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Robust Power Control Algorithm for Multiple Primary Users and Secondary Users in Cognitive Radio Networks

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Abstract

Conventional distributed power control algorithms in cognitive radio (CR) are based on the assumption of perfect channel state information (CSI) which may lead to performance degradation in practical systems. In this paper, the robust distributed power control problem is investigated in CR systems by considering the uncertainty of channel gain. The objective is to maximize the sum rate of secondary users (SUs) subject to the primary users (PUs) interference power constraints and the transmission power constraint of SUs. The channel gain fluctuation is described using ellipsoid sets and the interference power constraints can be converted into robust interference power constraints. Lagrangian duality method is applied to solve the robust power control problem in a distributed way. Simulation results show that the effectiveness of the proposed robust algorithm.

Keywords: *Cognitive radio, power control, imperfect channel state information, Lagrangian duality method*

1. Introduction

Cognitive radio (CR), as a promising technology to enhancing the utilization efficiency of the scarce radio spectrum, has attracted tremendous interests recently. A key feature of the CR network is to allow a secondary user (SU) to simultaneously share a licensed spectrum as long as the secondary transmission does not interfere with the primary link. As a result, the challenge of the CR network is to protect the primary users (PUs) from harmful interference induced by the SUs as well as to meet the quality of service (QoS) demands of SUs [1].

Power control plays a key role in reducing interference in wireless communications. There are some papers addressing the distributed power control problems in CR from different aspects [2-4]. All these work make the assumption that the parameters defining the objective function and constraints are constant or perfectly known. However, in practical systems, perfect CSI is difficult to obtain due to the loose cooperation between PUs and SUs, as well as many other factors such as inaccurate channel estimation, limited feedback or lack of channel reciprocity. The worst-case approach has been used to design robust power for SUs in a multiple-input single-output (MISO) CR system [5-6]. In [5], the software assisted method and a geometric method were considered for single SU and single PU to find suboptimal solution for the certainty and uncertainty models. For more SUs or PUs, [7] made some approximations for the uncertainty channel model between SUs and PUs. In [8], the worst-case of uncertainty was considered and the initially non-convex uncertainty problems are transformed into a second order cone programming (SOCP) or other convex problems, which can be solved by software. A robust distributed uplink power allocation algorithm for underlay CR networks was proposed in [9]. The objective is to maximize the sum utility of SUs when channel gains from SUs to primary base station (PBS) and interference caused by Pus to the secondary base station (SBS) are

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uncertain. In [10], the authors studied the problem of joint beamforming and power allocation in a cognitive MIMO system using game theory, in this work the imperfect CSI was taken into account by the robust interference constraint. The optimization problem in the formulated robust game is converted into a SOCP problem.

Comparing with previous work on robust power control in CR, in this paper, we study the problem of power control in cognitive downlink where multiple SUs coexisting with multiple PUs. The aim is to maximize the sum rate of SUs under the interference power constraints at PUs, the total transmission power constraint of SUs. An ellipsoid model was adopted to describe the CSI uncertainty and the objective function can be transformed to a concave function through a log transformation. Moreover, a subgradient iteration algorithm is proposed. Simulation results show that the effectiveness of the proposed robust algorithm.

The rest of this paper is organized as follows. In Section 2, the system mode of CR is introduced and the robust power control problem is formulated. In Section 3, the objective function is transformed to a concave function through a log transformation and a subgradient iteration algorithm is proposed. Numerical results are presented in Section 4. Concluding remarks are made in Section 5.

2. System Model and Problem Formulation

We consider a coexisting system where a primary network consisting of a PBS and M PUs coexists with a secondary network with a SBS and K SUs as shown in Figure 1. In the secondary network, SUs operate in the frequency band allocated to the PUs, thus the channels between the base stations and users are inherently interference channels. We consider the downlink of this CR network in which all the BSs and users are equipped with only one antenna. For simplicity, we assume a flat fading model for all channels. Let h_k, $k \in [1, K]$, and g_m, $m \in [1, M]$ denote the channels from the SBS to the k -th SU and to the m -th PU, respectively. Similarly, channels between PBS and the m -th PU and between PBS and the k -th SU are denoted by $\hat{h}_m$ and $\hat{g}_k$, respectively. Then the received signal at SU_k is represented as

$$y_k = \sqrt{p_k} h_k s_k + h_k \sum_{i=1, i \neq k}^{k} \sqrt{p_i} s_i + \hat{g}_k \sum_{m=1}^{M} \sqrt{p_m} s_m + n_k \tag{1}$$

where s_k and s_m are the transmitted signals from SBS to SU_k and that from PBS to PU_m, respectively. Likewise, p_k and p_m account for the transmission power of SU_k and PU_m. And n_k is an additive white Gaussian noise with zero mean and variance σ_k^2.

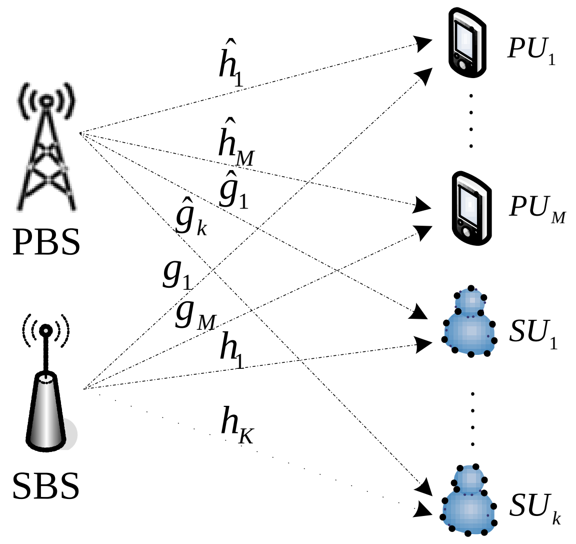


Figure 1. Cognitive Radio System Model. Solid Line Denotes Transmission Channel and Dotted Line Denotes Interference Channel

To protect the communications of PUs, interference power should not exceed a predefined threshold $p_{m,th}$ at PU_m. Therefore, the total received power at a specified PU_m must meet [4].

$$\sum_{k=1}^{K} |g_m|^2 p_k \le p_{m,th} \tag{2}$$

In practice, the CSI between the SBS and the PU is often imperfect due to lack of cooperation. Under ellipsoid approximation [10], the true channel coefficient g_m can be written as

$$g_m = \tilde{g}_m + \Delta_m \tag{3}$$

where $\tilde{g}_m$ is the estimated channel vector at the SBS and Δ_m is a norm-bounded uncertain vector, namely,

$$|\Delta_m|^2 \le c_m \varepsilon_m^2 \tag{4}$$

Here, c_m and ε_m^2 are uncertainty parameter and estimation error, respectively.

The achievable rate at SU_k writes

$$\begin{aligned} R_k &= \log(1 + SINR_k) \\ &= \log(1 + \frac{|h_k|^2 p_k}{\sum_{i=1,i\ne k}^{K} |h_k|^2 p_i + \sum_{j=1}^{M} |\hat{g}_k|^2 p_m + \sigma_k^2}) \end{aligned} \tag{5}$$

where $\sum_{j=1}^{M} |\hat{g}_k|^2 p_m$ is the interference introduced by PUs.

To improve the system performance of the secondary network, we employ a power control strategy to maximize its sum rate under the interference constraints at PUs, the total transmission power constraint of SUs. The optimization problem can be formulated as follows

$$\max_{p_k} \sum_{k=1}^{K} R_k$$
$$\text{s.t.} \sum_{k=1}^{K} |g_m|^2 p_k \le p_{m,th} ,$$
$$\sum_{k=1}^{K} p_k \le P_T . \tag{6}$$

So, the robust power control problem that considers the channel uncertainty can be written as

$$\max_{p_k} \sum_{k=1}^{K} R_k$$
$$\text{s.t.} \sum_{k=1}^{K} |\tilde{g}_m + \Delta_m|^2 p_k \le p_{m,th} ,$$
$$\sum_{k=1}^{K} p_k \le P_T .$$
$$|\Delta_m|^2 \le c_m \varepsilon_m^2 . \tag{7}$$

3. Power Control Game

In this section, we will derive the optimal power allocation strategy at the SBS to maximize the sum rate of the secondary network. We can see that the objective function shown above is a non-linear non-convex function which can be transformed to a concave function through a log transformation. Assume that the distance between two arbitrary SUs is large enough and SINR is much larger than 1, then the achievable rate of SU_k can be approximated as

$$R_k = \log\left(\frac{|h_k|^2 p_k}{\sum_{i=1,i\ne k}^{K} |h_k|^2 p_i + \underbrace{\sum_{j=1}^{M} |\hat{g}_k|^2 p_m + \sigma_k^2}_{\sigma^2}}\right)$$
$$= \log\left(\frac{|h_k|^2 p_k}{\sum_{i=1,i\ne k}^{K} |h_k|^2 p_i + \sigma^2}\right) \tag{8}$$

Define $\tilde{p}_k = \log p_k$, we have

$$R_k = \log\left(\frac{|h_k|^2 e^{\tilde{p}_k}}{\sum_{i=1,i\ne k}^{K} |h_k|^2 e^{\tilde{p}_i} + \sigma^2}\right) \tag{9}$$

It can be verified that the above function is a strictly concave function [11] with respect to $\tilde{p}_k$ .Therefore, the original optimization problem (7) can be transformed as follows

$$\max_{\tilde{p}_k} \sum_{k=1}^{K} \log\left(\frac{|h_k|^2 e^{\tilde{p}_k}}{\sum_{i=1,i\ne k}^{K} |h_k|^2 e^{\tilde{p}_i} + \sigma^2}\right)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \left| \tilde{g}_m + \Delta_m \right|^2 e^{\tilde{p}_k} \le p_{m,th} ,$$
$$\sum_{k=1}^{K} e^{\tilde{p}_k} \le P_T ,$$
$$\left| \Delta_m \right|^2 \le c_m \varepsilon_m^2 . \tag{10}$$

Following similar steps in [12], defining $\hat{\Delta}_m = \frac{1}{\sqrt{c_m}} \Delta_m$, the interference constraint in (2) is equivalent to

$$\sum_{k=1}^{K} \max_{\left|\hat{\Delta}_m\right| \le \varepsilon_m} \left| \tilde{g}_m + \hat{\Delta}_m \right| \sqrt{e^{\tilde{p}_k}} \le \sqrt{p_{m,th}} \tag{11}$$

Using the triangle inequality and the Cauchy-Schwarz inequality with $\left|\hat{\Delta}_m\right| \le \varepsilon_m$, it follows that

$$\begin{aligned} \sum_{k=1}^{K} \left| \tilde{g}_m e^{\tilde{p}_k} + \hat{\Delta}_m \sqrt{c_m} e^{\tilde{p}_k} \right| &\le \sum_{k=1}^{K} \left(\left| \tilde{g}_m e^{\tilde{p}_k} \right| + \left| \hat{\Delta}_m \sqrt{c_m} e^{\tilde{p}_k} \right| \right) \\ &\le \sum_{k=1}^{K} \left| \tilde{g}_m e^{\tilde{p}_k} \right| + \left| \hat{\Delta}_m \sqrt{c_m} e^{\tilde{p}_k} \right| \\ &\le \sum_{k=1}^{K} \left| \tilde{g}_m e^{\tilde{p}_k} \right| + \varepsilon_m \left| \sqrt{c_m} e^{\tilde{p}_k} \right| \end{aligned} \tag{12}$$

where the equality is achieved when

$$\hat{\Delta}_m = \varepsilon_m \frac{\sqrt{c_m} e^{\tilde{p}_k}}{\left| \sqrt{c_m} e^{\tilde{p}_k} \right|} \tag{13}$$

This indicates

$$\sum_{k=1}^{K} \max_{\left|\hat{\Delta}_m\right| \le \varepsilon_m} \left| \left(\tilde{g}_m + \hat{\Delta}_m \right) e^{\tilde{p}_k} \right| = \sum_{k=1}^{K} \left| \tilde{g}_m e^{\tilde{p}_k} \right| + \varepsilon_m \left| \sqrt{c_m} e^{\tilde{p}_k} \right| \tag{14}$$

So (11) is equivalent to

$$\sum_{k=1}^{K} \left| \tilde{g}_m \right| e^{\tilde{p}_k} + \varepsilon_m \left| \sqrt{c_m} \right| e^{\tilde{p}_k} \le \sqrt{p_{m,th}} \tag{15}$$

By combining (10) and (15), the optimization problem can be converted into

$$\max_{\tilde{p}_k} \quad \sum_{k=1}^{K} \log\left(\frac{\left|h_k\right|^2 e^{\tilde{p}_k}}{\sum_{i=1, i \ne k}^{K} \left|h_k\right|^2 e^{\tilde{p}_i} + \sigma^2} \right)$$
$$\text{s.t.} \quad \sum_{k=1}^{K} \left| \tilde{g}_m \right| e^{\tilde{p}_k} + \varepsilon_m \left| \sqrt{c_m} \right| e^{\tilde{p}_k} \le \sqrt{p_{m,th}} ,$$
$$\sum_{k=1}^{K} e^{\tilde{p}_k} \le P_T . \tag{16}$$

Now we get a convex optimization problem which can be solved by Lagrangian duality method [10]. Introducing dual variables λ_m corresponding to the interference power constraint at PU_m and ν corresponding to the total transmit power constraint, the dual problem of (18) reads

$$\min_{\lambda_m,\nu}\max_{\tilde{p}_k} \sum_{k=1}^{K}\log\left(\frac{|h_k|^2 p_k}{\sum_{i=1,i\neq k}^{K}|h_k|^2 p_i+\sigma^2}\right) - \sum_{m=1}^{M}\lambda_m\left(\sum_{k=1}^{K}|\tilde{g}_m| p_k+\varepsilon_m\left|\sqrt{c_m}\, p_k\right|-\sqrt{p_{m,th}}\right) - \nu\left(\sum_{k=1}^{K}p_k-P_T\right) \quad (17)$$

Since the problem is convex, the duality gap is zero. Therefore, solving the dual problem is equivalent to solving the original problem. Applying the KKT optimality conditions, the optimal power allocation strategy is determined as

$$p_k^* = \frac{1}{\ln 2\left(\sum_{m=1}^{M}\lambda_m|\tilde{g}_m|+\nu\right)+\sum_{i=1,i\neq k}^{K}\frac{|h_i|^2}{\sum_{i=1,i\neq l}^{2}p_l+\sigma^2}} \quad (18)$$

The dual variables can be efficiently solved by subgradient method which updates λ_m and ν with step sizes $\delta_m(t)$ and $\xi(t)$, respectively.

$$\lambda_m(t+1)=\left[\lambda_m(t)+\delta_m(t)\left(\sum_{k=1}^{K}|\tilde{g}_m| p_k+\varepsilon_m\left|\sqrt{c_m}\right| p_k-\sqrt{p_{m,th}}\right)\right]^+ \quad (19)$$

$$\nu(t+1)=\left[\nu(t)+\xi(t)\left(\sum_{k=1}^{K}p_k-P_T\right)\right]^+ \quad (20)$$

where t is the iteration time and $[X]^+=\max\{0,X\}$.

To summarize, the algorithm solving the dual problem to determine the optimal power allocation is as follows

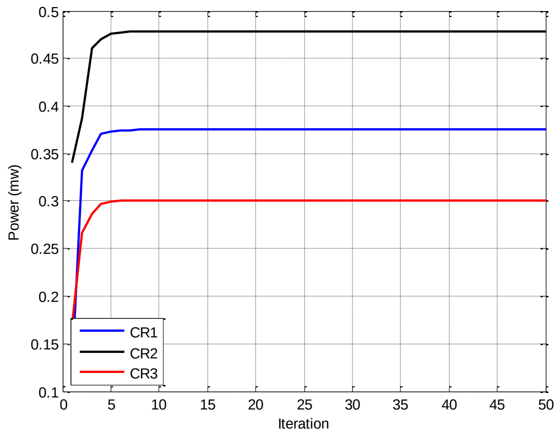
Algorithm 1 The subgradient iteration algorithm

1: Set the initial value $\lambda_m(0)>0$, $\nu(0)>0$, $P(0)=diag\{P_T/K,\cdots,P_T/K\}$.
2: At each iteration t, $t=0,1,2,\cdots$
  1) Obtain the optimal power solution based on (18);
  2) Update λ_m and ν using (19) and (20);
  3) $t=t+1$.
3: Required precision is satisfied.
4: end

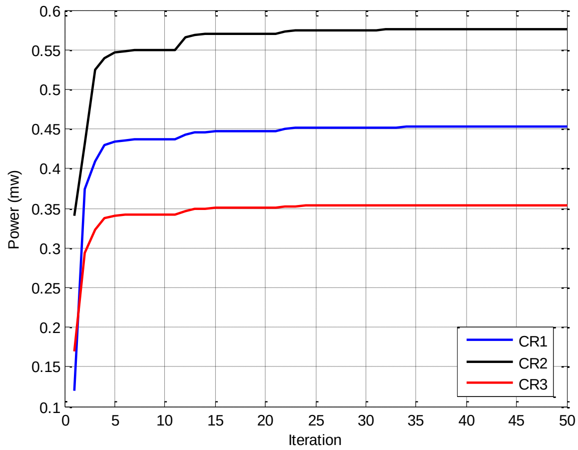
4. Numerical Results

In this section, we present the numerical results to show the convergence of the proposed robust power control algorithm. We also compare the performance of robust and non-robust algorithms in guaranteeing the target SINR requirement at each SU. Assume that there are three SUs and three PUs in the cognitive network, *i.e.*, $K = 3$ and $M = 3$. The uncertainty of channel gains due to imperfect estimation is submitted to Gaussian random distribution. The estimation error is set to $\varepsilon_m^2 = \left|\tilde{g}_m\right|^2$. The target SINR value for each SU is $10\ dB$. The maximum transmit power for each SU is $p_{\max} = 1\ mW$ and the maximum interference level at PU_m is $p_{m,th} = 10 \times 10^{-10}\ mW$.

To illustrate the advantages of the proposed algorithm, we compare the convergence of the proposed algorithm with non-robust algorithm (perfect CSI). From Figure 2 (a) and (b), we can see that the power values obtained by the robust algorithm are lager than those achieved under non-robust algorithm. Robust algorithm takes a longer time than non-robust algorithm to converge because the parameters in robust algorithm are updated at a slower rate.



(a) Non-robust algorithm



(b) Robust algorithm

Figure 2. The Convergence of Power for Each SU

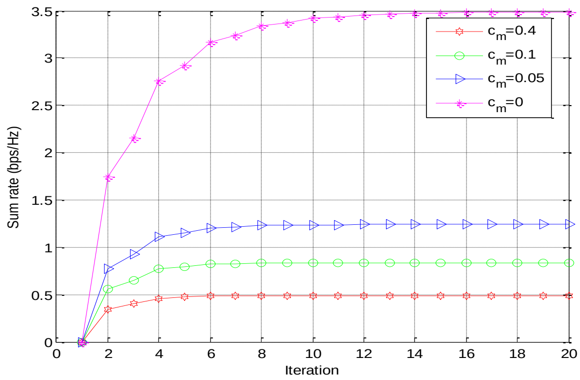


Figure 3. The Convergence of Sum Rate for Different c_m

The uncertainty parameter is set to $c_m = 0, 0.05, 0.1, 0.4$, respectively. When $c_m = 0$, the CSI is perfect. The convergence of sum rate for SUs is shown in Figure 3. It is shown that the sum rate increases as the number of iterations evidently. As the value of the uncertainty parameter c_m increases, the sum rate becomes less. The sum rate of SUs under imperfect CSI converges with the increase of transmission power of SUs.

5. Conclusion

In this paper, we have studied the robust power control problem in CR networks with multiple PUs and multiple SUs under imperfect CSI. An ellipsoid model was adopted to describe the CSI uncertainty. To ensure the QoS of PUs while maximizing the sum rate of SUs, the Lagrangian duality method was applied and a subgradient iteration algorithm is proposed. Simulation results have shown that the iteration algorithm converges to the optimal power solution efficiently satisfying the total transmission power constraint for SUs and interference power constrains at PUs. The sum rate of SUs is also presented to illustrate the performance of the secondary network with the proposed robust algorithm.

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