Extended Kalman Filter for Rate Estimation in Doppler Radar Cardiopulmonary Monitoring System

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Abstract
Remote monitoring of cardiopulmonary activities using quadrature direct conversion Doppler radar shows remarkable promise in medical and security applications. One major challenge for such monitoring is demodulation of the IQ signals and heart and respiration rate determination. This paper presents a rate finding algorithm based on extended kalman filter (EKF) and principal component analysis. We present a state space model of the quadrature IQ signals and use it with the EKF to simultaneously estimate and track heart and respiration rate by a unified statistical approach. We evaluate the performance of the algorithm for practical data obtained from the implemented hardware and present the results to illustrate the feasibility and accuracy of the algorithm. Results are compared with reference ECG results which clearly demonstrate that the proposed algorithm can be successfully applied for heart and respiration rate detection.

Keywords: Remote monitoring, Doppler radar, Cardiopulmonary, Extended Kalman filter, Quadrature receiver

1. Introduction
Microwave Doppler radar with quadrature direct conversion is a promising method for non-contact detection and monitoring of human cardiopulmonary activities. This monitoring system can be efficient for regular health care, emergency, military, security as well as in the case of neonates, infants or burn victims where contact sensors are not suitable. Microwave Doppler radar has been used for physiologic sensing since the early 1970s \cite{1}. This system included bulky, heavy and expensive waveguide components which were not practicable everywhere. However, integration of Doppler radar transceivers on a single chip is now achievable due to the recent advancements of micro-fabrication and wireless technology. Further, robust digital signal processors have opened up enormous possibilities for processing and extracting the information from noisy data. Several research works are published on remote sensing and monitoring of cardiopulmonary activities using quadrature direct conversion Doppler radar \cite{2-5}. Demodulation of the noisy quadrature (IQ) outputs of the direct conversion
Doppler radar is a great challenge for accurate monitoring. A significant work done by Park et al. [4] suggested arctangent demodulation to solve the problem of demodulation sensitivity to target position. They combined the quadrature outputs using arctangent demodulation with DC offset compensation. Successful arctangent demodulation of quadrature channels depends on correction of channel imbalances and removal of unwanted DC. Channel imbalance can be corrected using Gram-Schmidt procedure. However, accurate prior knowledge of the amplitude and phase errors is required. Further, removal of unwanted DC offsets from the quadrature signal is also difficult. A frequency tuning technique with double-sideband transmission has been proposed for Ka-band radar [6]. However, this technique might have limitations in practical use due to the requirement of complex hardware with a tunable intermediate frequency. Kalman filters (KF) and extended Kalman filters (EKF) have been widely used in biomedical signal processing of electrocardiogram (ECG) and electroencephalogram (EEG) signals [7-10]. In addition to various estimation and processing problems, KF and EKF have been used successfully for heart and respiration rate estimation [7, 8] from ECG and arterial blood pressure (ABP) signals. Hence, due to the remarkable success of EKF and model-based signal processing framework to solve processing and tracking problems in biomedical signal processing, we are motivated to utilize the power of EKF in Doppler radar monitoring system. In our previous work [11], we presented the algorithm and simulation results for EKF estimation of heart and respiration rates from Doppler radar IQ signals, which is updated and verified for the practical data in the present work. To the best of our knowledge, EKF has not been applied by any other research group to solve the estimation and tracking problems in Doppler radar cardiopulmonary monitoring system. IQ signals from the Doppler radar monitoring system contain the heart and respiration rate information that must be estimated, but have a complicated nonlinear relationship with the observed signal. We design the statistical state-space model for IQ signals which incorporates the significant state variables of the system. The model is used for EKF to estimate the heart and respiration rate monitoring. Although there are other Bayesian filters such as the unscented Kalman filter (UKF), in this work, we have chosen the EKF for its simplicity and better numerical stability. In our proposed algorithm, channel selection is performed using principle component analysis.

Remaining of this paper is organized as follows. We explain the operation principle of quadrature direct conversion Doppler radar cardiopulmonary monitoring system. Then, we present and describe the proposed algorithm. Next section contains the review of EKF followed by the state-space model of the IQ signals in the later section. Performance evaluation of the proposed algorithm from experimental results is described. Finally, we give our conclusion.
2. Quadrature Doppler Radar Transceiver

![Doppler radar cardiopulmonary monitoring system block diagram](image)

Figure 1. Doppler radar cardiopulmonary monitoring system block diagram. The LO and RF output signals are provided from the same source. The LO signal is split by a two-way 90º power splitter to obtain in- and quadrature phase signals. These two signals are mixed with the reflected RF signal and lowpass filtered to get I and Q signals.

According to Doppler theory, if a radio signal is reflected from a target with a time-varying position but with zero net velocity, it will be phase modulated (PM). In that case, the modulation is proportional to the time varying target position. If the change in target position is small compared to the wavelength of the radio signal, the phase change will be small and the PM signal can be directly demodulated by mixing it with the original signal. Human chest has a periodic motion for heart beat and respiration with a net zero velocity, and therefore, a Doppler radar with the chest as a target will receive a signal similar to the transmitted signal with its phase modulated by the time varying chest position. Figure 1 shows the block diagram of the Doppler radar cardiopulmonary monitoring system. Typically the transceiver transmits a radio wave and receives a phase modulated signal reflected from the target. The LO and RF output signals are generated from the same source. A 90º power splitter is used to divide the LO signal. These two LO signals are mixed with the reflected RF signal to provide two orthonormal baseband output signals. Use of quadrature receiver eliminates the problem of null points in single channel receivers [3]. For 2.4 GHz radar operating frequency, the null points occur at every 3 cm which is difficult to avoid by adjusting the target position. Use of quadrature channels ensures that at least one output would not be in null point. In order to represent the IQ signals mathematically, let us consider \( \omega_h(t) \) and \( \omega_r(t) \) to be the heart beat and respiration frequencies of the target, respectively. After mixing the reflected and LO signals and lowpass filtering the mixed signals, the baseband I and Q signals can be expressed as follows, respectively [3]:

\[
B_I(t) = A \cos(\theta + \pi/4 + 4\pi A_r \sin \omega_r(t)/\lambda + 4\pi A_h \sin \omega_h(t)/\lambda + \Delta \phi(t)) \\
B_Q(t) = A \cos(\theta - \pi/4 + 4\pi A_r \sin \omega_r(t)/\lambda - 4\pi A_h \sin \omega_h(t)/\lambda + \Delta \phi(t))
\]  

where, \( \theta \) is the constant phase shift related to the nominal distance between the antenna and the target, the phase change at the target surface, and the phase delay between the
mixture and the antenna. $\Delta \phi(t)$ is the residual phase noise. When $\theta + \pi/4$ is an integer multiple of $\pi$, I channel signal will be in null point [3]. At the same time, the Q channel signal will be in optimum point. On the other hand, the condition will be reversed if $\theta - \pi/4$ is odd multiple of $\pi/2$. At a frequency of 2.4 GHz, I or Q channel null points occur at every 3 cm. By using the quadrature receiver, it can be assured that at least one output would not be in null point. If $\theta$ becomes integer multiple of $\pi$, both I and Q channel will neither be in null nor in optimum points. However, still the heart rate can be detected provided that target displacement due to heart beat and respiration is smaller than the wavelength of the radio signal. We have implemented the quadrature Doppler radar monitoring hardware with operating frequency 2.4 GHz which is shown in Figure 2.

![Figure 2. 2.4 GHz Doppler Radar Cardiopulmonary Monitoring System Hardware](image)

3. Signal Processing Technique

Figure 3 shows the block diagram of the proposed digital signal processor. IQ signals obtained from the Doppler radar monitoring hardware are A/D converted. Prior to A/D conversion, the signals are lowpass filtered to avoid aliasing effect and out of band noise. The quadrature IQ signals are then fed into EKF module for state estimation. In order to avoid null case mentioned in the previous section, PCA channel selection is done. For the channel selection purpose, a 5-sec sliding window is passed over the signal. PCA is done on the data inside the window. The sliding window is shifted over the samples in one-sample increment. The mean is subtracted from the quadrature channel data. Covariance matrix for I and Q channel data, eigenvalues and eigenvectors of the covariance matrix are calculated. The channel with the largest eigenvalue is then selected for heart and respiration rate estimation. In the rate determination process, EKF state estimation of heart and respiration rate from I and Q signals are compared. If they are identical, then the channel selection is not utilized. However, if they differ, rate estimation is done from PCA selected channel.
4. Extended Kalman Filter Review

Kalman filter is an optimal state estimation method for stochastic signals that estimates the state of a discrete time-controlled process by using a form of feedback control [12]. The filter estimates the process state at some time and then obtains feedback in the form of noisy measurements. The equations for the Kalman filter fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. The EKF is a nonlinear extension of the conventional KF, which has been developed particularly for systems having nonlinear dynamic models. For a discrete nonlinear system with the state vector $x_n$ and observation vector $y_n$, the dynamic model and its linear approximation near a desired reference point may be formulated as follows [13]:

$$
x_{n+1} = f(x_n, u_n, w_n)$$

$$\approx f(\hat{x}_n, u_n, w_n) + A_n(x_n - \hat{x}_n) + W_n w_n$$

$$y_n = g(x_n, v_n)$$

$$\approx g(\hat{x}_n, v_n) + H_n(x_n - \hat{x}_n) + V_n v_n$$  \hspace{1cm} (3)

where $\hat{x}_n$ is a posteriori estimate of the state at step $n$. The random variables $w_n$ and $v_n$ represent the process and measurement noises, respectively, with covariance $Q_n = E\{w_n w_n^T\}$ and $R_n = E\{v_n v_n^T\}$. Also,

$$A_n = \left. \frac{\partial f(\hat{x}_n, u_n, w_n)}{\partial x} \right|_{x=\hat{x}_n}$$

$$W_n = \left. \frac{\partial f(\hat{x}_n, u_n, w_n)}{\partial w} \right|_{w=\hat{w}_n}$$

$$H_n = \left. \frac{\partial g(\hat{x}_n, v_n)}{\partial x} \right|_{x=\hat{x}_n}$$

$$V_n = \left. \frac{\partial g(\hat{x}_n, v_n)}{\partial v} \right|_{v=\hat{v}_n}$$  \hspace{1cm} (4)
The time update and measurement update equations can be summarized as follows:

\[
\begin{align*}
\hat{x}_{n+1} &= f(\hat{x}_n, u_n, w_n) \\
\hat{P}_{n+1} &= A_n P_n A_n^T + W_n Q_n W_n^T \\
K_n &= P_n H_n^T \left( H_n P_n H_n^T + V_n R_n V_n^T \right)^{-1} \\
\hat{x}_n &= \hat{x}_{n+1} + K_n \left( y_n - g(\hat{x}_{n+1}, v_n) \right) \\
P_n &= (I - K_n H_n) P_{n+1}
\end{align*}
\]  

(5)

where \( \hat{x}_{n+1} = E\{x_{n+1} | y_1, y_2, \ldots, y_n\} \) is the a priori estimate of the state vector, \( x_n \), at the \( n^{th} \) update, using the measurements \( y_1 \) to \( y_n \), and \( \hat{x}_n = E\{x_n | y_1, y_2, \ldots, y_n\} \) is the a posteriori estimate of the state vector after adding the \( n^{th} \) observation, \( y_n \). \( P_{n+1} \) and \( P_n \) are defined in the same manner as the a priori and a posteriori estimates of the error covariance matrix in the \( n^{th} \) step, before and after using the \( n^{th} \) measurements, respectively.

5. System Model

In order to apply EKF recursion, we have expressed the relationship between the variables of interest and the observed signal in state space form.

\[
\begin{align*}
x(n+1) &= f(x(n)) + w(n) \\
y(n) &= g(x(n)) + v(n)
\end{align*}
\]  

(6)

where \( x(n) \) and \( y(n) \) are the state and observation vectors of the system, \( w(n) \) is the process noise with a covariance matrix \( Q \) and \( v(n) \) is the observation or measurement noise with a covariance of \( R \). Our observation model of (1) and (2) can be represented as follows:

\[
\begin{align*}
y_1(n) &= A \cos(\theta + \pi/4 + 4\pi A_k \sin(\omega_f(n) n T_s / \lambda) \\
&\quad + 4\pi A_h \sin(\omega_h(n) n T_s / \lambda + \Delta \phi(n))) \\
&= F_1(\theta, \omega_f(n), \omega_h(n)) \\
y_Q(n) &= A \cos(\theta - \pi/4 + 4\pi A_k \sin(\omega_f(t) n T_s / \lambda) \\
&\quad + 4\pi A_h \sin(\omega_h(n) n T_s / \lambda + \Delta \phi(n))) \\
&= F_2(\theta, \omega_f(n), \omega_h(n))
\end{align*}
\]  

(7)

where \( \omega(n) \) is the instantaneous frequency in units of radians per sample, \( \theta \) is the phase, and \( T_s \) is the sampling interval.

The state vector \( x(n) \) includes the unknown parameters of the system.

\[
x(n) = \begin{bmatrix} \theta_f(n) \\ \omega_f(n) \\ \theta_h(n) \\ \omega_h(n) \end{bmatrix}
\]  

(8)
Instantaneous phases are represented by first-order difference equations.

\[
\begin{align*}
\theta_i(n+1) &= \theta_i(n) + s[\omega_i(n)]T_s \\
\theta_b(n+1) &= \theta_b(n) + s[\omega_b(n)]T_s
\end{align*}
\] (9)

where \( s[\omega] \) is a function to limit the range of the instantaneous frequency to known limits. A simple clipping function is used as \( s[\omega] \).

\[
s[\omega] = \begin{cases} 
\omega_{\text{min}} & \omega < \omega_{\text{min}} \\
\omega & \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}} \\
\omega_{\text{max}} & \omega > \omega_{\text{max}}
\end{cases}
\]

(10)

Variations in the respiratory rate \( \omega_i(n) \) and heart rate \( \omega_b(n) \) are both modeled as first order autoregressive process with mean and soft nonlinearity that limits the frequencies to some known physiologic ranges [8].

\[
\begin{align*}
\omega_i(n+1) &= \bar{\omega}_i + \alpha_r \{ s_r[\omega_i(n) - \bar{\omega}_i] \} + w_r(n) \\
\omega_b(n+1) &= \bar{\omega}_b + \alpha_h \{ s_h[\omega_b(n) - \bar{\omega}_h] \} + w_h(n)
\end{align*}
\] (11)

where \( \bar{\omega}_r \) and \( \bar{\omega}_h \) are the \textit{a priori} estimates of the expected respiratory and heart rates, respectively; \( \alpha_r \) and \( \alpha_h \) are the controlling factors of the bandwidths of the frequency fluctuations and \( w_r(n) \) and \( w_h(n) \) are white noise processes that model the random variation in the respiratory and heart rates, respectively. The instantaneous respiratory and heart rates in units of Hz can be written as:

\[
\begin{align*}
f_i(n) &= \frac{s_r[\omega_i(n) \bar{\omega}_i]}{2\pi T_s} \\
f_b(n) &= \frac{s_h[\omega_b(n) - \bar{\omega}_b]}{2\pi T_s}
\end{align*}
\] (12)

Table 1 lists the numerical values of the parameters that are used in our model.

**Table 1. Model Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum respiratory rate</td>
<td>( f_{r,\text{min}} )</td>
<td>0.15</td>
</tr>
<tr>
<td>Maximum respiratory rate</td>
<td>( f_{r,\text{max}} )</td>
<td>0.40</td>
</tr>
<tr>
<td>Mean respiratory rate</td>
<td>( \bar{f}_r )</td>
<td>0.25</td>
</tr>
<tr>
<td>Minimum heart rate</td>
<td>( f_{h,\text{min}} )</td>
<td>1.00</td>
</tr>
<tr>
<td>Maximum heart rate</td>
<td>( f_{h,\text{max}} )</td>
<td>2.00</td>
</tr>
<tr>
<td>Mean heart rate</td>
<td>( \bar{f}_h )</td>
<td>1.40</td>
</tr>
</tbody>
</table>
6. Results and Discussion

In this section, we demonstrate the results of the proposed algorithm and compare the heart rate estimation with the reference ECG results. The algorithm is implemented using Matlab on a computer with AMD Athlon dual core processor of 3 GHz speed and 1 GB RAM. To test the performance of our algorithm, we choose 5 volunteers and performed the experiment on each person for 30 minutes. The test IQ signals from the hardware were taken using a directional antenna facing the test target seated at varying distances of 10 to 50 cm. The raw IQ signals were initially lowpass filtered with cutoff frequency 30 Hz, to minimize out of band noise and aliasing error. Analog to digital conversion was done using AD7888 12bit ADC with a sampling frequency of 125 Hz. Figure 4 shows sample IQ signals obtained from the Doppler radar monitoring system for a stationary target seated 50 cm away from the antenna and the reference ECG signal for a time period of 10 sec. Figure 5 shows EKF estimation tracking results of the heart and respiration rate for 600 seconds. Heart rate for the ECG signal is calculated from time interval between two consecutive peaks and the calculated value is assumed constant for that interval. Since the amplitude corresponding to the respiration signal is typically about 100 times greater than that of the heart rate signal, the respiration rate estimation process is not so much difficult as heart rate detection. Hence, our analysis is mainly concerned with heart rate monitoring. Figure 5(a) shows the eigenvalues of the windowed IQ signals, which is used for channel selection. Since, Q signals show larger eigenvalues than I signals, we can assess that the Q channel is closer to optimum point than I channel and would give better information for EKF estimation. As a result, our algorithm chooses the EKF estimation from the Q signals for the whole duration of 600 seconds presented here. From Figures 5(b) and 5(c), we can observe that the heart rate estimation from I and Q signals has some dissimilarities between them in some places of the tracking while they agree elsewhere. For the 600 sec data, the standard deviations of the difference between the EKF and ECG heart rates are found to be 0.0299 and 0.021 Hz (1.794 and 1.26 BPM) for I and Q signals, respectively. These results manifest that the EKF estimation can follow the ECG reference heart rate well with some acceptable fluctuations. This experiment was repeated for different target positions ranging from 10 to 50 cm with EKF output consistently providing better agreement with ECG results. Similar results were obtained for all test persons. The experimental results demonstrate the ability of EKF to track the heart and respiration rate, to an acceptable level of accuracy, from the quadrature Doppler radar cardiopulmonary monitoring system IQ signals without using any other beat detection algorithm.
Figure 4. Raw IQ signals obtained from the Doppler radar cardiopulmonary monitoring system and reference ECG signal. Data are normalized along the y axis.

Figure 5. Heart and respiration rate estimation results. Eigenvalues of windowed IQ signals are shown in a), which represents the IQ signal variances. Respiration rate is found to be identical both from I and Q signals.
3. Conclusion

In the present study, we proposed a digital signal processor using EKF to extract the respiration and heart rate information from I and Q quadrature signals of the direct conversion Doppler radar cardiopulmonary monitoring system. The algorithm is able to decompose the IQ signals and track the heart and respiration rate information continuously. Experimental IQ data collected from the implemented hardware are analyzed using the proposed algorithm and results are compared with the reference ECG results for heart rate detection. The representative results demonstrate the feasibility and accuracy of the algorithm to extract the rate information from IQ signals. Future works include improvement of the EKF dynamic model by incorporating residual phase noise, phase delay between the mixer and antenna, and other constant phase shift related to the nominal distance to the target and phase change at the surface of the target. In addition, we need to compare the performance of the proposed algorithm against the current practices in order to assess its ability.

References

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