

# Construction Method of Location Fingerprint Database Based on Gaussian Process Regression Modeling

Jianhui Han\* and Xiang Chen

*The college of Computer Science and Technology, Harbin University of Science and Technology, Harbin 150080, China  
363992791@qq.com*

## Abstract

*In the terms of indoor positioning, the location of the fingerprint technology that based on receiving wireless LAN WIFI signal strength (RSS) has been widely used. In the process of location of fingerprint offline training, the traditional method has more manpower and time. In this paper, we propose a location of fingerprint database constructing method that based on Gaussian process regression (GPR), compared with the process of the traditional method for collecting a large number of fingerprint, now we based on the propagation law of space radio signals, the Gaussian process regression model is applied to the construction of the location fingerprint database, and forecast the location of fingerprint inside the located area through the study of collected samples, by doing this we can reduce the collecting density of fingerprint samples, improve the efficiency of position fingerprint positioning technology.*

**Keywords:** *Wireless LAN; Indoor positioning; GPR; location fingerprint; RSS*

## 1. Introduction

With the arrival of the era of mobile internet, location-based services have become a basic demand [1]. Under the outdoor environment, the global positioning system could substantially satisfy people's most of position service requirement, yet under the large complex indoor environment, such as museums, railway stations, hypermarkets, hospitals, mines and other areas. Due to the 'Urban Canyon' [2] effect, leading to a sharp decrease in GPS accuracy. As a result, a great development occurs at the indoor positioning technology, domestic and foreign researchers proposed the indoor positioning technology and application systems that based on Radio Frequency Identification abbreviation is RFID [3], ultrasonic, infrared, WIFI etc. With the rapid development of wireless internet, the coverage of wireless network is being more and more widely. Due to the WIFI-based positioning technology [4] can make up for the shortage of GPS indoor positioning application, expanding the scope of location-based services, as result the technology become the prevalent choice of indoor location.

At present, roughly there are four kinds of positioning technology that based on distance-measurement: Based on Time of Arrival [5], based on Time Difference Of Arrival [6], based on Angle Of Arrival [7] and based on Received Signal Strength Indication [8]. Of which, the method that based on TOA and TDOA requires base station and under-test equipment's clock keep in synchronization. A small deviation of clock would lead to a prodigious deviation on distance, while the AOA method will be more accurate under the line-of-sight situation, so commonly the RSSI-based (Received Signal Strength Indication) method is used on positioning. And RSSI-based [9] method basically has two kinds: the one is the Radio Frequency Propagation Loss model combined with geographic information, and then converted into distance information. But this method is easily affected by environment, such as the loss caused by penetrating the walls, as well as multipath transmission of signal, which lead to the failed construction of accurate

positioning model. Another method is the Location Fingerprint technology [10], the LF positioning technology can deal with the problems that caused by non-line-of-sight propagation and multipath transmission, the method is that we form the location fingerprint database by combining the RSSI values with geographic coordinates, and then realize positioning function through using the fingerprint to match the algorithm. Its shortcoming is offline construction of position fingerprint database in the early stage and fingerprint database update and maintenance in the later period is going to take more manpower and time.

In this paper, according to the space wireless signal propagation law and variation tendency, the Gaussian process regression [11] model is going to be applied to the construction of position fingerprint database, and forecast the location of fingerprint inside the located area through the study of collected samples, by doing this we can reduce the collecting density of fingerprint samples, improve the efficiency of position fingerprint positioning technology.

## 2. The Gaussian Process Regression Model Sketch

### 2.1. Determine the Objective Function

The Gaussian process model assumes that the observed output is the Gaussian process of the input parameter, we noted it as:

$$y(x) \square gp(m(x), k(x, x')) \tag{1}$$

The  $m(x)$  is the mean function, said that at the point  $x$ , the average of  $y$  is  $E(y(x))$ ,  $k(x, x')$  is the covariance function.

In the Gaussian Process Regression modeling, covariance function determines the similarity among samples. The covariance function that was used in the Gaussian Process is kernel function. Kernel function can improve the nonlinear processing ability, and we can reduce dimension reasonably if we select the appropriate kernel function, the high quality data regression effect will be achieved at length.

The square covariance function is commonly used.

$$k(x, x') = \delta_f^2 \exp\left(-\frac{1}{2l^2}|x, x'|^2\right) \tag{2}$$

The square covariance function formula defines the characteristic length scale of the covariance function. The characteristic length is a size scale that could estimate the related degree between two input points. Square covariance function is the most widely used in the field of nuclear.

Due to the existence of noise interference in the observed output in the reality, we add the noise model to the Gaussian Process model that selected above. We assume that the noise component in the observed values is the independent identically distributed Gaussian white noise, its mean value is 0, variance is  $\delta_n^2$ , change the observed value to

$$y'(x) = y(x) + \xi \tag{3}$$

Among them, the covariance of observed values is

$$\text{cov}(y'_p, y'_q) = k(x_p, x_q) + \delta_n^2 \delta_{pq} \tag{4}$$

Of which

$$\delta_{pq} = \begin{cases} 1 & p = q \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

According to the law of central limit, the sum of sufficient independent identically variables that distributed randomly is obeying Normal distribution, that is to say, the noise that generated by the sum of many independent identically distributed elements is obeying the Gaussian distribution, so we choose the Gaussian distribution to describe the noise.

## 2.2. The Calculation of the Likelihood Function

We assume that  $D=\{(x_i, r_i) \mid i=1,2,\dots,n\}$  is the observed sample,  $X=\{x_i \mid i=1,2,\dots,n\}$ ,  $Y=\{y_i \mid i=1,2,\dots,n\}$  is the input sample and output sample respectively. So by the definition of Gaussian Process assumption we can get the likelihood function of sample  $D$  is  $P(D|h)=P(y'(X)=Y|h)$ , that is

$$P(D | h) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma + \delta_n^2 I|} \exp(L) \quad (6)$$

$$L = (Y - m(X))^T |\Sigma + \delta_n^2 I|^{-1} (Y - am(X)) \quad (7)$$

## 2.3. Parameter Learning

The (6) contains hypothetical unknown parameters, we called them ready-learning parameters. According to the Bayesian Reasoning, those learning parameters can be ascertained by maximizing  $P(D|h)$ .

## 2.4 Forecasting the Output

After the end of the parameter learning, the filtrate of optimal hypothesis in the hypothesis space is realized. On the next stage we can make the use of this optimal hypothesis, calculating the conditional probability density of tested points that under the condition of the observed samples have been obtained to predict the output.

Set the  $x^*$  as the tested input data,  $y(x^*)$  is the prediction of the output value, with the help of the characters of optimal hypothesis and Gaussian Process, the joint distribution of observed value and tested output value can noted as:

$$\begin{bmatrix} y(X) \\ y(x^*) \end{bmatrix} \square N \left( \begin{bmatrix} m(X) \\ m(x^*) \end{bmatrix}, \begin{bmatrix} \Sigma + \delta_n^2 I & \Sigma_* \\ \Sigma_*^T & \Sigma_{**} \end{bmatrix} \right) \quad (8)$$

Thereinto  $\Sigma_{**}$  is the covariance matrix of the tested data,  $\Sigma_*$  is the covariance matrix between the tested data and the observed data, the elements of covariance matrix is defined by the covariance function  $k(x_p, x_q)$ .

Calculated by the conditional probability, we can predict that the conditional density distribution of output is in accordance with the normal distribution

$$y(x^*) | D \square N(\mu, \delta^2) \quad (9)$$

Thereinto

$$\mu = m(x^*) + \Sigma_*^T (\Sigma + \delta_n^2 I)^{-1} (Y - m(X)) \quad (10)$$

$$\delta^2 = \Sigma_{**} - \Sigma_*^T (\Sigma + \delta_n^2 I)^{-1} \Sigma_* \quad (11)$$

## 3. RSS Gaussian Process Regression Model

### 3.1. Model Selection

Gaussian Process Regression has the characters that continuous spatial positions, random likelihood function model, prediction of uncertainty and parameter estimation of consistency, thus it can simulate the received signal strength well.

The parameter of the Gaussian Process Regression model can be acquired by calibration parameter in the process of studying. These parameters include the spatial correlation among different location and the spatial correlation among tested noise.

In the RSS Gaussian Process Regression model, we assumed that the relationship of RSS and points of location is Gaussian Process.

$$RSS(x) \square gp(m(x), k(x, x')) \tag{12}$$

$x$  is the spatial location point,  $m(x)$  is the mean function, said the RSS average of point  $x$ ,  $k(x, x')$  is covariance function, it means the covariance of signal strength  $RSS_x$  and  $RSS_{x'}$  that received at the point  $x$  and  $x'$ .

In this paper we choose the constant function  $C$  as the mean function, and the square exponential function as covariance function, as shown in (2), it illustrates  $l$  a scale that measure the correlation of received signal strength between two points with the measurement of unit distance. The greater of  $l$ , the correlation of received signal strength between two points of measurement of unit distance is greater.  $C, l, \delta_f$  is the ready-learning parameter therein.

### 3.2 Algorithm Implementation

#### 3.2.1. Parameter Learning

The likelihood function that calculated from (6) contains ready-learning parameter  $C, l, \delta_f, \delta_n$ , they are noted as

$$\theta = (C, l, \delta_f, \delta_n) \tag{13}$$

In this paper, we use the conjugate gradient algorithm to minimize the negative logarithm likelihood function  $-\log P(D|h)$ .

In the numerical optimization, we commonly use iterative method to solve unconstrained optimization problems. Here supposed that a unconstrained optimization problems

$$\min f(x) \tag{14}$$

The basic idea of iteration method is that given an initial point, then generate an iterative sequence according to a certain iteration regulation, if it is a limited sequence, the last point of sequence is the minimum point in the unconstrained optimization problems. Otherwise, when the sequence is infinite point sequence with limit point, the point is the minimum point of the problem.

Set the  $x_k$  as the  $k^{\text{th}}$  times iteration point,  $d_k$  is  $k^{\text{th}}$  search direction,  $\alpha_k$  is the  $k^{\text{th}}$  step-size factor, then after  $k^{\text{th}}$  iteration we can get a new iteration point  $x_{k+1}$ :

$$x_{k+1} = x_k + \alpha_k d_k \tag{15}$$

The common search methods for solving nonlinear unconstrained optimization are Newton method and conjugate gradient method, and the optimization algorithm that based on these two kinds of algorithm. Conjugate gradient method is an algorithm that has least storage and high speed of convergence, the PRP nonlinear conjugate algorithm is adopted in this paper.

$$-\log P(D | h) = \frac{1}{2} Y^T (\Sigma + \delta_n^2 I)^{-1} Y + \frac{1}{2} \log |\Sigma + \delta_n^2 I| + \frac{n}{2} \log 2\pi \tag{16}$$

Remove the items that are irrelevant with parameter learning of the polynomial in the (16), the optimization problem is defined as

$$\min f(\theta) \tag{17}$$

$$f(\theta) = \frac{1}{2} Y^T (\Sigma + \delta_n^2 I)^{-1} Y + \frac{1}{2} \log |\Sigma + \delta_n^2 I| \tag{18}$$

Specific iterative algorithm is as follows:

- (1) Take the  $\theta_0, d_0 = \nabla f(\theta_0)$  as initial point, accuracy  $\epsilon > 0$ , and set  $k = 0$ .

(2) If  $|\nabla f(\theta_0)| \leq \epsilon$ , the algorithm is terminated, get the solution  $\theta_k$  of the problem. Otherwise skip to the step c.

(3) Calculate the search direction  $d_k$ , the calculated method is in detail as follow:

$$d_k = \begin{cases} -\nabla f(\theta_k) & k = 0 \\ -\nabla f(\theta_k) + \beta_{k-1}d_{k-1} & k \geq 1 \end{cases} \quad (19)$$

$$\beta_{k-1} = \frac{\nabla f(\theta_k)^T (\nabla f(\theta_k) - \nabla f(\theta_{k-1}))}{\nabla f(\theta_{k-1})^T \nabla f(\theta_{k-1})} \quad (20)$$

(4) The step length  $\alpha_k$  is determined by adopting inexact linear search. This paper takes the inexact linear search which based on Wolfe. Given the  $\rho \in (0, 0.5)$ ,  $\sigma \in (\rho, 0.5)$ , acquire the  $\alpha_k$  that meet the conditions of two inequalities simultaneously.

$$f(\theta_k + \alpha_k d_k) \leq f(\theta_k) + \rho \alpha_k \nabla f(\theta_k)^T d_k \quad (21)$$

$$\nabla f(\theta_k + \alpha_k d_k)^T d_k \geq \sigma \nabla f(\theta_k)^T d_k \quad (22)$$

(5) Set  $\theta_{k+1} = \theta_k + \alpha_k d_k$ ,  $k := k+1$ , skip to the step (2).

### 3.2.2. The RSS Value of Location $x^*$

In (6) and (7), the inverse matrix  $(\Sigma + \delta_n^2 I)^{-1}$  need to be calculated, when make the use of adjoints matrix or Gaussian elimination method to calculate the inverse matrix, the time complexity is  $O(n^3)$ , this will require a large amount of computing time and space. So for the starter we should carry on the matrix decomposition, determine the inverse matrix of each decompose matrix, at length calculate the final inverse matrix by using the prior inverse matrix.

Because the sum of the positive semi-definite matrix and positive definite matrix is still the positive definite matrix, so  $\Sigma + \delta_n^2 I$  is the real symmetric positive definite matrix. Toward a real symmetric positive definite matrix, we can use Cholesky decomposition method, thus a matrix can be decomposed into the product of upper triangular matrix and lower triangular matrix. We have known that the inverse matrix of a upper triangular matrix is still a upper triangular matrix, and we can calculate the inverse matrix of upper triangular matrix by using simple recursion method. Similarly, the lower triangular matrix can be calculated by recursion algorithm, or can get the transpose of the upper triangular matrix at the first time, after calculating the inverse matrix of the transpose, transpose transformation is carried on this inverse matrix, the result is what we need.

Assumed that  $A = (a_{ij})$  is real symmetric positive definite matrix, the result of the Cholesky decomposition is  $A = (GG^T)$ , of which the  $G = (g_{ij})$  is lower triangular matrix, and there goes:

$$a_{ij} = g_{i1}g_{j1} + g_{i2}g_{j2} + \dots + g_{ij}g_{jj} \quad i > j \quad (23)$$

$$a_{ii} = g_{i1}^2 + g_{i2}^2 + \dots + g_{ii}^2 \quad (24)$$

Then the recurrence formula is obtained:

$$g_{ij} = \begin{cases} \left( a_{ii} - \sum_{k=1}^{i-1} g_{ik}^2 \right) & (i = j) \\ \frac{1}{g_{jj}} \left( a_{ij} - \sum_{k=1}^{j-1} g_{ik}g_{jk} \right) & (i > j) \\ 0 & (i < j) \end{cases} \quad (25)$$

Because of  $a_{ii} = g_{i1}^2 + g_{i2}^2 + \dots + g_{in}^2$ , so

$$|g_{ij}| \leq \sqrt{a_{ii}} \quad (26)$$

It can be seen from (26) that the intermediate quantity in the Cholesky decomposition is under control <sup>completely</sup>, so the process of calculating is stable. In order to avoid the root operation in the process of calculation, we can make the fully use of recursion (25) to find the solution.

After acquiring the triangular matrix  $G$ , the inverse matrix  $G^{-1}$  can be obtained by using the recursive algorithm, finally, the result is

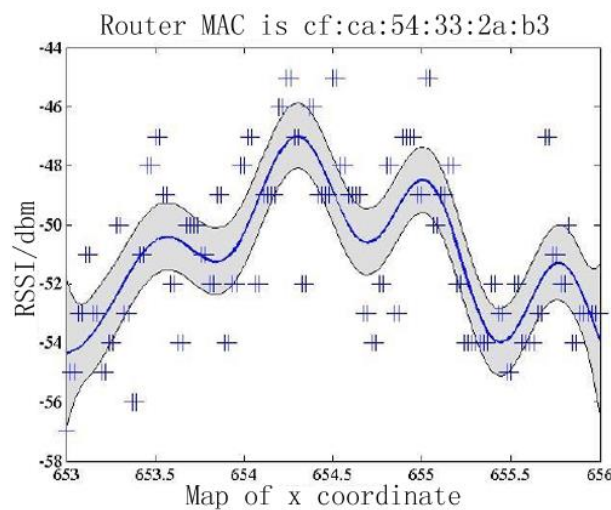
$$A^{-1} = (GG^H)^{-1} = (G^{-1})^H G^{-1} \quad (27)$$

#### 4. Experiment and Result Analysis

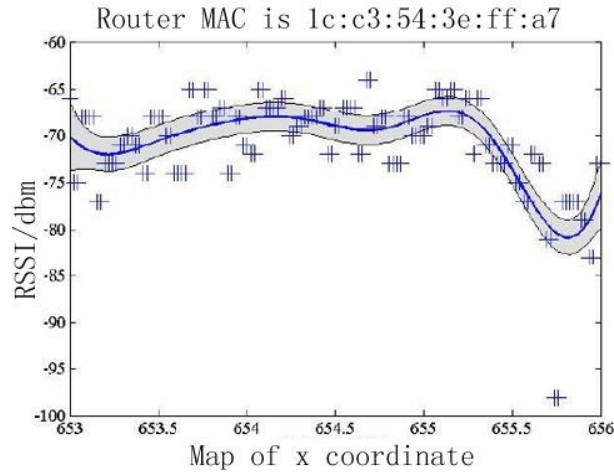
The experiment site in this paper is in the northeast of main building 15<sup>th</sup> floor in Harbin University of Science and Technology, this experiment mainly in empty area except rooms. The collecting device is a E40 laptop that manufactured by Lenovo.

On the stage of offline collection, select 300 patch samples with rectangular grid uniform in an open area, due to the fluctuations of RSS signal, so each point is collected 40 times, then we cite the RSS value as average of that point. Because every place has multiple AP signal, each location fingerprint is composed of RSS value of multiple AP plus a position coordinates, of which the MAC address of AP is used to differentiate the signal source. Because the number of AP in the corridor changes randomly, so in this paper we only choose out the 4 AP that have biggest signal strength as a reference.

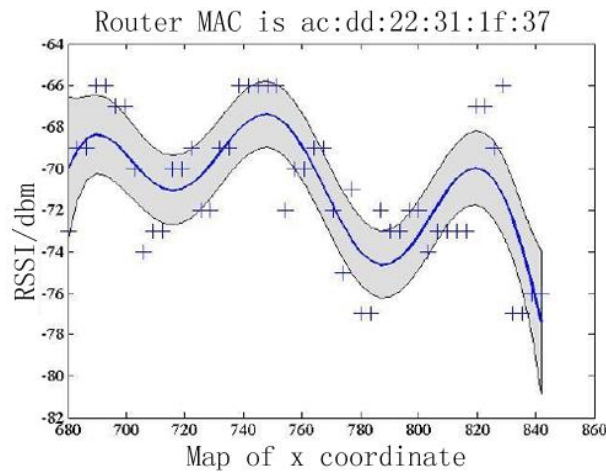
The collected samples were divided into two groups, 150 position fingerprints in one group will be used as the input data in the training, and the Gaussian Process Regression modeling is carried on respectively for each AP, the results of modeling as follows:



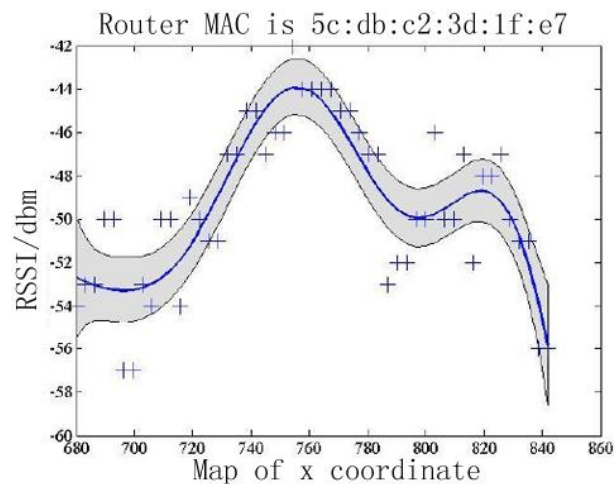
**Figure 1. AP1's Training Diagram**



**Figure 2. AP2's Training Diagram**



**Figure 3. AP3's Training Diagram**



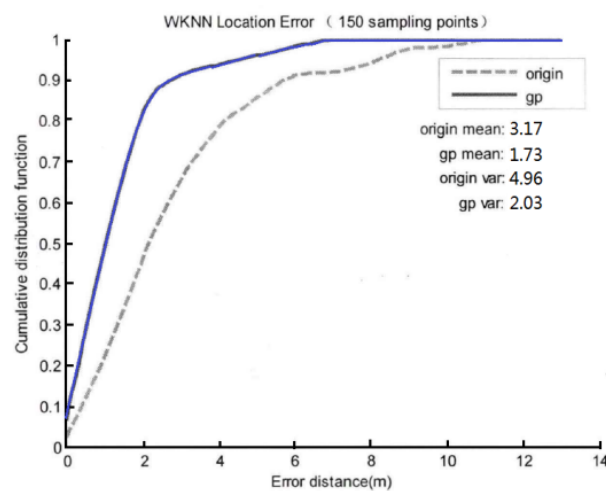
**Figure 4. AP4's Training Diagram**

As shown in Figure 1, Figure 2, Figure 3 and Figure 4, that is training effect diagram of each AP respectively, symbol “+” means the RSS values of the offline stage in the position, the heavy lines represent the training results of Gaussian Process Regression Line, this part is the position fingerprint that have been entered into the database, and the

shadow area represents that the position where the data with higher probability of occurrence.

From the predicted results above, we can build the GPR position fingerprint database, and we structure the plane grid points of positioning area at a certain spacing density, so those locations of the grid point coordinates and the corresponding prediction of the fingerprint can make up a fingerprint database. In this paper, we build a position fingerprint database with a density of 4 point per square meter according to the GPR functions that after training 150 samples, we call it “GPR position fingerprint database” temporarily, and then we build the original fingerprint database directly through 300 collected samples of position fingerprint.

Before the testing, first we select 100 tested points randomly in the position area, then note down the geographic coordinates of those points. The online position is carried on upon the 100 tested points. In this paper, we adopt WKNN algorithm to match the fingerprints of RSS values of tested points, the GPR fingerprint database and the original fingerprint database respectively, the test results as follows:



**Figure 5. Comparison of Position Error**

As shown in Figure 5, horizontal axis represents the position error, which means the distance between the predicted values and the true position, the vertical axis is the accumulated probability; Dashes are the results that based on the original fingerprint database. Solid line is the position results that based on GPR fingerprint database. As we can see, when the number of training sample is 150, the accuracy of original fingerprint positioning is 3.17m and the variance is 4.96; while the accuracy of GPR fingerprint position is 1.73m and the variance is 2.03.

Thus it can be seen that, the accuracy of position fingerprint database with 150 samples that based on GPR is higher than the one that based on original fingerprint database with 300 samples, and the GPR method needs less number of samples.

## 5. Conclusion

In this paper, the statistical analysis of the indoor WIFI signal strength has carried on, by using Gaussian Process Regression model we built the fingerprint database which is based on position fingerprint positioning technology. The experiments results show that in the case of the positions of AP are unnecessary to know, we could predict the fingerprint accurately and build the position fingerprint database by using less position fingerprint samples data. By doing this we can greatly reduce the human input in the fingerprint collection, and there is a dramatic increment in the applied efficiency of position fingerprint positioning technology.



## References

- [1] Q. Liang, "Application Research of Cultural Algorithms in Mobile Station Location Technology", *Journal of Harbin University of Science and Technology*, vol. 17, no. 1, (2012), pp. 74-77.
- [2] W. He, J. Xu and T. Ye, "Detection of GNSS Weak Signal in Cross Correlation Interference", *ACTA ELECTRONICA SINICA*, vol. 39, (2011), pp. 471-475.
- [3] X. Wen, Y. Huang and W. An, "Research on the Indoor Location Algorithm and Antenna Arrangement Scheme of RFID", *Journal of Xihua University*, (2015), pp. 79-85.
- [4] G. Shi, B. Wang and B. Wu, "Overview of Indoor Localization Method Based on WiFi and Mobile Smart Terminal", *Computer Engineering*, vol. 41, no. 9, (2015), pp. 39-50.
- [5] L. Girod and D. Estrin, "Robust range estimation using acoustic and multimodal sensing", *Conf Intelligent Robots and Systems*, (2001), pp. 1312-1320.
- [6] I. M. Escalona and F. B. Arroyo, "Impact of geometry on the accuracy of the passive-TDOA algorithm", *IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications*, (2008), pp. 1-6.
- [7] P. Brida, J. Machaj and J. Benikovsky, "A new complexangle of arrival location method for ad hoc networks", *Proceedings of the 7th Workshop on Positioning Navigation and Communication*, (2010), pp. 284-290.
- [8] L. Girod, V. Bychkovskiy and J. Elson, "Locating tiny sensors in time and space: a case study", *International Conference on Computer Design*, (2002), pp. 214-219.
- [9] C. Zhang, J. Guo and J. Gui, "Indoor positioning optimization techniques based on RSSI", *Computer Engineering and Applications*, vol. 51, no. 3, (2015), pp. 235-238.
- [10] Y. Li, K. Xu and J. Shao, "Updating Fingerprint Database by Crowdsourcing for Wi-Fi Indoor Localization", *Chinese Journal of Sensors and Actuators*, vol. 27, no. 12, (2014), pp. 1692-1698.
- [11] Z. He, G. Liu, X. Zhao and M. Wang, "Overview of Gaussian process regression", *Control and Decision*, vol. 28, no. 8, (2013), pp. 1121-1137.

