Robust DERIVATIVE Operator into Gradient Based SIGNAL Processing

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Abstract

Gradient is an important operator in signal processing. A gradient operation always is sensitive to noisy environment. In this paper, a new differential method is designed in the field of one-dimensional signal processing which is robust in low signal to noise ratio. The proposed scheme is suitable for signal differentiation in analog and digital signal processing applications. For low frequencies, this scheme performs the signal derivative, but for high frequencies recasts to a constant gain. This prevents differentiation of high frequency components of added noise. The proposed differential operator is applied to sound and image signals. Results show superiority of the proposed method.

Keywords: *Differentiation scheme; signal processing; noise prohibition*

1. Introduction

Gradient is one of operators that widely used in signal processing. Noisy signal affect to extracted features from signal but gradient-based features disturb signal extremely. Many papers include discussion in removing of gradient problems. In [1], a method of removing gradient artifacts is presented which is applied in noise cancellation from electroencephalography and functional magnetic resonance imaging signals. Smoothing signal using wavelet transform and giving derivative with minimum noise was studied in [2]. In practice, in some references [3, and 4] problem of denoising a signal are considered, as some signal specifications like derivative are measured suitability. Also in wavelet domain in [5] a approach has been presented to the reconstruction of finite signal derivatives from the extrema of a multiscale representation. This work focuses on signal approximation from the multiscale extrema representation of one of its derivatives.

In digital filters, the numerical differentiation is an unstable and risky operation, and should be under taken with great caution because it can greatly amplify noises [6, 7, 8 and 9]. This is mainly due to high frequency components of the noise signal. In the presence of noise, the noise amplification factor of a digital filter is given by the sum of the squares of the filter impulse response [9 and 10]. Also, noise amplification factors of various Savitzky-Golay digital differentiators are functions of the filter length. As the filter length increases or the degree of fitting polynomial decreases, the noise amplification factor deceases [11]. In [12] an autoregressive state-space approach is introduced for numerical differentiation of discrete-time signals, where the signal is parameterized via the autoregressive process using the singular value decomposition based subspace method. An autoregressive (AR) state-space model is then constructed, where the state transition matrix is obtained from the AR coefficients. The numerical algorithms are used in a matrix form, based on the state transition matrix.

As we know, noise disturbs signal and is caused major problems in signal derivatives. Various noises is studied in signal derivation as in [14] effect of jitter noise is checked while derivation of signal. From practical viewpoint, signal derivative widely used in speech, image, video, and other types of signal [13]. For an example, gradient-based image registration [15] relies on image gradients to perform the task of registration. Therefore registration endanger in low signal to noise ratio which [15] use a filter for smoothing gradient. A new scheme for robust gradient vector estimation in color images is presented in [16]. In [17] a new method that is lookup-table-based gradient field reconstruction discussed and in [18] another use of gradient for image restoration described. Another methods that are used for finding the gradient is used of Taylor series, this method is very good and useful but it isn't robust against noise for example in [19, 20, 21] theses methods are presented.

In this paper, a new differentiation operator is presented for analog and digital applications. Noise suppression is an important property of this approach. The proposed method is not based on gradient smoothing, it is a new derivative operator that is based on integrator so we expect smoothing is performed because of its nature. In Section 2, proposed differentiator is presented. In Section 3, the proposed differential operator is applied over sound and image signal. Final section includes the conclusions.

2. The Proposed Differential Operator

The proposed differential operator has been originated from Figure 1. This analog circuit able to obtain inverse function.

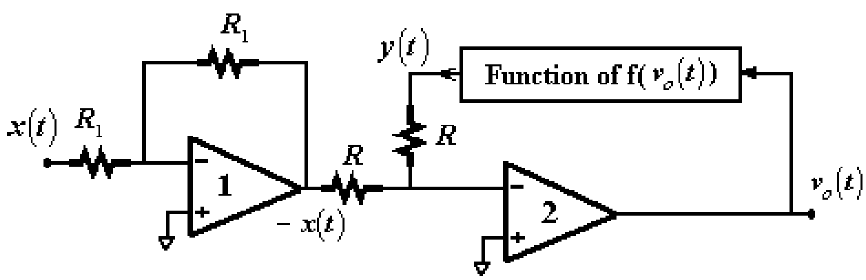


Figure 1. The Block Diagram of Analog Circuit for Obtaining Inverse Function

In Figure 1, function of $v_o(t)$ or one analog computer can be linear or nonlinear function. According this scheme $v_o(t) = f^{-1}(x(t))$. In the following, a novel form of differentiator based on the above scheme is presented.

Firstly, we suppose y(t) is response of universal linear differential equation:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = v_o(t). \tag{1}$$

where y(t) is an unknown dependent variable to be calculated. In the block diagram of Figure 1, KCL at inverting input of the op-amp (2) gives rise to:

$$\frac{-0 - \boldsymbol{y}(\boldsymbol{t})}{\boldsymbol{R}} + \frac{-0 - (-\boldsymbol{x}(\boldsymbol{t}))}{\boldsymbol{R}} = 0 \Rightarrow \\ \boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{x}(\boldsymbol{t}). \tag{2}$$

Substituting Eq.(2) in Eq.(1) gives us:

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \dots + a_0 x(t) = v_o(t). \qquad (3)$$

As Eq. (3) shows, $v_O(t)$ is a linear combination of differentials of x(t). To implement Eq. (1) with integrators, an n-fold integration is performed:

$$\iint \dots \int \left(a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t) \right) (dt)^n = \iint \dots \int (v_o(t))(dt)^n. \qquad (4)$$

Or,

$$y(t) = -\frac{a_{n-1}}{a_n} \int y(t) dt - \frac{a_{n-1}}{a_n} \iint y(t)(dt)^2 - \dots - \frac{a_0}{a_n} \iint \dots \int y(t)(dt)^n + \iint \dots \int (v_o(t))(dt)^n. \qquad (5)$$

The circuit of Figure 2 is used to implement Eq. (5).

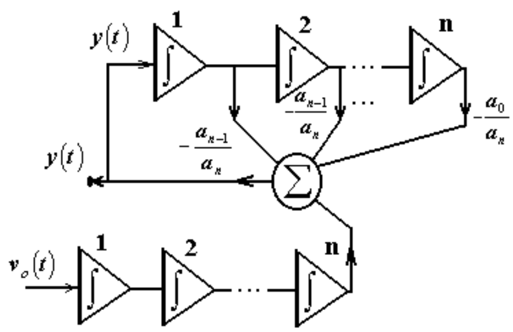


Figure 2. Implementation of Equation (5)

Each integrator is implemented with an op-amp RC circuit, and adder is also constructed using an op-amp adder circuit. Also, from viewpoint of circuit analysis, implementation of integrator is easier than the differentiator.

2.1 Finite Gain Adder

We consider the circuit of Figure 1 with the finite gain adder, as depicted in Figure 3.

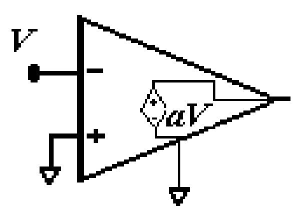


Figure 3. Equivalent Op-amp Circuit

Since the gain is finite, inverting input is not virtual ground. The KCL at the inverting input of the adder gives us:

$$\frac{-\boldsymbol{V}-\boldsymbol{y}(\boldsymbol{t})}{\boldsymbol{R}}+\frac{-\boldsymbol{V}-(-\boldsymbol{x}(\boldsymbol{t}))}{\boldsymbol{R}}=0 \Rightarrow \qquad (6)$$
$$\boldsymbol{V}=\frac{\boldsymbol{x}(\boldsymbol{t})-\boldsymbol{y}(\boldsymbol{t})}{2}.$$

Therefore:

$$\boldsymbol{v_o}(\boldsymbol{t})=\boldsymbol{a}\frac{\boldsymbol{x}(\boldsymbol{t})-\boldsymbol{y}(\boldsymbol{t})}{2} \qquad (7)$$

In order to compute $v_O(t)$, one must insert y(t) from Eq. (5) in Eq. (6).

2.2 Derivative Transfer Function

Suppose in Figure 1,

$$y(t)=\int dt v_o(t) \qquad (8)$$

Using Eq. (7), the transfer function of the system is:

$$H(s)=\frac{V_o(s)}{X(s)}=\frac{as}{s+a} \qquad (9)$$

When the adder gain is infinite, this transfer function reduces to H(s)=s, which is an ideal differentiator. For finite adder gain, on the other hand, the system behavior is different for low and high frequencies. For low frequencies, s<<a, therefore the system transfer function approximates an ideal differentiator. In an image signal usually low frequencies are more important than high frequencies. Therefore the system differentiates the image signal.

For high input frequencies, s>>a, and the system transfer function reduces to H(s)=a, i.e. a constant gain only. The noise signal usually includes low and high frequencies with equal energies. When a noisy signal is differentiated, high frequencies of noise cause serious problems. The derivative of high frequencies of noise will have large amplitudes. With the transfer function of (9), low frequencies of the main signal (and also noise) will be differentiated, whereas high frequencies will be multiplied by a constant gain. The adder gain must be chosen properly so that the system differentiates highest important frequencies in the main signal.

2.3 Digital Implementation of the Differentiator

In a digital signal processing system all equations must be implemented in the difference form. Suppose we want to implement the differentiation operator. The discrete form of Eqs. (7) and (8) are as follows:

$$\boldsymbol{y}[\boldsymbol{n}]=\boldsymbol{y}[\boldsymbol{n}-1]+\boldsymbol{v_o}[\boldsymbol{n}-1], \qquad (10)$$

And,

$$\boldsymbol{v_o}[\boldsymbol{n}]=\boldsymbol{a}\frac{\boldsymbol{x}[\boldsymbol{n}]-\boldsymbol{y}[\boldsymbol{n}]}{2} \qquad (11)$$

Where, n is the sample number. We assume $v_O(0)$ is zero. Equation (10) gives digital integration of v_O and Eq. (11) is the discrete form of Eq. (7).

2.4 Implementing in FPGA

We implement this method in FPGA to showed that this method is very simple and low cost but very useful for this purpose we use system generator in Simulink and implement it.

Figure 4 shows the top level for test and Figure 5 shows the detail of this algorithm:

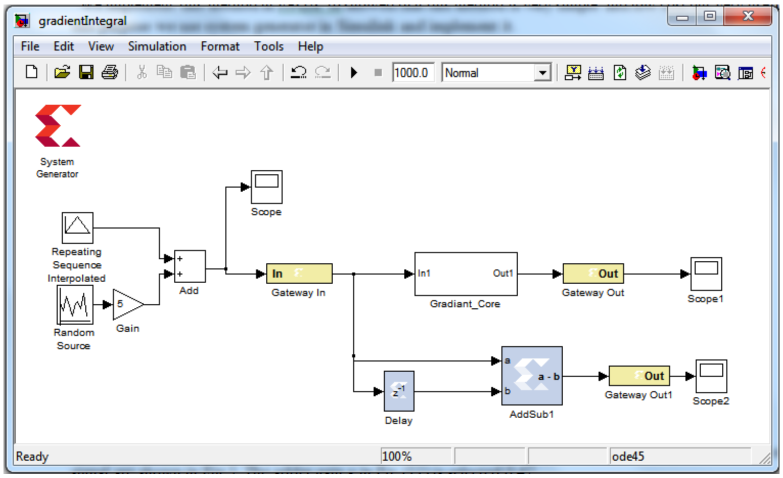


Figure 4. Top Level

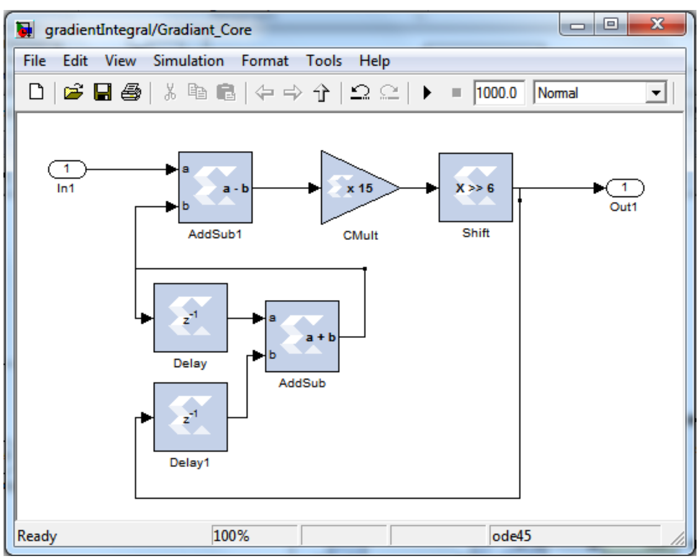


Figure 5. Algorithm Detail

The results are showed if Figures 6 and 7.

Figure 6 shows a noisy triangular waveform. The normal derivative and the proposed signal derivative of the signal are shown in Figure 7. The adder gain a in Eq. (11) is selected 0.47.

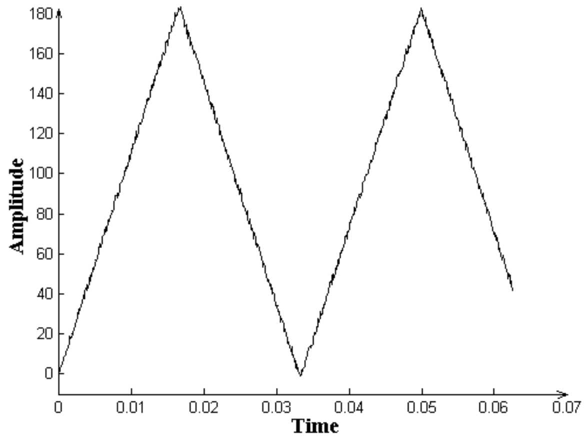


Figure 5. Noisy Triangular Wave

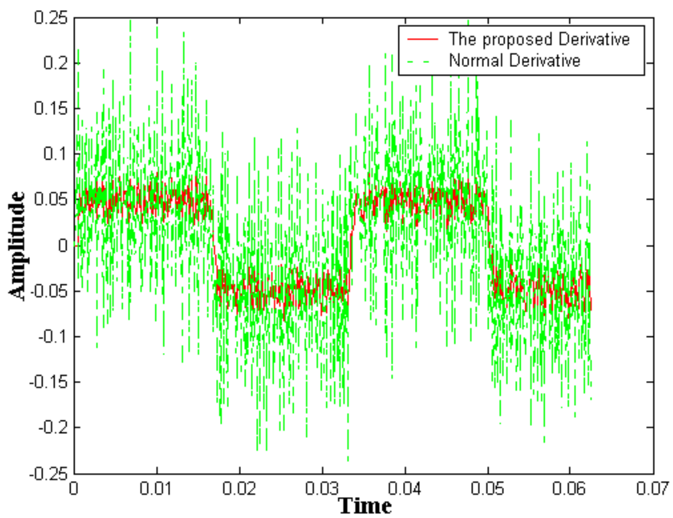


Figure 7. Comparing of the Proposed Method for Signal Derivation and Normal Signal Derivation

We define the performance as:

$$\textit{Performance} = 100\frac{\zeta_N - \zeta_p}{\zeta_N} \tag{12}$$

Where ζ_N is sum of square error between normal derivation of the noisy signal and the derivative of noise free signal, and ζ_p is sum of square error between the proposed derivation of the noisy signal and the derivative of noise free signal. Figure 8 shows the performance of the proposed method compared with the normal derivative of the input signal.

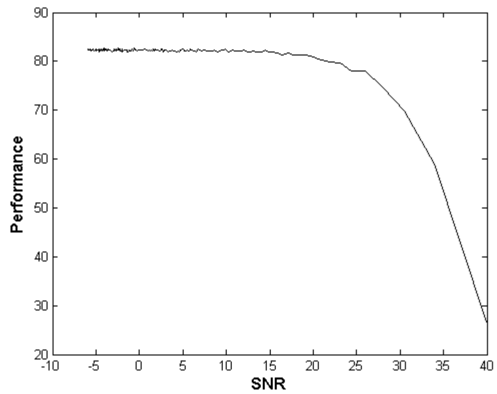


Figure 8. The Performance of the Proposed Method per to different SNR (signal to noise ratio)

In an environment without noise, the normal differentiator can be used, but in normal conditions which noise there exists, the proposed method is superior. Figure 9 shows a noisy sinusoidal signal and its derivation. As this figure shows, the proposed scheme attenuate high frequency noises in the signal derivative.

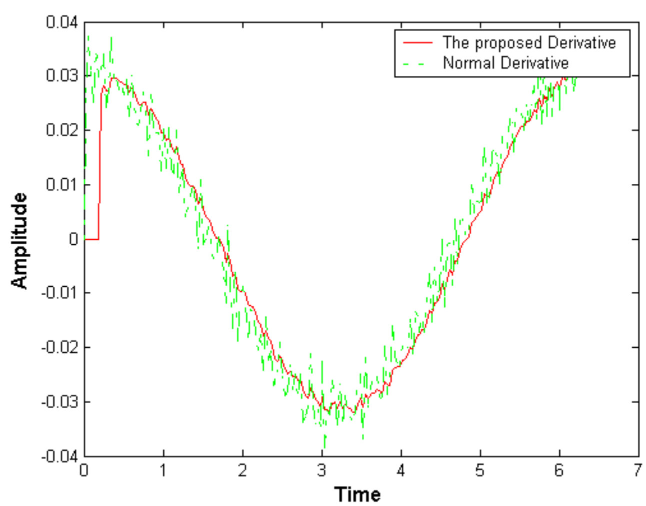


Figure 9. Comparison of the Proposed Method for Signal Derivation and the Normal Signal Derivation

The Z transform of Eqs. of (10) and (11) give,

$$\boldsymbol{H}(\boldsymbol{z})=\frac{\boldsymbol{V_o}(\boldsymbol{z})}{\boldsymbol{X}(\boldsymbol{x})}=\frac{0.5\boldsymbol{a}(\boldsymbol{z}-1)}{\boldsymbol{z}-(0.5\boldsymbol{a}-1)} \tag{13}$$

Root locus of H(z) (sample rate has been selected 0.01 sample per second) for a=1.5 is depicted in Figure 10. It can be obtain for stability of this function $\boldsymbol{a}<4$ until pole of $\boldsymbol{H}(\boldsymbol{z})$ is inside of unity circle. Increasing of $\boldsymbol{a}$ toward 4 is caused pole move to left of unity circle. With increasing of $\boldsymbol{a}$ system respond rapidly but move to instability.

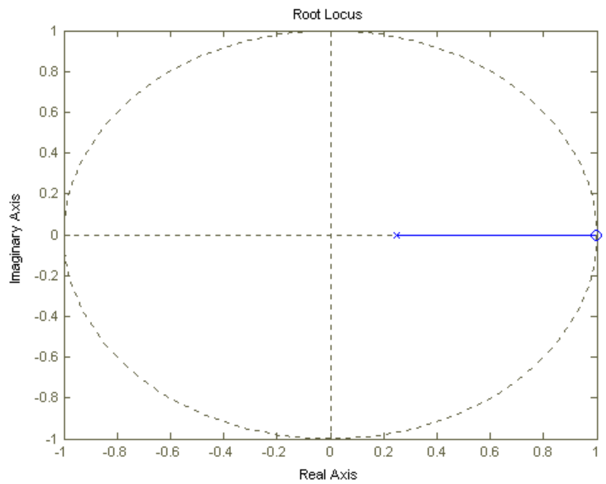


Figure 10. Root Locus of $H(z)$ for $a = 1.5$

End of this section appropriates to utilization of the proposed differentiator over sinusoidal chirp signal. Adaptive recovery of a chirp sinusoid buried in noise or extraction of specifications of chirp signal as derivation is of special interest to researchers because the chirp sinusoid represents a well-defined form of nonstationarity. The chirp signal is given by:

$$x(k) = \sqrt{p_s}\ \exp(\,j[(2\pi f_c + \psi k/2)k + \varphi]) \qquad (14)$$

Where, $\sqrt{p_s}$ denotes the signal amplitude, f_c is the center frequency, ψ is the chirp rate and φ is an arbitrary phase shift. The signal $x(k)$ is deterministic but nonstationary because of the chirping.

Figure 11 shows derivative of noisy chirp. It can be seen that the proposed method give better result relative conventional derivative. But for checking carefully, in Figure 12, a part of derivative signal is shown.

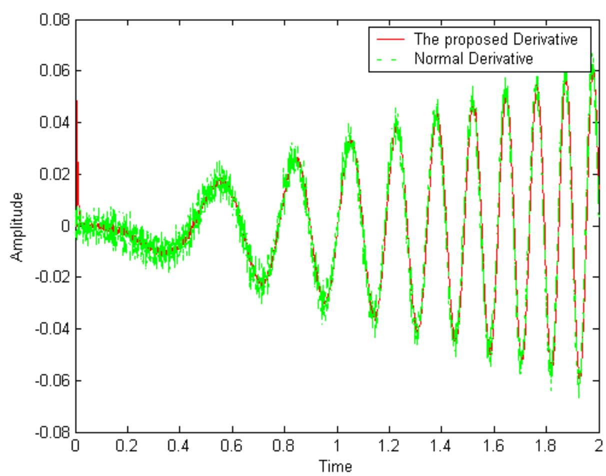


Figure 11. Derivative of Noisy Chirp

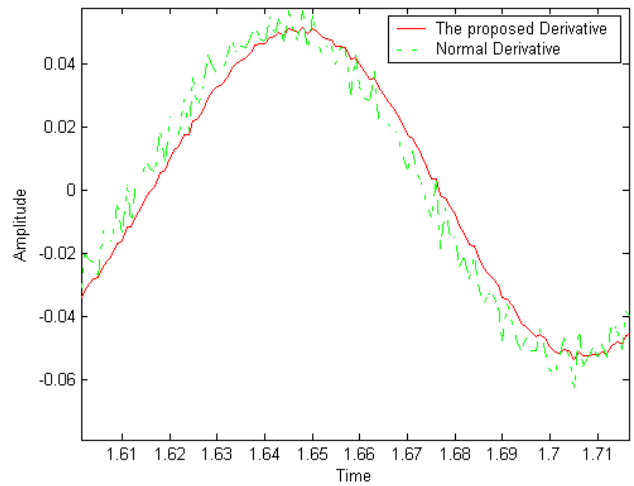


Figure 12. Fine Comparing of the Proposed Derivative and Normal Derivative Over Selected Part of Figure 9

In the next section, the proposed derivative method is applied over sound and image signal.

3. Sound and Image Gradient using the Proposed Derivative

In Figure 13, a Splash sound signal is shown. This signal has been captured with specifications of 8 bit resolution and 22050 samples/sec. In sound and speech processing, many features are based on derivation. Figure 14 shows Splash sound derivative using the proposed method and conventional numerical approach. Figure 15 shows part of figure 12 for better comparing. As shown in this figure, in real signal, signal derivative has many jump points. But the proposed method give excellent results. In this application a=0.47 has been selected (in Eq. (11)).

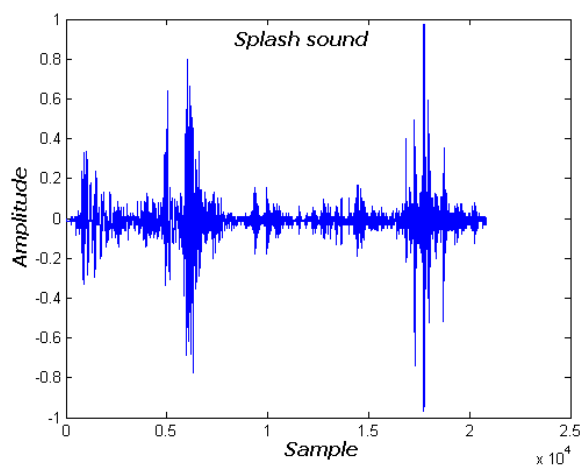


Figure 13. Splash Sound with Specifications of 8 bit Resolution and 22050 samples/sec

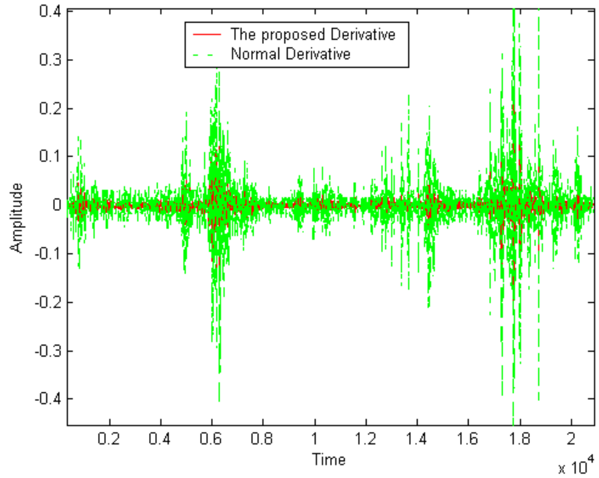


Figure 14. Splash Sound Derivative using the Proposed and Conventional Methods

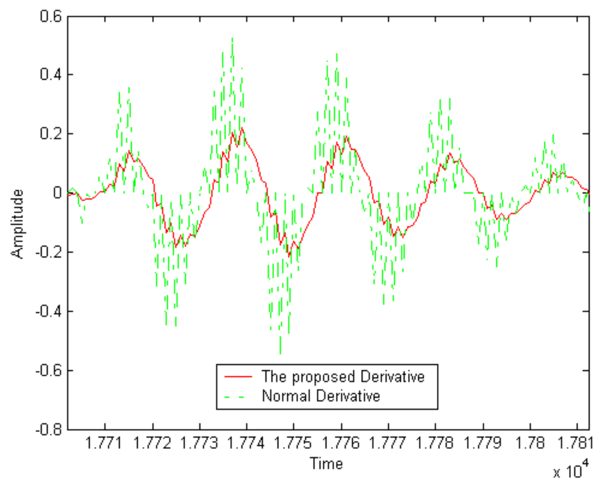
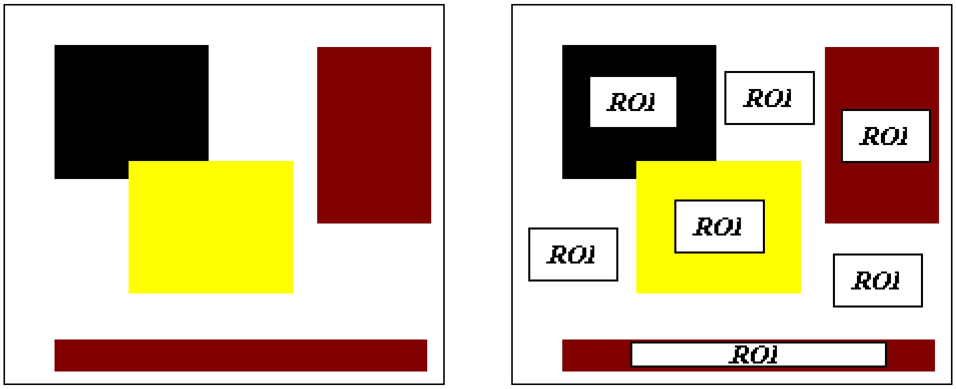


Figure 15. Fine Comparing of the Proposed Derivative and Normal Derivative Over Selected Part of Figure 12

Image gradient is one of main operator in the field of image processing. Finding edge is one of gradient application. For testing of new gradient operator, true position of edge is necessary. A synthetic image as shown in Figure 16 (a) can help us for comparing conventional method and the proposed gradient method. Figure 16 (b) shows regions that we expect result of image gradient has small value. This regions (region of interest (ROI)) use for obtaining a quantitative criterion in comparison of the proposed and conventional operators.



(a) (b)

Figure 16. a) Test Image for Comparing the Conventional Gradient Operator and the Proposed Operator over Image Edge Detection; b) Region of Interest for Checking Result of Gradient

After adding Gaussian noise to test image with zero mean and variance σ^2, following criteria is defined for evaluating of methods.

$$relative\ \ error = \frac{\varepsilon_{proposed} - \varepsilon_{normal}}{\varepsilon_{proposed}} \tag{15}$$

where ε_i is error for method i (proposed or normal gradient) according to following,

$$\varepsilon_i = \sum_{k=1}^{n} \left(\sum_{x,y \in ROI(k)} \nabla f(x,y) \right) \tag{16}$$

where $\nabla f(x,y)$ is image gradient and is equal $|\nabla_x f(x,y)| + |\nabla_y f(x,y)|$ and ∇_x, ∇_y are gradient in x and y directions respectively. In (16) n is number of region of interest. (16) give us sum of residual pixel value in total of ROI. Whatever ε_i be smaller then method is better. (15) help us for comparing of two method, the proposed and conventional methods. A small relative error (smaller than one) means the proposed method has lower error relative to the normal method. Figure 17 shows relative error versus variance of image noise. Noise variance is varied from 0.0001 to 1.000 which corrupted image has been shown in this two variance in Figure 18. In this application a=0.97 has been selected (in Eq. (11)). This result shows superiority of the proposed gradient relative normal gradient.

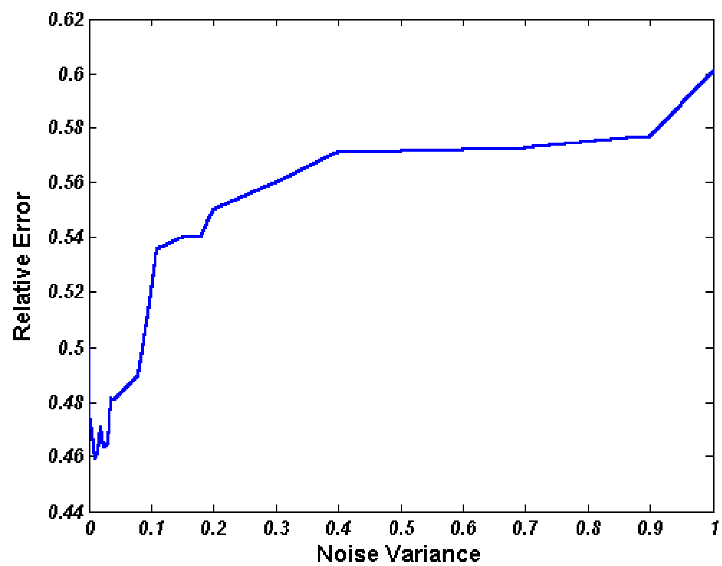


Figure 17. Relative Error versus Variance of Image Noise

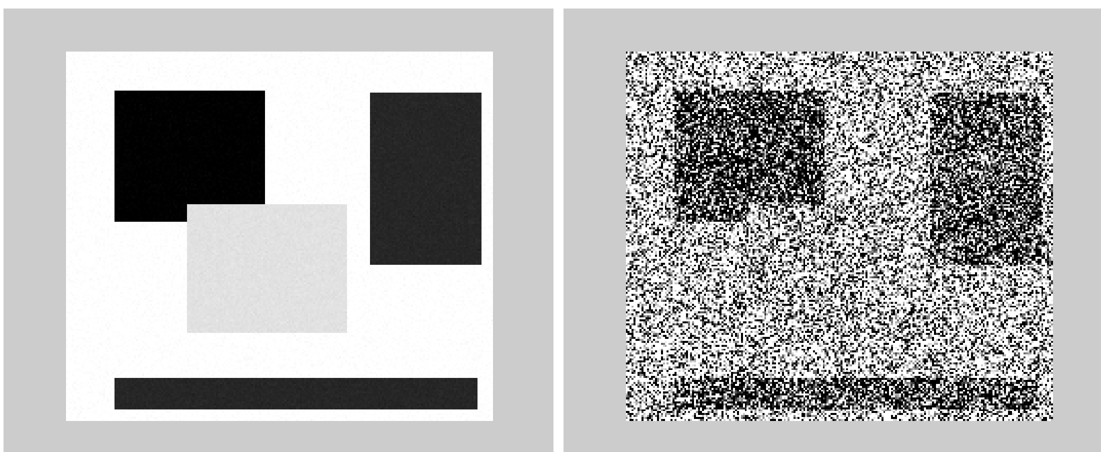


Figure 18. Corrupted Image with Gaussian Noise with Variances of 0.0001 (Left image) and 1.000 (Right Image)

4. Conclusions

When the derivative of a noisy signal is computed, high frequency components of noise causes serious problems. We proposed a new differentiation scheme, which in high frequencies recasts to a constant gain, whereas in low frequencies differentiates the signal. In image signals most of the signal energy is concentrated in low frequency components. On the other hand, noise energy is uniform distribution in the frequency domain. The proposed scheme prevents high frequency components of the noise to differentiate. Examples show that the method efficient for signal derivative.

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