Robust Predictive Control of Input Constraints and Interference Suppression for Semi-Trailer System

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Abstract

In this paper, an online receding robust predictive control scheme is proposed for input-constrained semi-trailer system with delay and disturbance. The controller method meets the requirements of control constraint and, based on dual-mode control, the method is obtained by online optimization of performance index. The performance index is the cumulative sum of quadratic weighted value of minimal states. Controller output is calculated by means of linear matrix inequality (LMI), and the controller itself can ensure the asymptotic stability and disturbance attenuation of semi-trailer closed-loop system. Finally, simulation results confirm the effectiveness of the method.

Keywords: semi-trailer system, time-delay and disturbance system, robust predictive control, dual-mode control, LMI

1. Introduction

With the development of transportation technology, driving safety and driving comfort have become an increasingly important indicator for vehicles [1]. A semi-trailer is an efficient means for long-distance transportation. Its fuel consumption per 100 ton kilometers is 40% lower than that of a bus, and its maintenance cost is lower, too. Therefore, vigorously developing semi-trailer transportation has become an important approach to improve transportation efficiency, because of factors such as the large mass and high center of gravity of semi-trailer, its driving stability and safety are great concerns. There are frequent accidents as rollover, folding and shimmy, not only damaging vehicles and drivers, but causing significant harm to other vehicles on road as well. Therefore ride performance of semi-trailer has become one of the important issues that need to be addressed. Ride performance is the ability of the vehicle system to ease and attenuate road shock so as to keep a certain comfort for drivers in a vibration environment when the vehicle is on the road, and also the ability of trucks to maintain the intactness of goods. However, there are conditions of longitudinal and transverse ramps or uneven road surface, such as sandstone road, mountainous road, forest and plateau. Under such conditions, once a vibration problem occurs, the longitudinal and lateral adhesion performance of semi-trailer’s tires will be reduced. For loaded goods, vibration can cause a negative impact on the reliability of materials [1, 2]. Semi-trailer’s vibration comes mainly from the ground roughness, followed by change of force from the engine, transmission system, steering system, and brake system as well as the imbalance and deflection of wheels, etc. These are the major factors affecting semi-trailer’s ride performance. The vibrations caused by the impact can also reduce the life of the semi-trailer and its steering stability. Therefore, to improve the semi-trailer’s ride performance, vibration must be reduced and semi-trailer’s anti-vibration performance must be improved.
In the driving process, because of time-delay [3] of the semi-trailer as well as external disturbance and input constraint, conventional control methods such as LQG/LTR [4, 5] and PD [6] cannot obtain good effect. Some engineers have turned their eyes to modern control theory. Model predictive control (MPC), based on predictive model [7], is an efficient control method to deal with control and state constraints [8~12]. Compared with single-state feedback control, predictive control replaces global one-time optimization with receding optimization, which means that the optimization process is not an offline one-time process but rather repeated online optimizations and receding implementations, so that uncertainties caused by model mismatch, time-varying and interference can get a timely remedy. This receding optimization strategy has taken into account both the ideal optimization for a sufficient long period of time in the future and the impact of actual uncertainties. Therefore, the establishment of a receding optimization strategy in a finite time domain will be more effective. Literature [13] gives a detailed summary of research on robust predictive control. In view of bounded disturbances of linear time-invariant system, $H_\infty$ controller with receding horizon is designed in literature [14], which can guarantee the stability of a closed-loop system and can meet the $H_\infty$ index. Since $H_\infty$ control directly designs controller in state space and it has strengths such as precise calculation and maximum optimization, it provides the uncertain MIMO system of model perturbation with a controller design method which can ensure robust stability of the control system and can optimize certain performance indicators [15]. Therefore, many scholars have combined the method with predictive control algorithm. In literature [16], an MPC algorithm is proposed for linear systems with input constraints and unknown time-delay. This algorithm is given under the assumption that the terminal matrix of time-delay is a constant matrix. Under this assumption, even though the optimal cost function should be transformed into a simple optimization problem of two equivalents, it can be relatively easy to obtain a controller with asymptotic stability. In literature [17], a new Lyapunov function is constructed for the same system to improve the rapidity of performance indexes. But the system has not taken into account disturbance factor. Literature [17-19] applies predictive control to network control, in which literature [18] considers a nonlinear system; literature [19] designs an output feedback predictive control method for a class of constrained linear systems; literature [20] designs a minimum-maximum predictive controller for a class of nonlinear network with delay and packet loss.

It is known that as the calculation amount of predictive control algorithm is large, the implementation of predictive control algorithm usually requires the use of high-performance computing devices. However, with the development of field control devices and embedded system as well as the continued improvements in the frequency of related microcontrollers, predictive control algorithm has also begun to penetrate underlying control and has been applied in PLC, FPGA and other devices [21].

Based on the above analysis, a predictive controller design method is put forward for semi-trailer system’s input constraints, uncertain time-delay and vibration. In predictive control, the dual mode control is used, and appropriate Lyapunov function is chosen to improve the performance of the system. A $H_\infty$ controller with receding horizon is designed based on that, which can guarantee the stability of the system and disturbance attenuation.
2. Description of Problems

Consider a semi-trailer system [21]: Consider as Figure 1:

![Figure 1. Semi-trailer Structure Chart](image)

where $x_1(k), x_2(k), x_3(k), u(k)$ respectively represent the angle difference between truck and trailer, trailer angle, vertical position of trailer tail and swinging angle of truck; truck length is $l$, trailer length is $L$, reverse speed is $v$. $a \in [0,1)$ refers to lagged variable coefficient and $T$ refers to sampling period. The swinging angle of truck is bounded by the condition $\|u(k)\| \leq \pi$. When $x_1(k)$ and $u(k)$ change slightly, the mathematical model is as follows:

$$\begin{align*}
    x_1(k+1) &= x_1(k) - \left(\frac{vT}{L}\right) x_1(k) + \left(\frac{vT}{L}\right) u(k) \\
    x_2(k+1) &= \left(\frac{vT}{L}\right) x_1(k) + x_2(k) \\
    x_3(k+1) &= vT \sin \left[\left(\frac{vT}{2L}\right) x_1(k) + x_2(k)\right] + x_3(k)
\end{align*}$$

(1)

As it is a non-rigid chain link from the truck to the trailer, let $a \in [0,1)$ be a lag coefficient, and add formula (1) to lags:

$$\begin{align*}
    x_1(k+1) &= x_1(k) - a \left(\frac{vT}{L}\right) x_1(k) + (1-a) \left(\frac{vT}{L}\right) u(k) \\
    x_2(k+1) &= a \left(\frac{vT}{L}\right) x_1(k) + (1-a) \left(\frac{vT}{L}\right) x_2(k) + x_2(k) \\
    x_3(k+1) &= vT \sin \left[\left(\frac{vT}{2L}\right) x_1(k) + x_2(k)\right] + x_3(k)
\end{align*}$$

(2)

In formula (2), $x_3(k)$ is with a linear form. Let it be an approximation linear and consider vibration of truck and trailer in the process of driving, and then formula (2) can be converted into formula (3) which is shown below:
\[
\begin{aligned}
\begin{cases}
    x_1(k+1) = x_1(k) - a_1 (vT/L) x_1(k) + (1 - a_1) (vT/L) \\
           & + x_1(k - d_1) + (vT/L) u(k) + b_1 w(k) \\
    x_2(k+1) = a_1 (vT/L) x_1(k) + (1 - a_1) (vT/L) \\
           & + x_1(k - d_1) + x_2(k) + b_2 w(k) \\
    x_3(k+1) = a_2 (vT^2/2L) x_1(k) + (1 - a_2) (vT^2/2L) \\
           & + x_1(k - d_1) + x_2(k) + x_3(k) + b_3 w(k)
\end{cases}
\end{aligned}
\]

(3)

Convert formula (3) into a matrix form:

\[
x(k+1) = A(k)x(k) + \sum_{j=1}^p A_j(k)x(k - d_j) + \\
B(k)u(k) + B_1(k)w(k)
\]

(4)

where \(x(k) \in \mathbb{R}^n\), \(u(k) \in \mathbb{R}^m\) and \(w(k) \in \mathbb{R}\) respectively refer to the system state, input and disturbance signal. \(d_j > 0 \) \((j = 1, \cdots, p)\) is an unknown time-delay constant. Suppose \(0 < d_j < \overline{d}_j\), where \(\overline{d}_j\) is a known constant. Meanwhile, for the convenience of calculation, \(\overline{d}_i \leq \overline{d}_j (i < j)\) is set. \(A(k), A_j(k)\) and \(B(k), B_1(k)\) refer to uncertain time-varying matrix with corresponding dimensionality:

\[
A(k) = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]

\[
A_j(k) = \begin{bmatrix}
    a_{j11} & a_{j12} & \cdots & a_{j1n} \\
    a_{j21} & a_{j22} & \cdots & a_{j2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{jn1} & a_{jn2} & \cdots & a_{jnn}
\end{bmatrix}
\]

\[
B(k) = \begin{bmatrix}
    b_{11}(k) \\
    b_{12}(k) \\
    \vdots \\
    b_{1p}(k)
\end{bmatrix}, \quad B_1(k) = \begin{bmatrix}
    b_{11}(k) \\
    b_{12}(k) \\
    \vdots \\
    b_{1p}(k)
\end{bmatrix}
\]

System (4) also satisfies the assumed condition: persistent disturbance \(w(k)\) has a known upper bound, \textit{i.e.}, \(\|w(k)\| < \overline{r}, \overline{r} > 0\)

An evaluation signal is defined:

\[
z(k) = [C_1 x(k), C_1 x(k - \overline{d}_1), C_2 x(k - \overline{d}_2), \cdots, \\
C_p x(k - \overline{d}_p), R^{1/2} u(k)]^T
\]

(5)
where \( C \cdot C_j (j = 1, \ldots, p) \in R^{m \times m} \) respectively refer to weighting matrix for quantity of state and quantity of time-delay state; \( R \in R^{m \times m} \) refers to a positive definite symmetric weighting matrix for input quantity.

Quadratic weighted accumulative value of minimized state is taken as the performance index. Meanwhile disturbance factor is introduced into the performance index. A controller is designed for system (4), so that the corresponding closed-loop system can meet the following performance:

1) The controller can satisfy the constraint condition, namely,
\[
\| u_j(k) \| \leq U_{j,\text{max}} \quad \forall k \geq 0, \ j = 1, 2 \cdots m
\]
and the following performance index can reach the minimum value.

\[
J(k) = J_{1}(k) + J_{2}(k)
\]
\[
= \sum_{i=0}^{N-1} \left( \| z(k+i/k) \|^2 - \gamma^2 \| w(k+i/k) \|^2 \right) + \sum_{i=0}^{N-2} \left( \| z(k+i/k) \|^2 - \gamma^2 \| w(k+i/k) \|^2 \right)
\]

where \( J_{1}(k) \) represents the finite time domain, and \( J_{2}(k) \) represents the infinite time domain, which can be converted into terminal cost function. \( x(k+i/k) \) indicates a state predictive value at \( k+i \) moment based on model (4) at \( k \) moment, and \( u(k+i/k) \) indicates the predictive control at the \( i \)th step of \( k \) moment.

2) When \( w(k) = 0 \), the system is asymptotically stable; when \( w(k) \neq 0 \), \( L_2 \) norm from disturbing \( w(k) \) signal to evaluation signal \( z(k) \) is no more than \( \gamma \), namely, \( \| z \| \leq \gamma \| w \| \).

3. Main Results

3.1. Controller Design

This controller is designed by the thinking of dual-mode control [22]. Assume that the parameters in system (4) are accurate, and it is a nominal model, i.e., the "closest" model to the actual system. When system constraints are not considered, closed-loop linear feedback form is used:

\[
u(k+i/k) = F(k+i/k) x(k+i/k)\]

The above state feedback control law allows unconstrained nominal system to have a stable closed-loop and to be optimal in a certain sense. When system constraints are considered, a set of auxiliary variables \( e(k+i/k) \) are added to control law (8), and then (8) becomes:

\[
u(k+i/k) = F(k+i/k) x(k+i/k) + e(k+i/k)\]

in which:

\[
F(k+i/k) = F(k+i/k-1), \ i = 1, 2 \cdots N - 2
\]
\[ F(k + N - 1/k) = F(k - 1) \]

As \( e(k + i/k) \) is a compensation term added to control law to meet the constraint condition, \( e(k + i/k) = 0 \) when the system constraints do not work.

Let the prediction horizon be \( N \). Combine formula (8) and (9) together:

\[
\begin{align*}
F(k + i/k) &= F(k + i/k) x(k + i/k) + e(k + i/k) \\
u(k + i/k) &= F(k)x(k + i/k) \\
& \quad i \geq N
\end{align*}
\]

Formula (10) and (11) are the dual-mode control strategy: in the prediction horizon, control law (10) with auxiliary variables is used to ensure that the system terminal state enters the robust invariant set structured by feedback control law and constraint; outside of the prediction horizon, control law (11) is used to ensure the stability of the closed-loop system. It should be noted that \( F(k + i/k) \), \( F(k) \) in formula (10) and formula (11) are called feedback gain in conventional control theory. There are used here to reflect the gain scheduling thinking of predictive control.

**Theorem 1**: Suppose \( F(k) = YQ \) in formula (10) and (11) for the system described in the equation (4). If there are variables \( Y, Q_1, Q_2, \ldots, Q_p, r, F(k + i/k), e(k + i/k), e, (i = 0, 1, \ldots, N - 1) \), then the control law (10) and (11) is used to minimize performance index online under the condition of satisfying control constraints, in which these variables are the optimal solutions for the following LMI problems.

\[
\begin{align*}
\min_{r, e, Q_1, \ldots, Q_p, Y, e(k + i/k)} & \sum_{i=0}^{N-1} \beta_i (k + N / k) e(k + i/k) \\
\begin{bmatrix}
1 \\
-m \beta_i & \cdots & \beta_0 (k + N / k)
\end{bmatrix} & > 0
\end{align*}
\]

\[
\begin{bmatrix}
-H & -Q_0 \\
0 & 0 & -\beta \\
K & 0 & 0 & -rR^{-1}
\end{bmatrix} < 0
\]
\[
\begin{bmatrix}
\gamma^2 r^{-1} & * & * \\
-C_x k i d k I (k + i - d_i / l) & 1 & * \\
\vdots & \vdots & \ddots \\
-C_p x k i d k I (k + i - d_p / l) & 0 & \ldots \\
F (k + i / l) x k i d k I (k + i / l) + e (k + i / l) & 0 & \ldots \\
\end{bmatrix}
\]
(15)

\[\begin{bmatrix}
U^2_{j,\max} & * \\
Y^\top & Q_0
\end{bmatrix} \geq 0
\]
(16)

\[\|F (k + i / l) x (k + i / l) + e (k + i / l)\| \leq U_{j,\max}
\]
(17)

where \( \wedge = \text{diag} \{ (Q_0, Q_1, \ldots, Q_p), \gamma^2 r^{-1} \} \),
\[H = \begin{bmatrix}
A Q + B Y & A'_i Q_i & \ldots & A'_p Q_p
\end{bmatrix}, \quad i = [1, 2, \ldots, l],
\]
\[N = \text{diag} \{ C Q_0, C Q_1, \ldots, C Q_p, 0, 0 \},
\]
\[\beta = \text{diag} \{ r, r, \ldots, r \},
\]
\[K = \text{diag} \{ Q_0, Q_1, \ldots, Q_p, 0, 0, 0 \},
\]
\[\theta = \text{diag} \{ (\bar{d}_1 - d_1) Q_0, (\bar{d}_2 - d_2) Q_1, \ldots, (\bar{d}_p - d_p) Q_p, 1, 1, 1 \},
\]
\[m_{x k i d k I} (k + N / l) \geq 0, \quad l \leq i \leq N - 1,
\]
includes all possible \( m (k + N / l) \) compact sets, and
\[m (k + i / l) = \begin{bmatrix}
x (k + i / l) \top, x (k + i - 1 / l) \top, \\
\vdots \\
x (k + i / l) \top, x (k + i - 2 / l) \top,
\end{bmatrix}, \quad i = 1, 2, \ldots, N
\]
\[\bar{\eta} = \text{diag} \{ Q_0^{-1}, \ldots, (d_1 - 1) Q_0^{-1} \ldots Q_0^{-1},
\]
\[\begin{bmatrix}
(\bar{d}_1 - d_1) Q_0^{-1}, (\bar{d}_2 - d_2 - 1) Q_1^{-1} \ldots Q_1^{-1}, \ldots \\
(\bar{d}_p - d_p - 1) Q_p^{-1} \ldots Q_p^{-1}
\end{bmatrix},
\]
The symbol \( * \) indicates symmetric items in the symmetric matrix, and the following relationships exist:
\[Q_0 = r P_0^{-1}, \quad Q_1 = r P_1^{-1}, \ldots, Q_p = r P_p^{-1}, \quad Y = F (k) Q_0^{-1}
\]
See appendix for the proof of Theorem 1.

Therefore, the following algorithm steps can be obtained by using Theorem 1:
Step 1 Let \( x(k/k) = x(k) \) at \( k \) moment.

Step 2 By solving LMI (8) -(13), the expressions of \( F(k), F(k + i/k), e(k + i/k) \)
\((i = 0, 1, \cdots, N - 1)\) can be obtained.

Step 3 \( u(k/k) = F(k/k)x(k/k) + e(k/k) \) is applied to system (4), and \( x(k+1) \) is calculated.

Step 4 Let \( k = k + 1 \), and repeat from Step 1 to Step 3.

3.2. Analysis of Stability and Disturbance Attenuation

Theorem 2: If the algorithm obtained by Theorem 1 has a feasible solution at the initial moment \( k = 0 \), the system is feasible for any \( k \geq 0 \), and the closed-loop system is with asymptotical stability and \( H \infty \) disturbance attenuation.

Proof: Assume that Theorem 1 has a feasible solution at the initial moment \( k = 0 \)
(expressioned by the symbol *): \( F^*(k), e^*(k + i/k), \eta^-(k) \). At this point, the following solution
is taken as a feasible solution at \( k + 1 \) moment:

\[
\begin{align*}
&u(k + i + 1/k + 1) = F^*(k + i + 1/k)x^*(k + i + 1/k) \\
&+ e^*(k + i + 1/k) \quad i = 0, 1, \ldots, N - 2 \\
&u(k + i + N/k + 1) = F^*(k)x^*(k + i + N/k), \quad (i \geq 0) \\
&\eta(k + 1) = \eta^*(k)
\end{align*}
\]

\[ J(k + 1) = \]
\[
\sum_{i=0}^{N-1} \left[ \|z^*(k + i + 1/k + 1)\|^2 - \gamma^2 \|e(k + i + 1/k + 1)\|^2 \right] + \\
+ m^2(k + N + 1/k + 1)\eta(k + 1)m(k + N + 1/k + 1)
\]
\[
= \sum_{i=0}^{N-1} \left[ \|z^*(k + i/k)\|^2 + \gamma^2 \|w^*(k + i/k)\|^2 \right] + \\
m^2(k + N + 1/k)\eta^*(k)m(k + N + 1/k)
\]

Equation (A2) is set up in the appendix, so the following can be obtained:

\[
\begin{align*}
&\|m(k + N + 1/k)\|_p F^*(k) \leq \|m(k + N/k)\|_p F^*(k) \\
&- \|z^*(k + N/k)\|^2 + \gamma^2 \|w^*(k + N/k)\|^2 \quad (19)
\end{align*}
\]

Add Equation (19) to equation (18):

\[
\begin{align*}
J(k + 1) \leq \sum_{i=0}^{N-1} \left[ \|z^*(k + i/k)\|^2 + \gamma^2 \|w^*(k + i/k)\|^2 \right] + \\
+ m^2(k + N/k)\eta^*(k)m(k + N/k) \leq J^*(k)
\end{align*}
\]

At this point, \( J^*(k + 1) - J^*(k) \leq J(k + 1) - J^*(k) \leq - \|z(k/k)\|^2 + \gamma^2 \|w(k/k)\|^2 \)
Herein $J^*(k+1)$ is obtained by optimizing Theorem 1 at moment $k+1$. At this point, let $J^*(k)$ serve as Lyapunov function in system (4). If $w(k) = 0$, the system is asymptotically stable; if $w(k) \neq 0$, the system has disturbance attenuation.

4. Simulation Research

Consider semi-trailer system (1). The parameters come from literature [22]. Take $l = 2.8m$, $L = 5.5m$, $v = -1.0m/s$ and $t = 2.0 \psi \in [1,1.5915]$, $a = 0.7$, $T = 0.1$, and then the system parameters are:

\[
A_1 = \begin{bmatrix}
1.0509 & 0 & 0 \\
-0.0509 & 1 & 0 \\
0.0509 & 0.4 & 1
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
0.01 \\
0.01 \\
-0.1429
\end{bmatrix}
\]

\[
A_i = \begin{bmatrix}
0.0218 & 0 & 0 \\
-0.0218 & 0 & 0 \\
0.0218 & 0 & 0
\end{bmatrix}, \quad B_i = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

In the simulation, the initial state is selected as $x(0) = [0.5 \pi \ 0.75 \pi \ -5]^T$, where the mean value of disturbance is taken as 0, and the variance is 0.5 white noise signal. The weighting matrix in evaluation signal is selected as $C = [2\ 2\ 2], C_1 = [1\ 1\ 1]$ and $R = 1$, time delay parameter $0 < a \leq d \leq 5$. Take $d = 3$ in the simulation, $L_2$, gain $\gamma = 0.8$. Several experiments show that the values of parameter $C$, $C_1$ and $L_2$ gain $\gamma$ have no significant effect on the control result.

The simulation results have shown the following: based on the robust predictive control algorithm proposed in the paper, Figure 2 indicates that the system is asymptotically stable and the system response speed is rather fast in situations without disturbance. Figure 3 shows that the control input switches back and forth largely at the beginning, and then tends to be stable gradually. Figure 4 shows that controller solved like this improves the performance index of the system, so that the upper bound value of performance index can quickly approach to the minimum value. When a disturbance occurs, Figure 5 shows the system can be rapidly stabilized. Figure 6 shows that control input fluctuates on a small scale and satisfies the constraint conditions. From the viewpoint of actual physical application, the controller suits well. Figure 7 shows that by using the method in this paper, there is no great change between disturbance performance index and non-disturbance performance index, and it can approach to the minimum value quickly.
Figure 2. State Curve of the System Without Disturbance

Figure 3. Control Input Curve without Disturbance

Figure 4. Upper Bound Value of Performance Index without Disturbance
Compared with literature [22], algorithm in the paper does not require a complex fuzzy model of semi-trail system and the controller’s design is simple with better control effects.
5. Conclusions

In this paper, uncertain time-delay and disturbance was considered and a robust predictive control method was proposed for the input-constrained semi-trailer system, which can represent a class of linear time-varying and uncertain time-delay system. Through the selection of dual mode control and Lyapunov function, it was ensured that the semi-trailer system has asymptotic stability and disturbance attenuation, and the disturbance is rapidly restrained by the quick online minimum performance index. These characteristics are very important for semi-trailer system traveling under different road conditions and with severe disturbance. Ultimately, the simulation verified the effectiveness of the method proposed. Furthermore, the robust predictive control method is also applicable for n-order system with time-varying uncertain and time-delay disturbance.

Acknowledgements

This work was supported by technical service project (2013H0306).

References


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