Performance of Spectrum Sensing Scheme Using Double Threshold Energy Detection in the Presence of Sensor Noise

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Abstract

In this paper the performance of cooperative spectrum sensing scheme using double threshold energy detection technique in the presence of sensor noise has been investigated. Each cognitive user takes a local decision on spectrum occupancy and sends one bit information to convey its decision to a fusion center. The fusion center collects all decisions and takes a final decision about the presence or absence of primary user using Majority logic. The performance of cooperative spectrum sensing has been investigated in Rayleigh fading channel.

Keywords: spectrum sensing scheme, double threshold energy detection, sensor noise, fusion center, Rayleigh fading channel

1. Introduction

In recent years the demand for wireless and mobile service has been growing exponentially. However, the fixed radio spectrum is not enough to fulfill the requirement. On the other hand, licensed band is experiencing low utilization efficiency under temporal and geographical variations. So there is a conflicting interest and that can be solved by Cognitive Radio (CR) technology [1-5]. Before accessing the licensed band, CR sensors perform spectrum sensing to limit the interference to the primary user [6]. But the performance of one cognitive user is often degraded due to fading and shadowing effects on wireless communication channels. Collaborative or co-operative spectrum sensing may be conducted to improve the spectrum sensing performance [7]. In collaborative spectrum sensing, a set of CR sensors perform spectrum sensing and send their local reports to the fusion center for further processing. Finally fusion center takes a decision in favour of the presence or absence of Primary User (PU).

Each cognitive radio needs a control channel to transmit their local decision. If all the cognitive sensors send their report, the required bandwidth is high even if one bit quantization is used. To meet the requirement and manage the bandwidth utilization, recently the decentralized detection technique has become a point of attraction. In [7], Ghasemi et al. evaluate detection probability and show that collaborative spectrum sensing improves the performance under single threshold. In [8], the average number of sensing bits has been derived. The paper showed a significant decrease of the average number of sensing bits to the common receiver at the expense of a little performance loss under double threshold. In [9], the authors propose a cooperative sensing method based on ‘n-ratio’ logic and derived ROC and detection probability. One major short fall of OR logic is that it can not distinguish between unused band and deep fade. On the contrary, Majority logic can be applied to overcome the short fall.
Contribution of the paper: In this paper we find the average number of normalized sensing bits and detection error probability under double threshold based energy detector assuming Majority logic at fusion centre in presence of Rayleigh fading. We also find average number of normalized sensing bits when CR sensors are at different fail sensing probability. We also propose a method to find the average number of normalized sensing bits in the presence sensor noise.

The rest of the paper is organized as follows. In Section 2, the system model is introduced. In Section 3, simulation model is discussed. The simulation results are shown in Section 4. Finally, in Section 5, we present the conclusions.

2. System Model

In this model, there is one primary user, one secondary base station and N secondary users. All N secondary users are cooperating with each other. Each secondary user performs energy detection for sensing spectrum and sends its local decision to the fusion center at the secondary base station. For local decision, secondary user has to distinguish between absence \( (H_0) \) and presence \( (H_1) \) of primary user.

![System Model Diagram](image)

**Figure 1. System Model**

The goal of spectrum sensing is to decide between the following two hypotheses,

\[
r(t) = \begin{cases} 
  n(t) & H_0 \\
  s(t) + n(t) & H_1 
\end{cases}
\]  

(1)

where \( r(t) \) is the signal received by a secondary user, \( s(t) \) is primary user’s transmitted signal and \( n(t) \) is additive white Gaussian noise (AWGN). \( H_0 \) and \( H_1 \) hypotheses test is used for detection of PU [10].

In AWGN channel, Energy received \( (O_i) \) by a secondary user has following distribution
\[ O_i = \begin{cases} \frac{X_{2u}^2}{X_{2u}^2(2\gamma)} & H_0 \\ \frac{X_{2u}^2}{X_{2u}^2(2\gamma)} & H_1 \end{cases} \]  

where \( X_{2u}^2 \) and \( X_{2u}^2(2\gamma) \) are central and non-central chi-square distribution, respectively. ‘u’ denotes time bandwidth product TW and \( \gamma \) denote SNR[11].

In cooperative spectrum sensing probability of detection \( (Q_d) \) and false-alarm \( (Q_f) \) may be written as follows,

\[ Q_d = 1 - (1 - P_d)^n \]  
\[ Q_f = 1 - (1 - P_f)^n \]

where \( P_d \) and \( P_f \) are the individual probabilities of detection and false-alarm and ‘n’ is the number of CR nodes[12].

In single threshold detection method, decision of \( H_0 \) and \( H_1 \) will depend only on one threshold \( \lambda \). In case of double threshold detection method, decision of \( H_0 \) and \( H_1 \) will depend on two thresholds \( \lambda_1 \) and \( \lambda_2 \) [13].

\[ D_i = \begin{cases} 0 & O_i \leq \lambda_1 \\ \text{No decision} & \lambda_1 < O_i < \lambda_2 \\ 1 & O_i > \lambda_2 \end{cases} \]

So each secondary user decides either 0 or 1 or “no decision” on the basis of collected energy \( O_i \). Decision goes in favour of ‘0’ if primary user is absent. Similarly Decision goes in favour of ‘1’ if the primary user is present. Probability of deciding ‘1’, probability of “no decision” and probability of deciding ‘0’ under hypothesis \( H_1 \) is represented by \( P_{d1} \), \( \Delta_1 \) and \( P_m \) respectively. Similarly, Probability of deciding ‘1’, probability of “no decision” and probability of deciding ‘0’ under hypothesis \( H_0 \) is represented by \( P_f \), \( \Delta_0 \) and \( P_{d0} \) respectively. The expressions for different probabilities are given below in respect to AWGN channel.

\[ P_{d1} = P\{O_i > \lambda_2 \mid H_1\} = Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda_2}\right) \]

\[ P_{d0} = P\{O_i < \lambda_1 \mid H_0\} = 1 - \frac{\Gamma(u, \lambda_1 / 2)}{\Gamma(u)} \]

\[ \Delta_1 = \{\lambda_1 < O_i < \lambda_2 \mid H_1\} = Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda_1}\right) - Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda_2}\right) \]

\[ \Delta_0 = P\{\lambda_1 < O_i < \lambda_2 \mid H_0\} = \frac{\Gamma(u, \lambda_1 / 2)}{\Gamma(u)} - \frac{\Gamma(u, \lambda_2 / 2)}{\Gamma(u)} \]

\[ P_m = P\{O_i < \lambda_1 \mid H_1\} = 1 - Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda_1}\right) \]
\[ P_j = P \{ O_j > \lambda_2 \mid H_0 \} = \frac{\Gamma(u, \lambda_2 / 2)}{\Gamma(u)} \]  
\hspace{1cm} (11)

The same model is used for Rayleigh fading channel which we have considered as flat. The PDF of SNR is given by.

\[ f(\gamma) = \frac{1}{\gamma} \exp \left( -\frac{\lambda}{\gamma} \right) \]  
\hspace{1cm} (12)

\( P_t \) will remain same as in case of AWGN. The average \( P_d \) (\( P_{d,\text{Ray}} \)) in Rayleigh fading channel [9] is given below. Using this equation we can find probability of deciding ‘1’ (\( P_{d,\text{Ray}} \)), probability of ‘No decision’ (\( \Delta_{\text{Ray}} \)) and probability of deciding ‘0’ (\( P_{\text{Ray,0}} \)).

\[
\overline{P}_{d,\text{Ray}} = e^{-\lambda/2} \sum_{n=0}^{u-2} \frac{1}{n!} \left( \frac{\lambda}{2} \right)^n + \left( \frac{1+\gamma}{\gamma} \right)^{u-1} \left[ e^{-\lambda/2(1+\gamma)} - e^{-\lambda/2} \sum_{n=0}^{u-2} \frac{1}{n!} \left( \frac{\lambda\gamma}{2(1+\gamma)} \right)^n \right]
\]  
\hspace{1cm} (13)

In this paper, the normalized average number of sensing bits is not for only one user. We assume that the common receiver receives \( K \) out of \( N \) local decisions reported from the cognitive sensors. The reporting channel is noiseless and no fading. To limit the interference to the primary user, the spectrum is assumed to be available only when majority of the reporting decisions are 0.

Let \( k \) denote the normalized average number of sensing bits, i.e.,

\[ k = \frac{K_{\text{avg}}}{N} = 1 - P_A \Delta_0 - P_B \Delta_1 \]  
\hspace{1cm} (14)

where \( K_{\text{avg}} \) is the average number of sensing bits. \( P_0 = P\{H=0\} \) and \( P_1 = P\{H_1\} \). If no CR sensor respond to the fusion center (i.e. \( K=0 \)), the situation is referred to as fail sensing, In such situation receiver requests all the CR users to perform spectrum sensing again.

Let \( b_0 \) and \( b_1 \) be the fail sensing probability under hypothesis \( H_0 \) and \( H_1 \). Thus the fail sensing probabilities are \( b_0 = \Delta_0^N \) and \( b_1 = \Delta_1^N \).

When there is no fail sensing, means \( K \geq 1 \), the \( Q_j \), \( Q_a \) and \( Q_m \) will be given as follows:

\[ Q_j = (1 - b_1)(1 - P_A) \]  
\hspace{1cm} (15)

\[ Q_a = (1 - b_1)(1 - P_B) \]  
\hspace{1cm} (16)

\[ Q_m = 1 - b_1 - Q_d \]  
\hspace{1cm} (17)

Here \( P_A = P\{H=0\mid H_0, K \geq 1\} \) and \( P_B = P\{H=0\mid H_1, K \geq 1\} \).

\[ P_A = F(\lambda_2^N) - b_0 \]  
\hspace{1cm} (18)

\[ P_B = G(\lambda_2^N) - b_1 \]  
\hspace{1cm} (19)

Here \( F(\lambda) \) and \( G(\lambda) \) are CDF of the collected energy under hypotheses \( H_0 \) and \( H_1 \) respectively[14].

According to this majority logic [15], if \( K_1 \) users decide hypothesis \( H_0 \) and \( K_2 \) users decide hypothesis \( H_1 \), where \( (K_1+K_2) \leq N \), the decision \( H \) will be

\[ H = \begin{cases} H_0 & K_1 \geq (N / 2) \\ H_1 & K_2 \geq (N / 2) \end{cases} \]  
\hspace{1cm} (20)
We also consider the observation at the $k$-th sensor ($k=1,2,\ldots,N$) at a given instant can be expressed as

$$r_k = Hs_k + n_{\text{sensor}}(k)$$

(21)

$s_k$ is the intensity of the phenomenon observed at $k$-th sensor and $n_{\text{sensor}}(k)$ is the sensor noise at the same CR sensor. We assume that the noise samples of sensor noise have complex Gaussian distribution and are independent from each other. The SNR at the $k$-th sensor can be defined as follows:

$$SNR_{\text{sensor}}^{(k)} = \frac{\left[E\{r_k \mid H_1\} - E\{r_0 \mid H_0\}\right]^2}{(\sigma_{\text{sensor}}^{(k)})^2} = \frac{s_k^2}{\sigma^{(k)2}}$$

(22)

3. Simulation Model

In order to verify the performance predicted by the analytical framework discussed in the previous section, we simulate different parameters at different conditions. In particular, each simulation run is carried out according to the following steps.

1. Threshold voltages ($\lambda_1$ and $\lambda_2$) are generated with respect to fail sensing probability.
2. Equally likely hypothesis $H \in \{H_0, H_1\}$ is generated.
3. The received signal of each CR sensor, $r(t) = s(t)+n(t)$ is generated under Rayleigh faded condition.
4. Next the received energy, i.e., the square of $r(t)$ of Step 3, at each CR sensor is compared with the respective threshold voltage and respective hypothesis. Each CR sensor sends one bit information to FC to take a decision. Here we assume each CR sensor sends its respective decision to FC.
5. At the FC, we use Majority logic for finding the decision about the presence of primary user.
6. Steps 1 to 5 are repeated a large number of times to reliably estimate the normalized average number of bits.

In the later part, we consider sensor SNR and simulate the model as follows:

7. Steps 1 to 2 are repeated.
8. Sensor SNR is generated.
9. The received signals of each CR sensor, $r(t) = Hs(k) + n_{\text{sensor}}(k)$ is generated.
10. Steps 4 to 5 are repeated.
11. Steps 7 to 10 are repeated a large number of times to reliably estimate the detection probability.
4. Results and Discussions

Figure 2 shows the normalized transmission bits in Rayleigh fading channel for different number of CR sensors at a particular fail sensing probability \( b_0 = 0.01 \), respectively. \( Q_f \) (Probability of failure detection) is upper and lower bounded. For each value of \( b_0 \), there will be upper and lower value of \( Q_f \). For example, when \( b_0 = 0.01 \), the upper and lower values of \( Q_f \) are 0.99 and 0.0099, respectively. The “No decision” region is \( \Delta_0 = (b_0)^{1/N} \). As \( N \) increases, this in turn reduces the number of sensing bits, the required bandwidth reduces. We also have tested our test bed using OR-rule [8], the result is shown in Figure 2. Double threshold based cooperative spectrum sensing has also been investigated in [16].

![Average Number of Sensing Bits as a Function of Q_f, b_0=0.01 and SNR=10 dB](image)

**Figure 2.** Average Number of Sensing Bits as a Function of \( Q_f, b_0 = 0.01 \) and \( \text{SNR}=10 \text{ dB} \)

![Probability of Detection (P_d) as a Function of Q_f, with b_0=0.01 and N=10](image)

**Figure 3.** Probability of Detection( \( P_d \)) as a Function of \( Q_f \), with \( b_0=0.01 \) and \( N=10 \)

Figure 3, shows that Probability of detection increases as SNR increases. So the probability of error in finding the PU decreases as SNR increases. In particular, when \( Q_f \) is 0.0099, the SNR increases from 10 dB to 15 dB and 15 dB to 20 dB, the detection probability increases from 0.1791 to 0.318 and 0.318 to 0.4279, respectively.
Figure 4. Average Number of Sensing Bits as a Function of $Q_f$, $N=10$ and $\text{SNR}=10 \text{ dB}$

Figure 4 shows normalized sensing bits at different $Q_f$. In Figure 4 we consider two different cases with fail sensing probabilities. For the first case, we took five CR sensors with fail sensing probability $b_0=0.01$ and rest five sensors with fail sensing probability $b_0=0.02$. For the second case we consider all the sensors have same fail sensing probability. In case of different fail sensing probability of the CR users, we find that the required normalized average number of sensing bits is near to the upper value of $b_0$.

Figure 5. Average Number of Sensing Bits as a Function of $Q_f$, $b_0=0.01$ and $\text{SNR}=10 \text{ dB}$

Figure 5, shows the normalized transmission bits for ten number of CR sensors at a particular $b_0=0.01$ while by considering the effects of sensor noise on average number of sensing bits. As sensor noise decreases the number of transmission bits decreases. When the $Q_f$ is 0.0297, the sensor noise decreases from 0 dB to -3 dB and -3 dB to -9 dB, the normalized average number of sensing bits decreases from 0.4217 to 0.4211 and from 0.4211 to 0.4109 (indicated by an arrow in figure).
In Figure 6, the detection probability for a PU is shown as a function of the number of CR sensors. This figure shows the effects of sensor noise and the number of CR sensors on the probability of detection. We assume that SNR of sensor -5.125dB. We find that three curves corresponding to $b_0=0.1, 0.01, 0.001$ overlap with each other. It indicates that the effect of $b_0$ on $P_d$ in this range is insignificant in the considered model.

![Figure 6. The Detection Probability as a Function of the Number of Selected Sensors $b_0=0.05$](image)

5. Conclusions

The performance of cooperative spectrum sensing scheme has been studied under double threshold condition. Simulation results indicate significant performance enhancement in terms of average number of sensing bits. Error probability decreases if SNR increases. We have also investigated the performance when the CR sensors are having different fail sensing probability. The required normalized average number of sensing bits is near to the upper value of the fail sensing probability. The number of transmission bits reduces as sensor noise increases and detection probability increases with the increase of number of sensor nodes.

References


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