Artificial Robust Control of Robot Arm: Design a Novel SISO Backstepping Adaptive Lyapunov Based Variable Structure Control

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Abstract

This paper examines single input single output (SISO) chattering free variable structure control (VSC) which controller coefficient is on-line tuned by fuzzy backstepping algorithm. VSC methodology is selected as a framework to construct the control law and address the stability and robustness of the close loop system based on Lyapunov formulation. The main goal is to guarantee acceptable trajectories tracking between the robot arm actual output and the desired input. The proposed approach effectively combines the design technique from variable structure controller is based on Lyapunov and fuzzy estimator to estimate the nonlinearity of undefined system dynamic in backstepping controller. The input represents the function between variable structure function, error and the rate of error. The outputs represent fuel ratio, respectively. The fuzzy backstepping methodology is on-line tune the variable structure function based on adaptive methodology. The performance of the SISO VSC which controller coefficient is on-line tuned by fuzzy backstepping algorithm (FBSAVSC) is validated through comparison with VSC and proposed method. Simulation results signify good performance of trajectory in presence of uncertainty and external disturbance.

Keywords: robot arm, variable structure controller, fuzzy backstepping controller, chattering phenomenon, adaptive methodology, Lyapunov based controller

1. Introduction

Robot manipulator is collection of links that connect by joints, these joints can be revolute and prismatic that revolute joint has rotary motion around an axis and prismatic joint has linear motion around an axis. Each joint provides one or more degrees of freedom (DOF). From the mechanical point of view, robot manipulator is divided into two main groups, which called; serial robot links and parallel robot links. In serial robot manipulator, links and joints is serially connected between base and final frame (end-effector). Parallel robot manipulators have many legs with some links and joints, where in these robot manipulators base frame has connected to the final frame. Most of industrial robots are serial links, which in serial robot manipulator the axis of the first three joints has a known as major axis, these axes show the position of end-effector, the axis number four to six are the minor axes that use to calculate the orientation of end-effector, at last the axis number seven to n use to avoid the bad
situation. One of the most important challenges in the field of robot arm is control of robot arm; because this system is multi input multi output (MIMO), nonlinear, time variant parameter and some dynamic parameters are uncertainty [1-2]. Presently, robot arms are used in different (unknown and/or unstructured) situation consequently caused to provide complicated systems, as a result strong mathematical theory are used in new control methodologies to design nonlinear robust controller. Classical and non-classical methods are two main categories of nonlinear plant control, where the conventional (classical) control theory uses the classical method and the non-classical control theory (e.g., fuzzy logic, neural network, and neuro fuzzy) uses the artificial intelligence methods. However both of conventional and artificial intelligence theories have applied effectively in many areas, but these methods also have some limitations [1-2]. Modeling of an entire robot arm is a very important and complicated process because robot arms are nonlinear, MIMO and time variant. One purpose of accurate modeling is to save development costs of real robot arms and minimizing the risks of damaging a robot when validating controller designs [1-10]. Dynamic modeling of robot arm is used to describe the behavior of this system, design of model based controller, and for simulation. The dynamic modeling describes the relationship between nonlinear output formulations to electrical or mechanical source and also it can be used to describe the particular dynamic effects to behavior of system [1, 11-22].

Controller (control system) is a device which can sense information from linear or nonlinear system (e.g., robot arm) to improve the systems performance [3-20]. In feedback control system considering that there are many disturbances and also variable dynamic parameters something that is really necessary is keeping plant variables close to the desired value. Feedback control system development is the most important thing in many different fields of engineering. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error[5-29]. At present, in some applications robot arms are used in unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (e.g., minimum error, good trajectory, disturbance rejection). One of the best nonlinear robust controllers is variable structure control which is used in nonlinear uncertain systems. One of the nonlinear robust controllers is variable structure controller, although this controller has been analyzed by many researchers but the first proposed was in the 1950 [41-62].This controller is used in wide range areas such as in control process, in aerospace applications and in IC engines because this methodology can solve some main challenging topics in control such as resistivity to the external disturbance and stability. Even though, this controller is used in wide range areas but, pure variable structure controller has two drawbacks: Firstly, output oscillation (chattering); which caused the heating in the mechanical parameters. Secondly, nonlinear dynamic formulation of nonlinear systems which applied in nonlinear dynamic nonlinear controller; calculate this control formulation is absolutely difficult because it depends on the dynamic nonlinear system’s equation [20-23]. Chattering phenomenon can causes some problems such as saturation and heats the mechanical parts of robot arm or drivers. To reduce or eliminate the oscillation, various papers have been reported by many researchers which one of the best method is; boundary layer saturation method [1]. In boundary layer linear saturation method, the basic idea is the discontinuous method replacement by linear continuous saturation method with small neighborhood of the switching surface. This replacement caused to considerable chattering reduction. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[21]. Slotine has presented variable structure method with boundary layer to improve the industry application [22]. R. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the
chattering and control the result performance[23]. Moreover, C. C. Weng and W. S. Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method[24]. Neural network, fuzzy logic, and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant, and uncertainty plant (e.g., robot arms). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory [30-41]. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model-based techniques as in classical controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [31-36] but also this method can help engineers to design easier controller. Control robot arms using classical controllers are based on robot arm dynamic modelling. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic model of robot arms. When the system model is unknown or when it is known but complicated, it is difficult or impossible to use classical mathematics to process this model[32]. The main reasons to use fuzzy logic technology are able to give approximate recommended solution for unclear and complicated systems to easy understanding and flexible. Fuzzy logic provides a method which is able to model a controller for nonlinear plant with a set of IF-THEN rules, or it can identify the control actions and describe them by using fuzzy rules [32]. In various dynamic parameters systems that need to be training on-line adaptive control methodology is used. Fuzzy adaptive method is used in systems which want to training parameters by expert knowledge. Traditional adaptive method is used in systems which some dynamic parameters are known. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, adaptive method is applied to variable structure controller. Mohan and Bhanot [40] have addressed comparative study between some adaptive fuzzy, and a new hybrid fuzzy control algorithm for robot arm control. They found that self-organizing fuzzy logic controller and proposed hybrid integrator fuzzy give the best performance as well as simple structure. H.Temeltas [46] has proposed fuzzy adaption techniques for VSC to achieve robust tracking of nonlinear systems and solves the chattering problem. Conversely system’s performance is better than variable structure controller; it is depended on nonlinear dynamic equation. C. L. Hwang et al. [47] have proposed a Tagaki-Sugeno (TS) fuzzy model based variable structure control based on N fuzzy based linear state-space to estimate the uncertainties. A MIMO FVSC reduces the chattering phenomenon and reconstructs the approximate the unknown system has been presented for a nonlinear system [42]. Yoo and Ham [58] have proposed a MIMO fuzzy system to help the compensation and estimation the torque coupling. This method can only tune the consequence part of the fuzzy rules. Medhafer et al. [59] have proposed an indirect adaptive fuzzy variable structure controller to control nonlinear system. This MIMO algorithm, applies to estimate the nonlinear dynamic parameters. Compared with the previous algorithm the numbers of fuzzy rules have reduced by introducing the variable structure surface as inputs of fuzzy systems. Y. Guo and P. Y. Woo [60] have proposed a SISO fuzzy system compensate and reduce the chattering. C. M. Lin and C. F. Hsu [61] can tune both systems by fuzzy rules. Shahnazi et al., have proposed a SISO PI direct adaptive fuzzy variable structure controller based on Lin and Hsu algorithm to reduce or eliminate chattering. The bounds of PI controller and the parameters are online adjusted by low adaption computation [44].

**Problem statements:** Even though, variable structure controller is used in wide range areas but, pure it also has chattering problem and nonlinear dynamic part challenges [8]. The boundary layer method is used to reduce or eliminate the chattering. To reduce the effect of
uncertainty in proposed method, SISO novel fuzzy backstepping adaptive method is applied in variable structure controller in robot arm.

Objectives: The main goal in this paper is to design a SISO fuzzy backstepping adaptive variable structure methodology which applied to robot arm with easy to design and implement. Robot arm has nonlinear dynamic and uncertain parameters consequently; following objectives have been pursuit in the mentioned research: To develop a chattering in a position pure variable structure controller against uncertainties and to develop a position fuzzy backstepping adaptive variable structure controller in order to solve the disturbance rejection.

In this research we will highlight the SISO adaptive fuzzy backstepping variable structure algorithm derived in the Lyapunov sense which it is applied on robot arm. Section 2, serves as an introduction to the variable structure formulation algorithm and its application to a robot arm. Part 3, introduces and describes the proposed methodology algorithms and proves Lyapunov stability. Section 4 presents the simulation results of this algorithm applied to a robot arm and the final section is describing the conclusion.

2. Variable Structure Formulation Applied to Robot Arm

Dynamic of robot arm: The equation of an n-DOF robot manipulator governed by the following equation [1, 4, 15-29, 63-74]:

\[ M(q)\ddot{q} + N(q, \dot{q}) = \tau \]  \hspace{1cm} (1)

Where \( \tau \) is actuation torque, \( M(q) \) is a symmetric and positive define inertia matrix, \( N(q, \dot{q}) \) is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form [1-29]:

\[ \tau = M(q)\ddot{q} + B(q)[\dot{q} \dot{q}] + C(q)[\dot{q}]^2 + G(q) \]  \hspace{1cm} (2)

Where \( B(q) \) is the matrix of coriolis torques, \( C(q) \) is the matrix of centrifugal torques, and \( G(q) \) is the vector of gravity force. The dynamic terms in equation (2) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component \( \dot{\ddot{q}} \) influences, with a double integrator relationship, only the joint variable \( \ddot{q} \), independently of the motion of the other joints. Therefore, the angular acceleration is found as to be [3, 41-62]:

\[ \ddot{q} = M^{-1}(q).[\tau - N(q, \dot{q})] \]  \hspace{1cm} (3)

This technique is very attractive from a control point of view.

Variable structure methodology: Based on variable structure discussion, the control law for a multi degrees of freedom robot manipulator is written as [18-24, 63-74]:

\[ U = U_{\text{Nonlinear}} + U_{\text{dis}} \]  \hspace{1cm} (4)

Where, the model-based component \( U_{\text{Nonlinear}} \) is compensated the nominal dynamics of systems. Therefore \( U_{\text{Nonlinear}} \) can calculate as follows:

\[ U_{\text{Nonlinear}} = [M^{-1}(B + C + G) + \dot{\dot{S}}]M \]  \hspace{1cm} (5)

A simple solution to get the variable structure condition when the dynamic parameters have uncertainty is the switching control law:
\[ U_{\text{dis}} = K(\bar{x}, t) \cdot \text{sgn}(s) \quad (6) \]

where the switching function \( \text{sgn}(S) \) is defined as

\[
\text{sgn}(s) = \begin{cases} 
1 & s > 0 \\
-1 & s < 0 \\
0 & s = 0 
\end{cases} \quad (7)
\]

and the \( K(\bar{x}, t) \) is the positive constant.

the lyapunov formulation can be written as follows,

\[
V = \frac{1}{2} S^T M S \quad (8)
\]

the derivation of \( V \) can be determined as,

\[
\dot{V} = \frac{1}{2} S^T M S + S^T M \dot{S} \quad (9)
\]

the dynamic equation of IC engine can be written based on the structure variable surface as

\[
M \dot{S} = -VS + M \dot{S} + B + C + G \quad (10)
\]

it is assumed that

\[
S^T(M - 2B + C + G)S = 0 \quad (11)
\]

by substituting (10) in (9)

\[
\dot{V} = \frac{1}{2} S^T M S - S^T \dot{S} B + C S + S^T(M \dot{S} + B + C S + G) = S^T(M \dot{S} + B + C S + G) \quad (12)
\]

suppose the control input is written as follows

\[
\bar{U} = U_{\text{nonlinear}} + U_{\text{dis}} = [M^{-1}(B + C + G) + \dot{S}]M + K \cdot \text{sgn}(S) + B + C S + G \quad (13)
\]

by replacing the equation (13) in (12)

\[
\dot{V} = S^T(M \dot{S} + B + C + G - M \dot{S} - B + C S + G - K \text{sgn}(S)) = S^T(M \dot{S} + B + C S + G - K \text{sgn}(S)) \quad (14)
\]

it is obvious that

\[
|\dot{M}S + B + C S + G| \leq |\bar{M}S| + |B + C S + G| \quad (15)
\]

the Lemma equation in robot arm system can be written as follows

\[
K_u = [|\bar{M}S| + |B + C S + G| + \eta_i]_i, i = 1, 2, 3, 4, \ldots \quad (16)
\]

the equation (11) can be written as

\[
K_u \geq [|\bar{M}S + B + C S + G|]_i + \eta_i \quad (17)
\]

therefore, it can be shown that
\[ \dot{V} \leq -\sum_{i=1}^{n} \eta_i |S_i| \]  \hspace{1cm} (18)

Consequently the equation (18) guarantees the stability of the Lyapunov equation.

Figure 1 is shown pure variable structure controller, applied to robot arm.


First part is focused on eliminating the oscillation (chattering) in pure variable structure controller based on linear boundary layer method. To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. This replace is caused to increase the error performance [20-24, 63-74].

\[ B(t) = \{x, |S(t)| \leq \emptyset\}; \emptyset > 0 \]  \hspace{1cm} (19)

Where \( \emptyset \) is the boundary layer thickness. Therefore, to have a smooth control law, the saturation function \( \text{Sat}(S/\emptyset) \) added to the control law:

\[ U = K(\bar{x}, t) \cdot \text{Sat}\left(\frac{S}{\emptyset}\right) \]  \hspace{1cm} (20)

Where \( \text{Sat}\left(\frac{S}{\emptyset}\right) \) can be defined as
Based on above discussion, the control law for a robot arm is written as [18-24]:

\[ U = U_{eq} + U_r \]  \hfill (22)

Figure 2 is shown classical variable structure which eliminates the chattering using linear boundary layer method.

**Figure 2: Chattering Free Block Diagram of a Variable Structure Controller: Applied to Robot Arm**

Second step is focused on design SISO fuzzy estimation backstepping adaptive variable structure based on Lyapunov formulation. The first type of fuzzy systems is given by

\[ f(x) = \sum_{i=1}^{M} \theta^i \mathcal{E}^i(x) = \theta^T \mathcal{E}(x) \]  \hfill (23)

Where \( \theta = (\theta^1, ..., \theta^M)^T, \mathcal{E}(x) = (\mathcal{E}^1(x), ..., \mathcal{E}^M(x))^T, \) and \( \mathcal{E}^i(x) = \prod_{l=1}^{n} \frac{\mu_{A_1^l(x_i)}}{\sum_{l=1}^{M} (\prod_{l=1}^{n} \mu_{A_1^l(x_i)})}, \theta^1, ..., \theta^M \) are adjustable parameters in (23).
\( \mu_{A^l_i}(x_1), \ldots, \mu_{A^m_i}(x_n) \) are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

\[
f(x) = \frac{\sum_{l=1}^{M} \theta^l \left[ \prod_{i=1}^{n} \exp \left(-\frac{(x_i - \alpha_i^l)^2}{\delta_i^l}\right) \right]}{\sum_{l=1}^{M} \left[ \prod_{i=1}^{n} \exp \left(-\frac{(x_i - \alpha_i^l)^2}{\delta_i^l}\right) \right]}
\]  

(24)

Where \( \theta^l, \alpha_i^l \) and \( \delta_i^l \) are all adjustable parameters. From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust \( \theta^l \) in (23). We define \( f^*(x|\theta) \) as the approximator of the real function \( f(x) \).

\[
f^*(x|\theta) = \theta^T \varepsilon(x)
\]  

(25)

We define \( \theta^* \) as the values for the minimum error:

\[
\theta^* = \arg \min_{\theta \in \Omega} \sup_{x \in U} | f^*(x|\theta) - g(x) |
\]  

(26)

Where \( \Omega \) is a constraint set for \( \theta \). For specific \( x \), \( \sup_{x \in U} | f^*(x|\theta^*) - f(x) | \) is the minimum approximation error we can get.

We used the first type of fuzzy systems (23) to estimate the nonlinear system (11) the fuzzy formulation can be write as below:

\[
f(x|\theta) = \theta^T \varepsilon(x)
\]  

(27)

\[
= \frac{\sum_{l=1}^{n} \theta^l \left[ \mu_{A^l_i}(x) \right]}{\sum_{l=1}^{n} \left[ \mu_{A^l_i}(x) \right]}
\]

Where \( \theta^1, \ldots, \theta^n \) are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of \( \theta - \theta^* \). A MIMO (multi-input multi-output) fuzzy system is designed to compensate the uncertainties of the nonlinear system. The parameters of the fuzzy system are adjusted by adaptation laws. The tracking error and the sliding surface state are defined as:

\[
e = q - q_d
\]  

(28)

\[
s = \dot{e} + \lambda_e
\]  

(29)
We define the reference state as

\[ \dot{q}_r = \dot{q} - \dot{s} = \dot{q}_d - \lambda e \]  
(30)

\[ \ddot{q}_r = \ddot{q} - \ddot{s} = \ddot{q}_d - \lambda \dot{e} \]  
(31)

The general MIMO if-then rules are given by

\[ R^l: \text{if } x_1 \text{ is } A^l_1, x_2 \text{ is } A^l_2, \ldots, x_n \text{ is } A^l_n, \text{then } y_1 \text{ is } B^l_1, \ldots, y_m \text{ is } B^l_m \]  
(32)

Where \( l = 1, 2, \ldots, M \) are fuzzy if-then rules; \( x = (x_1, \ldots, x_n)^T \) and \( y = (y_1, \ldots, y_n)^T \) are the input and output vectors of the fuzzy system. The MIMO fuzzy system is defined as

\[ f(x) = \Theta^T \varepsilon(x) \]  
(33)

Where

\[ \Theta^T = (\theta_1, \ldots, \theta_m)^T = \begin{bmatrix} \theta^1_1, \theta^2_1, \ldots, \theta^M_1 \\ \theta^1_2, \theta^2_2, \ldots, \theta^M_2 \\ \vdots \\ \theta^1_m, \theta^2_m, \ldots, \theta^M_m \end{bmatrix} \]  
(34)

\[ \varepsilon(x) = (\varepsilon^1(x), \ldots, \varepsilon^M(x))^T, \quad \varepsilon^1(x) = \prod_{i=1}^n \mu_{A^i_1}(x_i)/\sum_{i=1}^M (\prod_{i=1}^n \mu_{A^i_1}(x_i)) \]  
and \( \mu_{A^i_1}(x_i) \) is defined in (32). To reduce the number of fuzzy rules, we divide the fuzzy system into three parts:

\[ F^1(q, \dot{q}) = \Theta^1 \varepsilon(q, \dot{q}) \]  
(35)

\[ = \begin{bmatrix} \theta^1_1 \varepsilon(q, \dot{q}), \ldots, \theta^1_m \varepsilon(q, \dot{q}) \end{bmatrix}^T \]

\[ F^2(q, \dot{q}_r) = \Theta^2 \varepsilon(q, \dot{q}_r) \]  
(36)

\[ = \begin{bmatrix} \theta^2_1 \varepsilon(q, \dot{q}_r), \ldots, \theta^2_m \varepsilon(q, \dot{q}_r) \end{bmatrix}^T \]

\[ F^3(q, \ddot{q}) = \Theta^3 \varepsilon(q, \ddot{q}) \]  
(37)

\[ = \begin{bmatrix} \theta^3_1 \varepsilon(q, \ddot{q}), \ldots, \theta^3_m \varepsilon(q, \ddot{q}) \end{bmatrix}^T \]

The control input is given by

\[ \tau = M^* \ddot{q}_r + P_m(\theta) + P_{net}(\theta) + F^1(q, \dot{q}) + F^2(q, \dot{q}_r) + F^3(q, \ddot{q}) - K_B \dot{s} - W \text{sgn}(s) \]  
(38)
Where $M^\top, P_m(\theta) + P_{net}(\theta)$ are the estimations of $M(q)$ and are positive constants; $W = \text{diag} \left[ W_1, ..., W_m \right]$ and $W_1, ..., W_m$ are positive constants. The adaptation law is given by

$$
\dot{\theta}_1^j = -\Gamma_{1j} s_j \epsilon(q, \dot{q})
$$

$$
\dot{\theta}_2^j = -\Gamma_{2j} s_j \epsilon(q, \dot{q}_r)
$$

$$
\dot{\theta}_3^j = -\Gamma_{3j} s_j \epsilon(q, \ddot{q})
$$

Where $j = 1, ..., m$ and $\Gamma_{1j} - \Gamma_{3j}$ are positive diagonal matrices.

The Lyapunov function candidate is presented as

$$
V = \frac{1}{2} s^T M s + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_1^j \phi_1^j + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_2^j \phi_2^j + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_3^j \phi_3^j
$$

Where $\phi_1^j = \phi_1^j - \phi_1^j, \phi_2^j = \phi_2^j - \phi_2^j$ and $\phi_3^j = \phi_3^j - \phi_3^j$ we define

$$
F(q, \dot{q}, \dot{q}_r, \ddot{q}) = F^1(q, \dot{q}) + F^2(q, \dot{q}_r) + F^3(q, \ddot{q})
$$

From (26) and (27), we get

$$
M(q)\ddot{q} + P_m(\theta) + P_{net}(\theta) = M^\top \ddot{q} + P_m(\theta) + P_{net}(\theta) + F(q, \dot{q}, \dot{q}_r, \ddot{q}) - K_D s - W sgn(s)
$$

Since $\ddot{q}_r = \ddot{q} - s$ and $\ddot{q}_r = \ddot{q} - \dot{s}$, we get

$$
M_s + (P_m(\theta) + P_{net}(\theta) + K_D) s + W sgn(s) = -\Delta F + F(q, \dot{q}, \dot{q}_r, \ddot{q})
$$

Then $M_s + P_m(\theta) + P_{net}(\theta) T s$ can be written as

$$
M_s + P_m(\theta) + P_{net}(\theta) T s = -\Delta F + F(q, \dot{q}, \dot{q}_r, \ddot{q}) - K_D s - W sgn(s)
$$

Where $\Delta F = \bar{M} \ddot{q}_r + P_m(\theta) + P_{net}(\theta), \bar{M} = M - M^\top, C_1 = P_m(\theta) + P_{net}(\theta) - P_m(\theta) + P_{net}(\theta)$ The derivative of $V$ is

$$
\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T M s + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_1^j \dot{\phi}_1^j + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_2^j \dot{\phi}_2^j + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_3^j \dot{\phi}_3^j
$$

We know that $s^T M \dot{s} + \frac{1}{2} s^T M s = s^T (M_s + P_m(\theta) + P_{net}(\theta) s)$ from (45). Then

$$
\dot{V} = -s^T [-K_D s + W sgn(s) + \Delta F - F(q, \dot{q}, \dot{q}_r, \ddot{q})] + \sum_{j=1}^{m} \frac{1}{\Gamma_{1j}} \phi_1^j \dot{\phi}_1^j + \sum_{j=1}^{m} \frac{1}{\Gamma_{2j}} \phi_2^j \dot{\phi}_2^j + \sum_{j=1}^{m} \frac{1}{\Gamma_{3j}} \phi_3^j \dot{\phi}_3^j
$$

We define the minimum approximation error as

$$
\omega = \Delta F - [F^1(q, \dot{q}, \Theta^1) + F^2(q, \dot{q}_r, \Theta^2) + F^3(q, \ddot{q}, \Theta^3)]
$$
We plug (46) in to (47)

\[ \dot{V} = -s^T [-K_D s + W sgn(s) + \Delta F - F(q, \dot{q}, \dot{q}_r, \dot{\dot{q}})] + \sum_{j=1}^{m} \frac{1}{r_{ij}} \varphi_j^T \hat{\varphi}_j + \sum_{j=1}^{m} \frac{1}{r_{3j}} \varphi_j^{13T} \hat{\varphi}_j \]

\[ = -s^T [-K_D s + W sgn(s) + \omega + F^1(q, \dot{q}, \dot{q}_r, \dot{\dot{q}}) + F^2(q, \dot{q}_r, \dot{\dot{q}}) + F^3(q, q) \Theta 3 \ast - F^1(q, \dot{q}, F^2(q, \dot{q}_r) + F^3(q, q) + f] + \sum_{j=1}^{m} \frac{1}{r_{ij}} \varphi_j^{1T} \hat{\varphi}_j + \sum_{j=1}^{m} \frac{1}{r_{3j}} \varphi_j^{13T} \hat{\varphi}_j \]

\[ = -s^T K_D s - s^T W sgn(s) - s^T \omega - \sum_{j=1}^{m} s_j \varphi_j^{1T} \varepsilon(q, \dot{q}) - \sum_{j=1}^{m} s_j \varphi_j^{2T} \varepsilon(q, \ddot{q}_r) - \sum_{j=1}^{m} s_j \varphi_j^{3T} \varepsilon(q, \dddot{q}) + \sum_{j=1}^{m} \frac{1}{r_{ij}} \varphi_j^{1T} \hat{\varphi}_j + \sum_{j=1}^{m} \frac{1}{r_{3j}} \varphi_j^{13T} \hat{\varphi}_j \]

\[ = -s^T K_D s - s^T W sgn(s) - s^T \omega - \sum_{j=1}^{m} \varphi_j^{1T} (s_j \varepsilon(q, \dot{q}) - \frac{1}{r_{ij}} \dot{\varphi}_j) - \sum_{j=1}^{m} \varphi_j^{2T} (s_j \varepsilon(q, \ddot{q}_r) - \frac{1}{r_{3j}} \dot{\varphi}_j) - \sum_{j=1}^{m} \varphi_j^{3T} (s_j \varepsilon(q, \dddot{q}) - \frac{1}{r_{ij}} \dot{\varphi}_j) \]

\[ = -s^T K_D s - s^T W sgn(s) - s^T \omega - \sum_{j=1}^{m} \varphi_j^{1T} (s_j \varepsilon(q, \dot{q}) + \frac{1}{r_{ij}} \dot{\varphi}_j) - \sum_{j=1}^{m} \varphi_j^{2T} (s_j \varepsilon(q, \ddot{q}_r) + \frac{1}{r_{3j}} \dot{\varphi}_j) - \sum_{j=1}^{m} \varphi_j^{3T} (s_j \varepsilon(q, \dddot{q}) + \frac{1}{r_{ij}} \dot{\varphi}_j) \]

Then \( \dot{V} \) becomes

\[ \dot{V} = -s^T K_D s - s^T W sgn(s) - s^T \omega \]

\[ = - \sum_{j=1}^{m} (s_j^2 K_{Dj} + W_j \|s_j\| + s_j \omega_j) \]

\[ = - \sum_{j=1}^{m} [s_j (s_j K_{Dj} + \omega_j) + W_j \|s_j\|] \]

Since \( \omega_j \) can be as small as possible, we can find \( K_{Dj} \) that \( |s_j^2 K_{Dj}| > |\omega_j| (s_j \neq 0) \).

Therefore, we can get \( s_j (s_j K_{Dj} + \omega_j) > 0 \) for \( s_j \neq 0 \) and \( \dot{V} < 0 \) (s \neq 0). Figure 3 is shown the fuzzy estimator variable structure.
Third step is focused on design Mamdani’s fuzzy [30-40] backstepping adaptive fuzzy estimator variable structure. As mentioned above pure variable structure controller has nonlinear dynamic equivalent limitations in presence of uncertainty and external disturbances in order to solve these challenges this work applied Mamdani’s fuzzy inference engine estimator in variable structure controller. However proposed MIMO fuzzy estimator variable structure has satisfactory performance but calculate the variable structure surface slope by try and error or experience knowledge is very difficult, particularly when system has structure or unstructured uncertainties; SISO Mamdani’s fuzzy backstepping variable structure function fuzzy estimator variable structure controller is recommended. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of robot arm. This method is used as feedback linearization in order to solve nonlinearities in the system. To use of nonlinear fuzzy filter this method in this research makes it possible to create dynamic nonlinear backstepping estimator into the adaptive fuzzy estimator variable structure process to eliminate or reduce the challenge of uncertainty in this part. The backstepping controller is calculated by:

$$U_{BS} = U_{eqBS} + M \cdot I$$  \hspace{1cm} (49)

Where $U_{BS}$ is backstepping output function, $U_{eqBS}$ is backstepping nonlinear equivalent function which can be written as (50) and $I$ is backstepping control law which calculated by (51)

$$U_{eqBS} = [B + C + G]$$  \hspace{1cm} (50)

$$I = [\dot{\theta} + K_1(K_1 - 1) \cdot e + (K_1 + K_2) \cdot \dot{e}]$$  \hspace{1cm} (51)

Based on (11) and (50) the fuzzy backstepping filter is considered as

$$(B + C + G) = \sum_{l=1}^{M} \theta_l^T \zeta(x) - \lambda S - K)$$  \hspace{1cm} (53)
Based on (49) the formulation of fuzzy backstepping filter can be written as;

\[ U = U_{eqB}S_{fuzzy} + MI \]  \hspace{1cm} (54)

Where \( U_{eqB,S_{fuzzy}} = [(B + C + G)] + \sum_{t=1}^{M} \theta^T \zeta(x) + K \)

The adaption law is defined as

\[ \dot{\theta}_j = y_{sj}S_j \zeta_j(S_j) \]  \hspace{1cm} (55)

where the \( y_{sj} \) is the positive constant and \( \zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \ldots, \zeta_j^M(S_j)]^T \)

\[ \zeta_j^1(S_j) = \frac{\mu(A)_j^l(S_j)}{\sum_{i}(\mu(A)_j^l(S_j))} \]

The dynamic equation of IC engine can be written based on the variable structure surface as;

\[ MS = -VS + MS + VS \]  \hspace{1cm} (57)

It is supposed that

\[ S^T(\dot{M} - 2V)S = 0 \]  \hspace{1cm} (58)

The derivation of Lyapunov function (\( V \)) is written as

\[ \dot{V} = \frac{1}{2}S^TMS - S^TVS + \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = S^T(-\lambda S + \Delta f - K) + \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = \sum_{j=1}^{m} [S_j(\Delta f_j - K_j)] - S^T \lambda S + \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = \sum_{j=1}^{m} [S_j(\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = \sum_{j=1}^{m} [S_j(\Delta f_j - (\theta_j^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j)))] - S^T \lambda S + \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = \sum_{j=1}^{m} [S_j(\Delta f_j - ((\theta_j^T \zeta_j(S_j)))] - S^T \lambda S + \sum_{j=1}^{m} \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

\[ = \sum_{j=1}^{m} [S_j(\Delta f_j)] - S^T \lambda S + \sum_{j=1}^{m} \frac{1}{\gamma_{sj}} \varphi_j^T \phi_j \]

Where \( \dot{\theta}_j = y_{sj}S_j \zeta_j(S_j) \) is adaption law and \( \phi_j = -\dot{\theta}_j = -y_{sj}S_j \zeta_j(S_j) \), consequently \( \dot{V} \) can be considered by

\[ \dot{V} = \sum_{j=1}^{m} [S_j \Delta f_j - ((\theta_j^T \zeta_j(S_j)))] - S^T \lambda S \]  \hspace{1cm} (59)

The minimum error can be defined by
\[ e_{mj} = \Delta f_j - \left( (\theta^T_j \zeta_j(S_j)) \right) \] (60)

\[ \dot{V} \text{ is intended as follows} \]

\[ \dot{V} = \sum_{j=1}^{m} |S_j e_{mj}| - S^T \lambda S \]

\[ \leq \sum_{j=1}^{m} |S_j| |e_{mj}| - S^T \lambda S \]

\[ = \sum_{j=1}^{m} |S_j| |e_{mj}| - \lambda_j S_j^2 \]

\[ = \sum_{j=1}^{m} |S_j| (|e_{mj}| - \lambda_j S_j) \] (61)

For continuous function \( U_{eqBS_{fuzzy}} \) and suppose \( \varepsilon > 0 \) it is defined the fuzzy backstepping controller in form of (53) such that

\[ \sup_{x \in U} \left| U_{eqBS_{fuzzy}} + M_l \right| < \varepsilon \] (62)

As a result SISO fuzzy backstepping adaptive fuzzy estimation variable structure is very stable which it is one of the most important challenges to design a controller with suitable response. Figure 4 is shown the block diagram of proposed SISO fuzzy backstepping adaptive fuzzy estimation variable structure.

![Supervisory Controller](image)

**Figure 4: Chattering free Block diagram of a SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller**
4. Results

Variable structure controller (VSC) and proposed SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller were tested to sinus response trajectory. The simulation was implemented in Matlab/Simulink environment. Links trajectory, disturbance rejection and error are compared in these controllers. It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems.

Trajectory: Figure 5 shows the links trajectory in VSC and proposed SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller without disturbance for sinus trajectory in general and zoom scaling because all 3 links have the same response so, all links are shown in a graph.

By comparing sinus response, Figure 5, in SMC and SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller, conversely the SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller 's overshoot (0%) is lower than VSC's (3.8%).

Disturbance rejection: Figure 6 is indicated the power disturbance removal in VSC and SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller. Besides a band limited white noise with predefined of 40% the power of input signal is applied to the sinus VSC and SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller; it found slight oscillations in VSC trajectory responses. All 3 links are shown in one graph.
Among above graph, relating to sinus trajectory following with external disturbance, VSC has slightly fluctuations. By comparing overshoot; SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller's overshoot (0%) is lower than VSC's (16%).

Errors in the model: SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller has lower error rate (refer to Table.1), VSC has oscillation tracking which causes chattering phenomenon at the presence of disturbances. Figure 7 is shown steady state and RMS error in VSC and MIMO fuzzy backstepping adaptive fuzzy estimator variable structure controller in presence of external disturbance.

<table>
<thead>
<tr>
<th>Table 1: RMS Error Rate of Presented controllers</th>
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<tbody>
<tr>
<td><strong>RMS Error Rate</strong></td>
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<tr>
<td>Without Noise</td>
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<tr>
<td>With Noise</td>
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</tbody>
</table>

![Figure 7: VSC Vs. SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller: Robot arm steady state error in presence of external disturbance](image)

In these methods if integration absolute error (IAE) is defined by (63), table 2 is shown comparison between these two methods.

\[
IAE = \int_0^\infty |e(t)| \, dt
\]  

(63)

<table>
<thead>
<tr>
<th>Table 2: Calculate IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method</strong></td>
</tr>
<tr>
<td>IAE</td>
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5. Conclusion

Refer to the research, a Lyapunov based SISO fuzzy backstepping adaptive fuzzy estimator variable structure controller design and application to robot arm has proposed in order to design high performance nonlinear controller in the presence of uncertainties and external disturbances. Regarding to the positive points in variable structure controller, fuzzy inference
system and adaptive fuzzy backstepping methodology it is found that the adaptation laws derived in the Lyapunov sense. The stability of the closed-loop system is proved mathematically based on the Lyapunov method. The first objective in proposed method is removed the chattering which linear boundary layer method is used to solve this challenge. The second target in this work is compensate the model uncertainty by SISO fuzzy inference system, in the case of robot arm, if we define $k_1$ membership functions for each input variable, the number of fuzzy rules applied for each joint is $K_1$ which will result in a low computational load. In finally part fuzzy backstepping methodology with minimum rule base is used to online tuning and adjusted the fuzzy variable structure method and eliminates the chattering with minimum computational load. In this case the performance is improved by using the advantages of variable structure, artificial intelligence compensate method and adaptive algorithm while the disadvantages removed by added each method to previous method.

References

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