Comparative Study between Wavelet Thresholding Techniques (Hard, Soft and Invariant-translation) in Ultrasound Images

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Abstract
In this paper method has been applied in the speckle noise reduction in medical imaging. Speckle Noise reduces the ability of an observer to distinguish fine details in diagnostic testing. It also limits the effective implementation of image processing such as edge detection, segmentation and volume rendering in 3 D. Therefore; treatment methods of speckle noise were sought to improve the image quality and to increase the capacity of diagnostic medical ultrasound images.

The methods we used in this work are new, they are used in image processing, among their applications noise reduction in medical imaging.

The use of wavelets has brought a new breath in the field of signal and image processing since its appearance. They make it possible to analyze and identify discontinuities of a signal to one or two dimensions and at different scales. This feature is used for image denoising.

In the field of medical imaging, denoising seeks to discern relevant information in the medical field as the shape, contour, etc. Thresholding by wavelet, we can denoise an image by eliminating fine details.

Keywords: Wavelets transform, image processing, ultrasound image, filtering

1. Introduction
In the literature of image processing, different denoising methods have been proposed and developed. First, the spatial filtering methods [1, 2], these are to reduce noise in areas that do not exhibit artifacts and enhance the perception of structures of interest. These filtering techniques use a low-pass filter to remove high frequencies, which has the disadvantage mitigate the edges of the image. To overcome these problems, new techniques, more efficient, have emerged in the years 80 and 90; statements include approaches based on PDEs variational [3, 4, 5], approaches using Markov random fields and approaches based on multiscale transforms, including wavelet transform. Recently, these approaches have demonstrated their power in the context of the statistical estimation. Through these sparse transformed, the energy of the desired signal is concentrated on a small number of coefficients, thereby providing a non-linear natural to estimate the signal. Indeed, simply thresholding the coefficients of the observed image and reverse transformed to obtain an estimate of the useful signal [6].

One solution is the use of wavelet bases in image processing has become widespread during the last twenty years [7]. Interest in compression and noise reduction has been shown since they have integrated the latest compression standard JPEG 2000 digital images. Their application to medical imaging dates back to 1992 and has spread widely since [8, 9]. In this context, the wavelets are used for compression and noise reduction, but also for the functional analysis of medical data (for diagnosis), the local tomography, segmentation and image enhancement, or the description of texture [10].
2. Denoising Using Wavelet Transform

Denoising can be viewed as an estimation problem trying to recover a true signal component $X$ from an observation $Y$ where the signal component has been degraded by a noise component $N$:

$$X = Y + N$$  \hspace{1cm} (1)

The estimation is computed with a thresholding estimator in an orthonormal basis:

$$\hat{X} = \sum_{m=1}^{N} \rho_m(\langle X, g_m \rangle) g_m$$  \hspace{1cm} (2)

Where $\rho_m$ is a thresholding function that aims at eliminating noise components (via attenuating or decreasing some coefficient sets) in the transform domain while preserving the true coefficients of the signal. If the function $\rho_m$ is modified to rather preserve or increase coefficient values in the transform domain, it is possible to enhance some features of interest in the true signal component with the framework of Equation (2).

Figure (1) illustrates a multi-scale enhancement and de-noising framework using wavelet transforms. An over complete dyadic wavelet transform using bi-orthogonal basis is used. Notice that since the DC-cap contains the overall energy distribution, it is usually kept untouched during the procedure. As shown in this figure, thresholding and enhancement functions can be implemented independently from the wavelet filters and easily incorporated into the filter bank framework.

![Figure 1. A Multi-scale Framework of De-noising and Enhancement using Discrete Dyadic Wavelet Transform. A Three Level Decomposition was](image-url)
3. Thresholding Operators for De-noising

As a general rule, wavelet coefficients with larger magnitude are correlated with salient features in the image data. In that context, de-noising can be achieved by applying a thresholding operator to the wavelet coefficients (in the transform domain) followed by reconstruction of the signal to the original image (spatial domain).

Typical threshold operators for de-noising include:

- **Hard thresholding:**
  \[
  \rho_T(x) = \begin{cases} 
  x, & |x| > T, \\
  0, & |x| \leq T 
  \end{cases}
  \]  

- **Soft thresholding (wavelet shrinkage):**
  \[
  \rho_T(x) = \begin{cases} 
  x - T, & x \geq T \\
  x + T, & x \leq -T \\
  0, & |x| < T 
  \end{cases}
  \]  

- **Affine (firm) thresholding:**
  \[
  \rho_T(x) = \begin{cases} 
  x, & |x| \geq T \\
  2x - T, & -T \leq x \leq -T/2 \\
  2x + T, & T/2 \leq x \leq T \\
  0, & |x| < T 
  \end{cases}
  \]  

The shapes of these thresholding operators are illustrated in Figure (2).

![Thresholding Functions](image)

**Figure 2.** Example of Thresholding Functions, Assuming that the Input Data was Normalized to the Range of [-1, 1]. (a) Hard Thresholding. (b) Soft Thresholding. (c) Affine Thresholding. The Threshold Level was Set to \( T = 0.5 \)

4. Selection of Threshold Value

Given the basic framework of de-noising using wavelet thresholding as discussed in the previous sections, it is clear that the threshold level parameter \( T \) plays an essential role. Values too small cannot effectively get rid of noise component, while values too...
large will eliminate useful signal components. There are a variety of ways to determine
the threshold value T as we will discuss in this section.

Depending on whether or not the threshold value T changes across wavelet scales and
spatial locations, the thresholding can be:

1. Global Threshold: a single value T is to be applied globally to all empirical
   wavelet coefficients at different scales $T=const$.

2. Level-Dependent Threshold: a different threshold value T is selected for each
   wavelet analysis level (scale). $T=T(j), j=1,\ldots,J$. J is the coarsest level for wavelet
   expansion to be processed.

3. Spatial Adaptive Threshold: the threshold value T varies spatially depending on
   local properties of individual wavelet coefficients. Usually, T is also level-
   dependent. $T=T_j(x,y,z)$.

While a simple way of determining T is a percentage of coefficients maxima, there are
different adaptive ways of assigning the T value according to the noise level (estimated
via its variance $\sigma$):

1. Universal Threshold: $T = \sigma \sqrt{2\log n}$, with n equal to the sample size. This
   threshold was determined in an optimal context for soft thresholding with random
   Gaussian noise. This scheme is very easy implement, but typically provides a
   threshold level larger than with other decision criteria, therefore resulting in
   smoother reconstructed data. Also such estimation does not take into account the
   content of the data, but only depends on the data size.

2. Minimax Threshold: $T=\sigma T_n$, where $T_n$ is determined by a minimax rule such that
   the maximum risk of estimation error across all locations of the data is
   minimized. This threshold level depends on the noise and signal relationships in
   the input data.

3. Stein Unbiased Estimated of Risk (SURE): Similar as minimax threshold but $T_n$
   is determined by a different risk rule.

4. Spatial Adaptive Threshold: $T=\sigma^2/\sigma x$, where $\sigma x$ is the local variance of the
   observation signal, which can be estimated using a local window moving across
   the image data or, more accurately, by a context-based clustering algorithm.

In many automatic de-noising methods to determine the threshold value T, an
estimation of the noise variance $\sigma$ is needed. Donoho and Johnstone proposed a robust
estimation of noise level $\sigma$ based on the median absolute value of the wavelet
coefficients as:

$$\sigma = \frac{\text{median}([W_j(x,y,z)])}{0.6745} \quad (6)$$

Where $W_j$ is the most detailed level of wavelet coefficients. Such estimator has become
very popular in practice and is widely used [11].

5. Experiment Results

For learning, a species of ultrasound images 292x400 pixels is used. we load our image
and resize it where (N=256 ) is the number of pixels which consist of two different grey
level values, and we add White Gaussian Noise with different values of sigma to obtain
denoised one.

To determine the threshold value T, the assignment of the value of noise as a function
of T (through its variance $\sigma$) is applied to different types of thresholding.
For this method, the orthogonal 2D wavelet transform on the noisy image is applied by adding a white Gaussian noise with different values of sigma as shown in the figures (3 and 4) and the reconstruction using hard and soft thresholding as shows the figures (5) and (6).

We try to compare the results obtained using the translation invariant wavelet transform applied to a noisy and binary image. The results are shown in the figure (7) and (8).

To measure the quality between the original image and the filtered, another comparison is performed by calculating the PSNR calculates the peak signal-to-noise ratio between two images. The higher PSNR means the quality of the image treated is better. The PSNR is calculated using the following equation:

$$PSNR = 10 \log_{10} \left( \frac{R^2}{MSE} \right)$$ (7)

In the above equation, R is the maximum variation in the data type of the input image. For example, if the input image is double, then R is 1. If it is of unit 8, R is 255. MSE is the mean square error (MSE) which is the square of the accumulated error between the original with noise and filtered image. The lower value of MSE means the error is small. MSE is calculated using the following equation:

$$MSE = \frac{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (I(m,n) - \tilde{I}(m,n))^2}{MN}$$ (8)

In the above equation, M and N are the number of rows and columns in the input images, respectively.

The mean square error (MSE) and peak ratio of signal to noise (PSNR) are the two error parameters used to compare the quality of image compression.

Figure 3. Original and Noised Image
Figure 4. Orthogonal 2D Wavelet Transform for the Noised and Denoised Image

Figure 5. Denoised Image (Hard Thresholding)

Figure 6. Denoised Image (Soft Thresholding)

Figure 7. Denoised Image (Soft Invariant)

Figure 8. Denoised Image (Hard Invariant)

The comparison results are shown in the following tables and figures.
Table 1. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3*Sigma

<table>
<thead>
<tr>
<th>Images</th>
<th>PSNR Sigma=0.06</th>
<th>PSNR Sigma=0.08</th>
<th>PSNR Sigma=0.5</th>
<th>PSNR Sigma=5</th>
<th>PSNR Sigma=30</th>
</tr>
</thead>
<tbody>
<tr>
<td>(noised, soft thresholding) T=3*sigma</td>
<td>25.4943</td>
<td>23.6629</td>
<td>13.9070</td>
<td>13.2766</td>
<td>13.0320</td>
</tr>
</tbody>
</table>

Table 2. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3/2*Sigma

<table>
<thead>
<tr>
<th>Images</th>
<th>PSNR Sigma=0.06</th>
<th>PSNR Sigma=0.08</th>
<th>PSNR Sigma=0.5</th>
<th>PSNR Sigma=5</th>
<th>PSNR Sigma=30</th>
</tr>
</thead>
<tbody>
<tr>
<td>(original, noised)</td>
<td>27.2259</td>
<td>24.3068</td>
<td>13.9578</td>
<td>12.0939</td>
<td>12.1704</td>
</tr>
<tr>
<td>(noised, hard thresholding) T=3/2*sigma</td>
<td>30.5460</td>
<td>27.6648</td>
<td>18.0594</td>
<td>15.3027</td>
<td>15.3385</td>
</tr>
<tr>
<td>(noised, hard invariant) T=3/2*sigma</td>
<td>33.1299</td>
<td>30.1836</td>
<td>19.3994</td>
<td>17.4704</td>
<td>17.4810</td>
</tr>
<tr>
<td>T=0.09</td>
<td>0.90</td>
<td>0.12</td>
<td>0.75</td>
<td>7.5</td>
<td>45</td>
</tr>
</tbody>
</table>

Figure 9. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for T=3*Sigma
Figure 10. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for $T=3/2*\sigma$.

Table 3. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for $T=3.5*\sigma$.

<table>
<thead>
<tr>
<th>Images</th>
<th>PSNR $\sigma=0.06$</th>
<th>PSNR $\sigma=0.08$</th>
<th>PSNR $\sigma=0.5$</th>
<th>PSNR $\sigma=5$</th>
<th>PSNR $\sigma=30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=3.5*\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(noised, soft thresholding)</td>
<td>25.0694</td>
<td>23.5698</td>
<td>14.7811</td>
<td>12.4771</td>
<td>13.1022</td>
</tr>
<tr>
<td>$T=3.5*\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T=3.5*\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(bruitée, invariante douce)</td>
<td>25.0928</td>
<td>23.5197</td>
<td>14.5937</td>
<td>12.4335</td>
<td>13.0885</td>
</tr>
<tr>
<td>$T=3.5*\sigma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T=0.21$</td>
<td>0.21</td>
<td>0.28</td>
<td>1.75</td>
<td>17.5</td>
<td>105</td>
</tr>
<tr>
<td>$T=0.28$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T=1.75$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T=17.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T=105$</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Figure 11. Comparaison between Original Image, Noised Image, Denoised by Wavelet (Hard and Soft Thresholding, Hard and Soft Invariant Thresholding) for $T=3.5\sigma$

After the comparison between the results obtained when we have used different values of sigma, it is clear that the value of PSNR is high for low values of $T$, this is for sigma=0.06 in the case of the comparison between (noised Image and hard Invariant Denoising) whereas high values of $T$ this technique will not be efficient, so we apply other filters such as median filter. This means that image quality is better using the technique of invariant translation denoising than other thresholding.

6. Conclusion

We have discussed in this paper the use of wavelet thresholding techniques in reducing noise which has a bad effect on medical imaging thus the degradation of its quality.

This manuscript is organized in five sections. Section 1 presents an introduction of the medical imaging and the methods used in reducing the noise. Section 2 describes the basics of wavelets when used as filter. The thresholding operators for denoising are studied in Section 3 and the selections of threshold value are represented in Section 4. In Section 5 we have applied the dwt2d and thresholding wavelet techniques on the medical image and were completed by comparing the results obtained. Section 6 draws the conclusion of this study.

In this paper we have demonstrated that the use of translation invariant wavelet could offer better results than hard hard and soft thresholding.

References


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