA using several selected ICI self-cancellation methods. SC-FDMA induces more interference problem than OFDM because of the DFT spreading phase offset mismatch caused by random phase noise. By using ICI self-cancellation methods, the system performance significantly improves only by simple data encoding. Simulation results show that in the SFBC SC-FDMA system, the ICI and SCI interference cannot be neglected because 2×1 SFBC SC-FDMA even make phase noise influence seriously due to the simultaneous generation of more SCI and ICI than 2×1 SC-FDMA only, so, interference compensation is necessary. It is found that 2×1 SC-FDMA with data-conjugate ICI self-cancellation method and 2×1 SC-FDMA with symmetric ICI self-cancellation method can achieve considerable performance improvement than 2×1 SFBC SC-FDMA system, besides, data-conjugate method is superior to the symmetric data-conjugate method.

Keywords: SFBC, MISO, SC-FDMA, phase noise, ICI, ICI self-cancellation, data-conjugate, symmetric data-conjugate

1. Introduction
As we know, OFDM (orthogonal frequency division multiplexing) is an efficient frequency division multiplexing schemes, which offers minimum spacing of the sub-bands without interference from adjacent channels in the synchronous case [1]. OFDM has the property of high-speed broadband transmission, high bandwidth efficiency and robustness to multi-path interference, frequency selective fading. However, OFDM has the disadvantage of high PAPR (peak to average power ratio) and serious ICI unlike single carrier system. Especially, the ICI (inter-carrier interference) caused by phase noise or frequency offset seriously degrades system performance.

There have been many previous studies on the phase noise and ICI [2-5]. There, the phase rotation-common phase error (CPE) and ICI caused by phase noise were analyzed in detail. To estimate and compensate ICI influence, ICI self-cancellation using the data-conversion method or data-conjugate method etc. was proposed to minimize the ICI effect on the system performance [6-11].
Recently, a new technique called SC-FDMA (single carrier – frequency division multiple access) has been achieved more and more attention for the uplink of the evolved UTRA (EUTRA) [12, 13]. This single-carrier based radio access scheme has the advantage of low PAPR so that it can support wide-area coverage in cellular systems. Compared with IFDMA (interleaved frequency division multiple access), SC-FDMA results in more spectral efficiency, provides a greater degree of commonality in design between uplink and downlink, and easily coexists with OFDM on the uplink [14, 15]. In the SC-FDMA system, each information symbol is sent simultaneously over all carriers and the each carrier for the symbol is assigned a corresponding orthogonal DFT spreading code. In comparison to other multi-carrier systems, there is no multiple access interference since user discrimination is done by applying an FDMA scheme. However, SC-FDMA may induce ICI and SCI (self-channel interference) problem because of the phase offset mismatch caused by random phase noise.

Besides, a great deal attention has been devoted to MIMO antenna array systems and space-time-frequency processing [16-19]. MIMO diversity technique which exist diversity gain and coding gain can resolve the high link budget problem in the high data rate transmission, especially in the multi-path fading channel. Besides, the space-time-frequency processing, especially Alamouti’s diversity technique offers significant increase in performance at a low decoding complexity.

In this paper, focused on the two transmit (Tx)/one receive (Rx) antenna configuration, we theoretically analyze and evaluate the phase noise influence on the MISO type SC-FDMA system, especially $2 \times 1$ SFBC SC-FDMA system, and apply several phase noise compensation method such as data-conjugate ICI self-cancellation method and symmetric data-conjugate ICI self-cancellation method to minimize interference effect. Simulation results show that in the SFBC SC-FDMA system, the ICI and SCI cannot be neglected and interference compensation is necessary. It is found that $2 \times 1$ SC-FDMA with data-conjugate ICI self-cancellation method and $2 \times 1$ SC-FDMA with symmetric data-conjugate ICI self-cancellation method can achieve considerable performance improvement than $2 \times 1$ SFBC SC-FDMA system, besides, data-conjugate method is superior to symmetric data-conjugate method.

2. $2 \times 1$ SFBC SC-FDMA System

![MISO SFBC SC-FDMA Transceiver (2×1)](image-url)
SFBC method is a kind of diversity method which utilizes space frequency diversity. The main characteristic of this method is applying transmission antennas to realize space diversity effect. In SFBC method, after pre-coding, the data matrix is composed of linear combinations of constellation symbols and their conjugates. So, encoding only requires linear processing, and also linear decoding process is required in the receiver side. By above method, modified OFDM system could be very efficient to achieve near optimum diversity gain in the non-frequency selective fading channels. However, because of the transmission of data symbols through several antennas simultaneously, another interference named SCI generates and seriously degrades system performance.

When SFBC diversity, and 2 transmitter antennas and 1 receiver antenna are considered, the high speed information data passes through serial to parallel converter and becomes parallel data streams with $\tilde{S}$ branches. Then, they are converted into two pairs of $\tilde{S}$ branches parallel data by the SFBC encoder. The encoding algorithm is where $B_s, B_{s+1}$ mean the $s$ th and $s+1$ th branches, $d_s$ is the $s$ th branch parallel information data.

Table 1. SFBC Encoding Algorithm

$$
B_{s} = \begin{bmatrix}
    d_{s} & d_{s+1} \\
    -d'_{s+1} & d'_{s}
\end{bmatrix}
$$

In the receiver, after FFT, sub-carrier de-mapping and DFT de-spreading, the signal with $\tilde{S}$ parallel branches is converted back into the $\tilde{S}$ branches by the SFBC decoder. So, using $\hat{d}_{s} = (\hat{z}_s + (\hat{z}_{s+1})')/2$, $\hat{d}_{s+1} = (\hat{z}_s' - (\hat{z}_{s+1})')/2$, the original data can be recovered from the two adjacent parallel branches data. Here, $\hat{z}_s'$ is the $s$ th parallel branch data after DFT de-spreading, $\hat{d}_{s}''$ is the $s$ th branch detected information data after the SFBC decoder.

Channel description between the transmitter antennas and receiver antenna is as Figure 2.

![Figure 2. Channel Definition in 2 x 1 Diversity Scheme](image-url)
In the $2 \times 1$ SFBC SC-FDMA system, the received signal can be expressed as

$$r(n) = \left[ \sum_{t=1}^{2} \left[ x'(n) \otimes h'(n) + v(n) \right] \right] \cdot e^{j\phi(n)}$$  \hspace{1cm} (1)$$

where, $t$ means antenna number.

After FFT, the $k$th sub-carrier signal is arranged as follows:

$$Y_k = \frac{1}{N} \sum_{n \in S} r[n] \cdot e^{-j\frac{2\pi nk}{N}}$$

$$= \frac{1}{N} \sum_{n \in S} \left\{ \sum_{t=1}^{2} \left[ x'(n) \otimes h'(n) + v(n) \right] \right\} \cdot e^{j\phi[n]} \cdot e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{t=1}^{2} \sum_{n \in S} X'_t \cdot H'_t \cdot Q_{t-k} + N_k$$  \hspace{1cm} (2)$$

Suppose adjacent two carriers have same channel characteristic, such as $H'_1 = H'_2 = 1$, after DFT de-spreading, the detected data of $\tilde{k}$ th and $\tilde{k} + 1$th branches can be expressed as follows.

$$\hat{z}'_k = \sum_{j=0}^{\tilde{s}-1} Y'_j \cdot e^{j2\pi\tilde{k}j/\tilde{S}} = \sum_{j=0}^{\tilde{s}-1} Y'_j \cdot e^{j2\pi\tilde{k}j/\tilde{S}} = \sum_{j=0}^{\tilde{s}-1} \left( \sum_{i=0}^{\tilde{s}-1} \sum_{i \in S} X'_i \cdot Q_{i-s} + N_s \right) \cdot e^{j2\pi\tilde{k}j/\tilde{S}}$$

$$= \sum_{i=0}^{\tilde{s}-1} \sum_{s=0}^{\tilde{s}-1} \left[ \sum_{j=0}^{\tilde{s}-1} d_{2s} \cdot p_{i,2s} + d_{2s+1} \cdot p_{i,2s} - d^*_s \cdot p_{i,2s+1} + d^*_s \cdot p_{i,2s} \cdot p_{i,2s+1} \right] \cdot Q_{i-s} \cdot e^{j2\pi\tilde{k}j/\tilde{S}} + N_k$$  \hspace{1cm} (3)$$

$$\hat{z}'_{k+1} = \sum_{j=0}^{\tilde{s}-1} Y'_j \cdot e^{j2\pi(k+1)j/\tilde{S}}$$

$$= \sum_{i=0}^{\tilde{s}-1} \sum_{s=0}^{\tilde{s}-1} \left[ \sum_{j=0}^{\tilde{s}-1} d_{2s} \cdot p_{i,2s} + d_{2s+1} \cdot p_{i,2s} - d^*_s \cdot p_{i,2s+1} + d^*_s \cdot p_{i,2s} \cdot p_{i,2s+1} \right] \cdot Q_{i-s} \cdot e^{j2\pi(k+1)j/\tilde{S}} + N_{k+1}$$  \hspace{1cm} (4)$$

The decision variable $\hat{d}'_k$ of the $\tilde{k}$ th parallel branch is found from the adjacent two branch detected data by the SFBC decoder. That is,

$$\hat{d}'_k = \left[ \hat{z}'_k + (\hat{z}'_{k+1})^* \right] / 2$$
\begin{align*}
&= \frac{1}{2} \sum_{i=0}^{\frac{S-1}{2}} \sum_{k=0}^{\frac{S}{2}-1} \sum_{v=0}^{\frac{S}{2}} 
&\quad \left[ d_{2v}^* \left( p_{i,2v} Q_{i,s} e^{j2\pi i k i S} + p_{i,2v+1}^* Q_{i,s}^* e^{-j2\pi (i + 1) i v S} \right) 
+ d_{2v+1}^* \left( p_{i,2v+1} Q_{i,s} e^{j2\pi i k i S} + p_{i,2v}^* Q_{i,s}^* e^{-j2\pi (i + 1) i v S} \right) 
+ d_{2v+1}^* \left( p_{i,2v+1}^* Q_{i,s}^* e^{j2\pi (i + 1) i v S} - p_{i,2v+1} Q_{i,s} e^{-j2\pi (i + 1) i v S} \right) \right] + \hat{N}_k \\
&= \frac{1}{S} \sum_{i=0}^{\frac{S-1}{2}} \sum_{k=0}^{\frac{S}{2}-1} \sum_{v=0}^{\frac{S}{2}} 
&\quad \left[ d_{k} + \frac{1}{2S} \sum_{i=0}^{\frac{S-1}{2}} \sum_{k=0}^{\frac{S}{2}-1} \sum_{v=0}^{\frac{S}{2}} 
&\quad \left( Q_0 - Q_0^* \right) \right]^{SCI \text{ and } ICI \text{ except } (i \neq (s, i, v) = (k, 0, /2))} + \hat{N}_k 
&= \frac{1}{S} d_{\tilde{k}+1} + \frac{1}{2S} \sum_{i=0}^{\frac{S-1}{2}} \sum_{k=0}^{\frac{S}{2}-1} \sum_{v=0}^{\frac{S}{2}} 
&\quad \left( Q_0 - Q_0^* \right) \right]^{SCI \text{ and } ICI \text{ except } (i \neq (s, i, v) = (\tilde{k}, 0, /2))} + \hat{N}_{\tilde{k}+1}
\end{align*}

Similarly, the decision variable \( \hat{d}_{\tilde{k}+1} \) of the \( \tilde{k} + 1 \) th parallel branch is found as follows,

\begin{align*}
\hat{d}_{\tilde{k}+1} &= \left[ \hat{z}_{\tilde{k}} - (\hat{z}_{\tilde{k}+1})^* \right] / 2 \\
&= \frac{1}{S} d_{\tilde{k}+1} + \frac{1}{2S} \sum_{i=0}^{\frac{S-1}{2}} \sum_{k=0}^{\frac{S}{2}-1} \sum_{v=0}^{\frac{S}{2}} 
&\quad \left( Q_0 - Q_0^* \right) \right]^{SCI \text{ and } ICI \text{ except } (i \neq (s, i, v) = (\tilde{k}, 0, /2))} + \hat{N}_{\tilde{k}+1}
\end{align*}

where, \( I \) th component corresponds to the original data \( d_{\tilde{k}} \) component without CPE, \( II \) th component corresponds to SCI caused by the adjacent data \( d_{\tilde{k}+1} \). \( III \) th component corresponds to the SCI and ICI caused by other data components except \( except(i \neq (s, i, v) = (\tilde{k}, 0, /2)) \).

Above results show that in the SFBC SC-FDMA system, the ICI and SCI interference cannot be neglected and interference compensation is necessary.

3. Phase Noise Compensation in MISO SC-FDMA System

3.1. Data-conjugate ICI Self-cancellation Method

Figure 3 shows the block diagram of the 2\times1 SC-FDMA system with data-conjugate ICI self-cancellation method. The high speed information data passes through serial to parallel converter and becomes parallel data streams with \( \frac{S}{2} \) branches. Then, they are converted into \( S \) branches parallel data by the data-conjugate method. That is, at first, after serial to parallel converter, the transmission data are remapped as the form of \( d_{2s} = d_s, \quad d_{2s+1} = -d_s^* \).

Here, \( d_s \) is the \( s \) th branch parallel information data, and \( d_{2s+1} \) is the \( 2\tilde{s} \) th branch parallel data after mapped by data-conjugate method. Every information data is mapped into pairs of adjacent branches by data-conjugate method, so the \( \tilde{s} / 2 \) branches data are carried on the \( \tilde{s} \) parallel branches. Two adjacent parallel branches are used to transmit one same data.
In the receiver, after FFT, sub-carrier de-mapping and DFT de-spreading, the signal with $\tilde{s}$ parallel branches is converted back into the $\tilde{s}/2$ parallel branches by the de-mapping of the data-conjugate ICI self-cancellation method. So, using $\hat{d}_{\tilde{s}} = (\hat{d}_{2\tilde{s}} - (\hat{d}_{2\tilde{s}+1})^*)/4$, the original data can be recovered from the two adjacent parallel branches data. Here, $\hat{d}_{2\tilde{s}}$ is the $2\tilde{s}$ th parallel branch data after DFT de-spreading, $\hat{d}_{\tilde{s}}$ is the $\tilde{s}$ th branch information data after data-conjugate method de-mapping.

First, the complex base-band OFDM signal at the transmitter is as follows.

$$x(n) = \sum_{k \in S} X_k e^{j \frac{2\pi}{N}kn}$$

$$= \sum_{k \in S} \left[ \tilde{s} \sum_{l=0}^{2l-1} (d_l \cdot p_{k,2l} - d_{l}^* \cdot p_{k,2l+1}) \right] \cdot e^{j \frac{2\pi}{N}kn}, \quad \text{for } 0 \leq n < N. \quad (7)$$

Then, after removing cyclic prefix, after FFT, the recovered output for the $k$th sub-carrier is as follows:

$$Y_k = \frac{1}{N} \sum_{n \in S} r[n] \cdot e^{-j \frac{2\pi}{N}nk} = \sum_{i=1}^{2} \sum_{k \in S} X_i \cdot H_i \cdot Q_{i-k} + N_k. \quad (8)$$

After sub-carrier de-mapping, DFT de-spreading, the transmitted symbol $d_{2\tilde{s}}$ is achieved as follows. Here, for the simple analysis, the channel is supposed to be AWGN.
\[
d_{2k+1}^* = \sum_{i=0}^{\tilde{s}-1} Y_i e^{j2\pi \tilde{k}i/\tilde{s}} = \sum_{i=0}^{\tilde{s}-1} Y_i e^{j2\pi \tilde{k}i/\tilde{s}} \\
= \sum_{i=0}^{\tilde{s}-1} \left(2 \sum_{t=1}^{\tilde{S}} X_{t} \cdot Q_{t-i} + N_s\right) e^{j2\pi \tilde{k}i/\tilde{s}} \\
= 2 \sum_{i=0}^{\tilde{s}-1} \sum_{t=0}^{\tilde{S}/2-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \left( d_{\tilde{v}} \cdot p_{i,2\tilde{v}+1} - d_{\tilde{v}}^* p_{i,2\tilde{v}+1} \right) \cdot Q_{t-i} e^{j2\pi \tilde{k}i/\tilde{s}} + N_{2k}^*. \tag{9}
\]

Similarly, the transmitted symbol \(d_{2k+1}\) is recovered as
\[
d_{2k+1}^* = 2 \sum_{i=0}^{\tilde{s}-1} \sum_{t=0}^{\tilde{S}/2-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \left( d_{\tilde{v}} \cdot p_{i,2\tilde{v}+1} - d_{\tilde{v}}^* p_{i,2\tilde{v}+1} \right) \cdot Q_{t-i} e^{j2\pi \tilde{k}i/\tilde{s}} + N_{2k+1}^*. \tag{10}
\]

The decision variable \(\hat{d}_k\) of the \(\tilde{k}\)th parallel branch is found from the difference between adjacent two branch detected data by the data-conjugate method de-mapping. That is,
\[
\hat{d}_k = [d_{2k}^* - (d_{2k+1}^*)^*]/4
\]
\[
= \frac{1}{2\tilde{s}} d_{k} \cdot (Q_0 + Q_0^*) - \frac{1}{2\tilde{s}} d_{k}^* \sum_{i=0}^{\tilde{s}-1} \left( Q_0 e^{-j2\pi i/\tilde{s}} + Q_0^* e^{j2\pi i/\tilde{s}} \right)

+ \frac{1}{2\tilde{s}} \sum_{i=0}^{\tilde{s}-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \left[ d_{\tilde{v}} (Q_{i-\tilde{v}} e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_{i-\tilde{v}}^* e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}}) \right]

- d_{\tilde{v}}^* (Q_{i-\tilde{v}} e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_{i-\tilde{v}}^* e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}})

+ \frac{1}{2\tilde{s}} \sum_{i=0}^{\tilde{s}-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \left[ d_{\tilde{v}} (Q_0 e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_0^* e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}}) \right]

- d_{\tilde{v}}^* (Q_0 e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_0^* e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}})

+ \frac{1}{2\tilde{s}} \sum_{i=0}^{\tilde{s}-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \sum_{\tilde{v}=0}^{\tilde{S}/2-1} \left[ d_{\tilde{v}} (Q_{i-\tilde{v}} e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_{i-\tilde{v}}^* e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}}) \right]

- d_{\tilde{v}}^* (Q_{i-\tilde{v}} e^{-j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}} + Q_{i-\tilde{v}}^* e^{j2\pi \tilde{k}(i-\tilde{v})/\tilde{s}}) \right].
\]
where \( N_k = \frac{1}{4} \left( N_{2k} - N_{2k+1}^* \right) \) is the AWGN of the \( \tilde{k} \) th branch parallel data in the receiver, \( I \) th component corresponds to the original data \( d_k \) component without CPE, \( II \) th component corresponds to the original data \( d_k \) with ICI caused by original data \( d_k \) components in the other sub-carriers. \( III \) th component corresponds to the SCI caused by other data superimposed with original data \( d_k \) in the same carriers, \( IV \) th component corresponds to the ICI caused by other data in the other sub-carriers.

### 3.2. Symmetric Data-conjugate ICI Self-cancellation Method

In the symmetric data-conjugate ICI self-cancellation method, after serial to parallel converter, the transmission data are remapped as the form of \( d_k^* = d_j \), \( d_{\tilde{s}-\tilde{s}-1}^* = -d_j^* \). In the receiver, after FFT, sub-carrier de-mapping and DFT de-spreading, the signal with \( \tilde{S} \) parallel branches is converted back into the \( \tilde{S}/2 \) parallel branches by de-mapping of the symmetric data-conjugate method. So, using \( \hat{d}_j = (d_j^* - (d_{\tilde{s}-\tilde{s}-1}^*)^*)/4 \), the original data can be recovered from the two symmetrical parallel branch data.

As before, the complex base-band SC-FDMA signal can be expressed as follows.

\[
x(n) = \sum_{k=0}^{\tilde{S}/2-1} \sum_{i=0}^{\tilde{S}/2-k-1} [d_j \cdot p_{k,j} - d_j^* \cdot p_{k,\tilde{s}-1-i,j}] \cdot e^{\frac{j2\pi kn}{N}} + \sum_{k=0}^{\tilde{S}/2-1} \sum_{i=0}^{\tilde{S}/2} \sum_{\tilde{s}=0}^{\tilde{S}/2-k-1} \sum_{\tilde{s}=0}^{\tilde{S}/2} \sum_{i=0}^{\tilde{S}/2-k} \sum_{i=0}^{\tilde{S}/2-k} X_k \cdot e^{\frac{j2\pi kn}{N}}
\]

\[
\hat{Y}_k = \frac{1}{N} \sum_{n=0}^{N-1} r[n] \cdot e^{-j2\pi nk/N} = \sum_{i=1}^{2} \sum_{\tilde{s}=0}^{\tilde{S}/2} \sum_{j=0}^{\tilde{S}/2-k} X_i \cdot H_i \cdot Q_{i-k} + N_k.
\]
Therefore, the transmitted symbol $d_k^*$ can be recovered as follows.

$$
\hat{d}_k = \sum_{s=0}^{S-1} Y_s \cdot e^{j2\pi \tilde{k}/S} = \sum_{s=0}^{S-1} Y_s \cdot e^{j2\pi \tilde{k}/S} = 2\sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot p_{t,\ell} \cdot d_{\ell}^* \cdot p_{t,\ell-1,\ell} \right) \cdot Q_{\ell-t} \cdot e^{j2\pi \tilde{k}/S} + N_k^*.
$$

Similarly, the transmitted symbol $d_{\tilde{k}-1-k}$ is expressed as

$$
\hat{d}_{\tilde{k}-1-k} = 2\sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot p_{t,\ell} \cdot d_{\ell}^* \cdot p_{t,\ell-1,\ell} \right) \cdot Q_{\ell-t} \cdot e^{j2\pi (\tilde{k}-1-k)/S} + N_{\tilde{k}-1-k}^*.
$$

The decision variable $\hat{d}_k$ of the $k$ th parallel branch is found from the difference between symmetrical two branch detected data by the symmetric data-conjugate method de-mapping.

$$
\hat{d}_k = [\hat{d}_{2k} - (\hat{d}_{2k+1})^*]/4
$$

$$
= \frac{1}{2S} \sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot Q_{0} \cdot e^{j2\pi (k-t)/S} + Q_{0}^* \cdot e^{-j2\pi (k-t)/S} \right)
$$

$$
+ \frac{1}{2S} \sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot Q_{0} \cdot e^{j2\pi (k-t)/S} + Q_{0}^* \cdot e^{-j2\pi (k-t)/S} \right)
$$

$$
+ \frac{1}{2S} \sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot Q_{0} \cdot e^{j2\pi (k-t)/S} + Q_{0}^* \cdot e^{-j2\pi (k-t)/S} \right)
$$

$$
+ \frac{1}{2S} \sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \left( d_{\ell} \cdot Q_{0} \cdot e^{j2\pi (k-t)/S} + Q_{0}^* \cdot e^{-j2\pi (k-t)/S} \right)
$$

$$
+ \frac{1}{2S} \sum_{\ell=0}^{S-1} \sum_{s=0}^{S-1} \sum_{t=0}^{S-1} \sum_{\bar{t}=0, \bar{t} \in k} \left( d_{\ell} \cdot Q_{0} \cdot e^{j2\pi (k-t)/S} + Q_{0}^* \cdot e^{-j2\pi (k-t)/S} \right).
$$
\[-d_i^* \left( Q_{i-s} \cdot e^{j2\pi [\tilde{\delta} - (\tilde{\delta} - 1)]} + Q_{i+s} \cdot e^{-j2\pi [\tilde{\delta} - (\tilde{\delta} - 1)]} \right) \right) + N_k^i \]

\[
= \frac{1}{S} d_k + II + III + IV + N_k^i, \quad \tilde{\nu}, \tilde{\delta} = 0, \ldots, \tilde{\delta} - 1.
\]

where \( N_k^i = \frac{1}{4} \left( N_k - N_{\tilde{\delta}-1-k}^* \right) \) is the AWGN of the \( k \) th branch parallel data in the receiver.

### 4. Performance Analysis

First, PAPR, PICE and BER performance are analyzed in the SC-FDMA and original OFDM system. Furthermore, \( 2 \times 1 \) SFBC SC-FDMA, \( 2 \times 1 \) SC-FDMA with data-conjugate ICI self-cancellation method and with symmetric data-conjugate ICI self-cancellation method are compared.

Simulation parameters are as follows: (1) Modulation method: QPSK modulation; (2) OFDM sub-carrier number: 64; (3) Data carrier number: 52; (4) DC carrier: 1; (5) Guard carrier: 13; (6) Guard interval: 0; (7) HPA: SSPA; (8) back-off: 0; (9) Channel: AWGN; (10) Phase noise model: the generalized model of PLL (ref [2, 3]); (11) Phase noise parameter: \( \sigma_\phi^2 = 0.06 \ rad^2 \); (12) Coding efficiency of each method: \( C_r \); Table.2 shows the Coding efficiency of each method. (13) required-transmitted-signal-to-noise-ratio: \( E_b/C_rN_0 \).

The coding efficiency of each method is as follows.

<table>
<thead>
<tr>
<th>Method</th>
<th>Coding efficiency (C_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original OFDM</td>
<td>1</td>
</tr>
<tr>
<td>SC-FDMA</td>
<td>1</td>
</tr>
<tr>
<td>SC-FDMA with data-conversion method</td>
<td>1/2</td>
</tr>
<tr>
<td>SC-FDMA with data-conjugate method</td>
<td>1/2</td>
</tr>
<tr>
<td>SC-FDMA with symmetric data-conversion method</td>
<td>1/2</td>
</tr>
<tr>
<td>SC-FDMA with symmetric data-conjugate method</td>
<td>1/2</td>
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<tr>
<td>2x1 SFBC SC-FDMA</td>
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</tbody>
</table>
Besides, PAPR of the SC-FDMA signal is defined as:

\[
PAPR = \max_{0 \leq r < T} \left| \frac{\mathbb{E}[x(t)^2]}{E[x(t)^2]} \right|
\] (17)

where \(E[\cdot]\) means the expectation operation and \(x(t)\) is the transmitted signal.

SSPA (solid-state power amplifier) model is expressed as follows.

\[
F_A[y(t)] = \frac{y(t)}{1 + \left( \frac{y(t)}{A_0} \right)^{2p}}^{1/2p}
\] (18)

\[
F_p[y(t)] = 0.
\] (19)

where, \(F_A[y(t)]\) and \(F_p[y(t)]\) are AM/AM and AM/PM characteristics of nonlinear HPA, \(A_0\) is saturation voltage of SSPA output, \(p\) is a smoothness factor related to nonlinearity and \(y(t)\) is the amplitude characteristic of the HPA input.

Besides, the PICR (peak interference to carrier ratio) is defined as follows.

\[
PICR = \max_{0 \leq k < N-1} \left( \frac{|I_k|^2}{|C_k|^2} \right)
\] (20)

where \(C_k\) is the \(k\)th sub-carrier desired component, \(I_k\) is the interference component of the \(k\)th sub-carrier.

Figure 4. BER in AWGN Channel with Phase Noise
Figure 4 shows the BER performance of original OFDM and SC-FDMA system in the AWGN channel when phase noise is considered. As seen in the figure, SC-FDMA induces more performance degradation compared with original OFDM when phase noise exists. When phase noise variance is $0.03\, rad^2$, SC-FDMA system has about $3.2\, dB$ $E_b/N_0$ penalty than original OFDM at BER=$10^{-4}$. When phase noise variance is $0.06\, rad^2$, unlike original OFDM, SC-FDMA generates error floor higher than BER=$10^{-3}$. That is, SC-FDMA system is more sensitive to the phase noise because of the DFT spreading phase offset mismatch caused by random phase noise.

Figure 5 shows the PAPR characteristic of SC-FDMA with several ICI self-cancellation methods. As seen in the above figure, SC-FDMA with data-conjugate method has the best PAPR property. SC-FDMA with symmetric data-conjugate method is slight worse, but much better than SC-FDMA without ICI cancellation scheme. Symmetric data-conversion method has a bit better PAPR property than SC-FDMA only. At $Pr(PAPR > PAPR_0) = 10^{-3}$, data-conjugate method, symmetric data-conjugate method and symmetric data-conversion method have about $0.9\, dB$, $0.7\, dB$, $0.1\, dB$ PAPR gain respectively compared with SC-FDMA system only, on the contrary, data-conversion method results in about $0.6\, dB$ PAPR penalty than SC-FDMA system only.

Figure 6 shows the PICR characteristic of SC-FDMA with several self-cancellation methods when phase noise variance is $0.06\, rad^2$. As seen in the above figure, PICR property is slightly improved in the order of data-conversion method, symmetric data-conversion method, symmetric data-conjugate method and data-conjugate method have nearly $2\, dB$, $2.1\, dB$, $2.2\, dB$ PICR gain at $10^{-2}$ respectively when compared with SC-FDMA without ICI self-cancellation scheme. Data-conversion method has slight performance degradation than other three ICI-self cancellation method, and has only about $1.7\, dB$ PICR gain at $10^{-2}$ compared with SC-FDMA only.

At all, according to the above several analyses, below, we select data-conjugate method and symmetric data-conjugate method for the further evaluation when phase
noise exists. BERs are found to discuss the system performance affected by phase noise in the SC-FDMA with data-conjugate method or symmetric data-conjugate method,

![Figure 6. PICR Comparison](image1)

respectively. BER versus the required-transmitted-signal-to-noise-ratio $E_b/C_rN_0$ ($C_r$ is coding efficiency in each method, listed in Table 1; $N_0$ is the spectral density coefficient for the white noise) is considered.

Figure 7 shows the PICR of MISO SC-FDMA. As seen in Figure 7, PICR property is significantly improved in the order of $2 \times 1$ SC-FDMA with symmetric data-conjugate method and $2 \times 1$ SC-FDMA with data-conjugate method. Symmetric data-conjugate and data-conjugate method respectively have about 2.8dB, 3.6dB PICR gain at $10^{-2}$ compared with $2 \times 1$ SFBC SC-FDMA. Data-conjugate method has better PICR property than symmetric data-conjugate method in MISO SC-FDMA system.

![Figure 7. PICR Comparison](image2)
Figure 8 shows the BER in the AWGN channel when phase noise variance is $0.06 \text{ rad}^2$. As seen in Figure 8, $2 \times 1$ SC-FDMA with data-conjugate method or symmetric data-conjugate method have considerable performance improvements compared with $2 \times 1$ SFBC SC-FDMA. At BER=$10^{-3}$, only about 1.9dB or 2.2dB $E_b/N_0$ penalties are found respectively in the $2 \times 1$ SC-FDMA with data-conjugate method and $2 \times 1$ SC-FDMA with symmetric data-conjugate method compared with AWGN theory, but error floor occurs in the $2 \times 1$ SFBC SC-FDMA system.

![Figure 8. BER in AWGN Channel with Phase Noise](image)

Figure 9 shows the BER in the AWGN channel when phase noise variance $0.06 \text{ rad}^2$ and SSPA with 0 back-off are considered. As seen in Figure 9, $2 \times 1$ SC-FDMA with data-conjugate method or symmetric data-conjugate method still have significant performance improvements compared with $2 \times 1$ SFBC SC-FDMA. At BER=$10^{-3}$, only
about 2.4dB and 3dB $E_b/C_rN_0$ penalties are found respectively in the $2 \times 1$ SC-FDMA with data-conjugate method and $2 \times 1$ symmetric data-conjugate method compared with AWGN theory, but also error floor occurs in the $2 \times 1$ SFBC SC-FDMA.

Figure 10 shows the BER according to the phase noise variance in AWGN channel when $E_b/C_rN_0$ is supposed to be 10dB. As seen in the figure, in the MISO system, data-conjugate and symmetric data-conjugate ICI self-cancellation methods improve BER performance significantly compared with SFBC SC-FDMA system. To reach BER=$10^{-3}$, about 0.04 $rad^2$ and 0.045 $rad^2$ more phase noise variance can be tolerated respectively in the $2 \times 1$ SC-FDMA with symmetric data-conjugate method and $2 \times 1$ SFBC SC-FDMA with data-conjugate method compared with $2 \times 1$ SFBC SC-FDMA.

Figure 11 shows the total relationship of the phase noise variance vs BER in AWGN channel when $E_b/C_0 = 0$ and QPSK modulation are considered. As seen in figure, ICI self-cancellation methods results in more performance improvement in the SISO system than MISO system with SSPA, $2 \times 1$ SFBC SC-FDMA induces more performance degradation than $2 \times 1$ SC-FDMA without SFBC diversity, it means SFBC diversity cannot resolve and even intensify phase noise influence due to the simultaneous generation of more SCI and ICI. Fortunately, data-conjugate and symmetric data-conjugate ICI self-cancellation methods improve system performance considerably in the MISO SC-FDMA systems although symmetric data-conjugate method produces performance degradation in the $2 \times 1$ SC-FDMA when phase noise variance is lower than 0.03 $rad^2$.

![Figure 12. BER in AWGN Channel with Phase Noise (16QAM)](image)

In the MISO system, data-conjugate method still outperforms symmetric data-conjugate method, and above ICI self-cancellation methods have significant performance improvement especially compared with $2 \times 1$ SFBC SC-FDMA system.

Figure 12 shows the BER performance in the AWGN channel when phase noise variance 0.03 $rad^2$ and 16QAM modulation are considered. As seen in the figure, in 16 QAM modulation, $2 \times 1$ SC-FDMA with data-conjugate method or symmetric data-conjugate method also have significant performance improvements compared with $2 \times 1$ SFBC SC-FDMA. At BER=10^{-2}, only about 2dB or 2.1dB $E_b/C_N$ penalties are found respectively in the $2 \times 1$ SC-FDMA with data-conjugate method and $2 \times 1$ SC-FDMA with symmetric data-conjugate method compared with AWGN theory, but about 9.5dB $E_b/C_N$ penalty occurs in the $2 \times 1$ SFBC SC-FDMA method.

5. Conclusion

In this paper, we analyze the ICI (inter-carrier interference) which caused by phase noise in MISO (multiple-input single output) type SC (single carrier) - FDMA (frequency division multiple access) system. Especially, in order to minimize phase noise effect, SFBC (space frequency block coding) SC-FDMA system apply several ICI cancellation method.
Simulation results show that more ICI and SCI (self-channel interference) are generated in $2 \times 1$ SFBC SC-FDMA than $2 \times 1$ SC-FDMA case. So ICI and SCI cannot be neglected and interference compensation is necessary. We apply data-conjugate method and symmetric data-conjugate method for ICI self-cancellation. Although data-conjugate method is superior to symmetric data-conjugate method, both methods can achieve considerable performance improvement.

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