Non-Linear Control Techniques Applied to High Frequency Inverters for Induction Plasma Generator

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Abstract

An induction plasma generator is very attractive for several industrial applications especially in material processing, so a high-frequency power supply class-E inverter is required to generate the magnetic field at frequency ranging from hundreds of kilohertz to tens of megahertz offer high efficiencies at high power densities. A resonant inverter based on zero voltage switching and an adaptive control via a backstepping based on Lyapunov theory and tuning functions control is used as a power supply for inductively Plasma generator. A second technique is applied as a linear quadratic regulator is used as a power supply for plasma generator. This type of control does not only keep the output current and the voltage in phase but also makes the system operating in zero-voltage-switching mode for constant and variable load. On the strategy design of the gains which are determined by minimizing a cost function which reduces the tracking error and the control signal. A recursive least squares (RLS) estimator identifies the plant parameters at different load conditions. The evaluation of the state variables is obtained by estimation using Kalman Filter. Simulation results verified that the proposed strategy has good performance to variable loads providing fast transient response.

Keywords: Class-E Inverter, Zero-voltage switching, Induction heating, Plasma generator, Buck-stepping Control, Tuning functions, LQ regulator, (RLS) estimator, Kalman Filter

1. Introduction

With the development of power semiconductor devices, many new circuit techniques and control schemes, research about high frequency circuits using advanced power devices such as MOSFETs, IGBTs and so we have been performed for high power applications, it has made it possible to implement high frequency inverters for induction heating, dielectric heating, and plasma generation. These applications generally require power levels from watts to megawatts at a single frequency ranging from hundreds of kilohertz to tens of megahertz [1]. Soft switching techniques have been used in power converters to reduce switching losses and alleviate electromagnetic interference (EMI); it has been studied based on its topology [2]. The various resonant inverters which are class-D, class-E, class-F and class E/F etc. inverter using power devices such as MOSFETs and IGBTs offer reduced switching loss by effective means of soft-switching technique [4]. The class-E inverter is an example of a resonant inverter which allows obtaining, almost the sinusoidal current–voltage at the frequency above (several of KHz to several of MHz) [3] and is a well-known resonant converter that can operate at these frequencies with very high efficiency and produce up to several kilowatts of power [1]–[4]. It is a single-ended or
push–pull topology where a transistor is soft switched, and therefore, its switching losses are significantly reduced.

Comparing class E to the two class-F tunings, several advantages and disadvantages are apparent. Class E has the advantage of being capable of strong switching operation even with a very simple circuit, whereas class F allows this only as a limiting case using a circuit with great complexity [4]. Whereas the class-E amplifier is limited only by the intrinsic switching speed of the active device, class-F amplifier tunings may find their switching speed dominated by the limited number of harmonics, which have been utilized in the waveforms. Additionally, class E has the advantage of incorporating the output capacitance of the transistor into the circuit topology. The simple class-E implementation will not work in the presence of large output capacitance since the harmonics that were intended to be open circuited at the transistor will instead be capacitive. Classes F and 1/F also have advantages. First, they present more desirable waveforms. It is desirable to have waveforms with low peak voltage and rms current, (it is clear that class-F and F⁻¹ amplifiers can perform better in these respects) [4].

This paper presents an approach to adaptive control of class E resonant inverter via a backstepping tuning function control design. This design removes several obstacles from adaptive linear control. Since the design is based on a single Lyapunov function incorporating both the state of the error system and the update law, the proof of global uniform stability is direct and simple. Moreover, all the error states except for the parameter error converge to zero [9]. In the second parts of this study, an adaptive linear quadratic regulator (LQR) is proposed such that the changing resonant frequency of the equivalent load can be tracked and meanwhile to keep the ZVS condition. The LQR regulator is a useful tool in modern optimal control design, consisting of explicit matrix design equation easily solved in a digital computer. In the proposed controller, a least square estimator (RLS) identifies the plant parameters which are used to compute the LQR gains periodically. The quadratic cost function parameters are chosen in order to reduce the energy of the control signal. A Kalman filter is used to estimate the inductor current state [13].

2. System Configurations

Figure below shows the system configurations of class-E series-parallel (LCL) resonant inverter for inductively coupled plasma generator. It consists of one or more switches (MOSFET IRFP) connected with output parasitic capacitance and a freewheeling drain-source diode.

![Class E Inverter for Inductively Coupled Plasma Generator](image-url)

**Figure 1.** Class E Inverter for Inductively Coupled Plasma Generator
The output resonant equivalent circuits were constructed by the output capacitor, matching transformer and plasma reactor. The load is modeled by the equivalent impedance which is varied during the heating process [7].

2.1. Operation of a class-E Resonant Inverter

As in this heading, they should be Times New Roman 11-point boldface, initially capitalized, flush left, with one blank line before, and one after.

The class-E resonant inverter is power topology especially suited for high frequency operation due to its low switching losses [8]. The key point of a class-E resonant inverter is the capacitance C voltage evolution after switch S is turned off. To minimize the switching losses, transistor S must be turned on while diode D is in conduction, thus providing to minimize the switching losses, transistor S must be turned on while diode D is in conduction, thus providing zero-voltage-switching. Capacitor C also operates as a turn-off snubber, further reducing the switching losses. The switch must operate with ZVS commutations. Failure to do so will result on capacitor discharging through the main switch thus increasing turn-on losses strongly. The voltage waveform applied to the resonant tank has strong harmonic content [8].

![Class E inverter](image)

**Figure 2. Class E inverter**

2.2. Steady-State Analysis Topology

The applied analysis method is based on a state-space description of the circuit (Figure 2) and calculations of its properties using a dedicated program written in MATLAB. The power switch of the inverter are turned on and off during each a constant interval T, the circuit variables namely, voltages $v_c$, $v_{cs}$, $v_{cp}$ and inductor currents $i_{Ls}$, $i_{Lo}$ are chosen as the state variables, such as:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Where $x = [x_1, x_2, x_3, x_4]$ is the state vector and $x_1 = v_{cp}$, $x_2 = v_{cs}$, $x_3 = i_{Ls}$, $x_4 = i_{Lo}$.

2.3. Simulation Results

The design procedure is explained with an example circuit of the Class E inverter. The specifications of the example circuit are described in the Table I.
Table 1. Design Parameters for the class-E and class-E/F₂ Inverter

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency</td>
<td>$f = 1 \text{ MHz}$</td>
</tr>
<tr>
<td>The peak transistor voltage</td>
<td>$V_{Tm} = 450 \text{V}$</td>
</tr>
<tr>
<td>the rms transistor current</td>
<td>$I_{Trms} = 5 \text{A}$</td>
</tr>
<tr>
<td>Parameters $L_0$, $L_n$</td>
<td>$L_n = 270 \mu\text{H}, L_0 = 16.8\mu\text{H}$.</td>
</tr>
<tr>
<td>parameter $C_s$, $C_p$</td>
<td>$C_s = 1.64 \text{nF}, C_p = 1.99 \text{nF}$</td>
</tr>
<tr>
<td>The load resistance</td>
<td>$R = 20.33 \Omega$ including $R_{L0}$</td>
</tr>
<tr>
<td>Input voltage</td>
<td>$V_I = 200 \text{V}$</td>
</tr>
</tbody>
</table>

We show in Figure 3 (a) that the peak transistor voltage is very important than the $V_{tr_{max}}$.

3. Adaptive Back-stepping Control Design

The control objective is to generate a feedback control input $u(t)$ for the plant with unknown parameters $\theta$, such that all closed loop signals are bounded, and the plant output $y(t) = x_i(t)$ tracks a given bounded reference output $y_r(t)$ with bounded derivatives $\dot{y}_r(t)$, $\ddot{y}_r(t)$ figure 7 [10-11-12].
Consider the plant of class E series resonant inverter:

\[ \dot{x}_1 = -a_1 x_1 - a_2 x_2 + b_1 u \]  \hspace{1cm} (6)

\[ \dot{x}_2 = -a_3 x_1 \]  \hspace{1cm} (7)

\[ y = c x_1 \]  \hspace{1cm} (8)

Where the specific parameters are: \( a_1 = R/l_1, a_2 = 1/l_1, a_3 = 1/C \) and \( b_1 = 1/l_1 \).

### 3.1. State Estimation Filters

We start by representing the plant (6), (7) in the observer canonical form:

\[ \dot{\hat{x}} = A \hat{x} + f(y, u) \]  \hspace{1cm} (9)

where

\[ f(x, u)^T = \begin{bmatrix} u & -y & 0 \\ 0 & 0 & -y \end{bmatrix} \] and the parameter vector \( \theta^T = [b_1 \ a_1] \).

For state estimation we employ the filters:

\[ \dot{\hat{\xi}} = A_\theta \hat{\xi} + Ky \]  \hspace{1cm} (10)

\[ \dot{\hat{\Omega}}^T = A_\theta \hat{\Omega}^T + f(y, u)^T \]  \hspace{1cm} (11)
Where the vector $K^T = [K_1 \ K_2]$ is chosen so that the matrix: $A_o = A - KC^T = \begin{bmatrix} -K_1 & 1 \\ -K_2 & 0 \end{bmatrix}$ is Hurwitz, and hence $P$ exists such that:

$$PA_o - A_o^T P = -I \ , \ P = P^T$$

(12)

With the help of the those filters our state estimate is

$$\hat{x} = \bar{x} + \Omega^T \theta$$

(13)

and the state estimation error

$$\epsilon = x - \hat{x}$$

(14)

In conclusion, from (15) and the expressions of KREISSELMEIR filters an equivalent expression for the virtual estimate $\hat{x}$ is:

$$\hat{x} = -A_o^T \eta - \sum_{i=1}^{3} a_i A_o^T \eta + b_o \lambda$$

(15)

An adaptive back-stepping control design procedure consists of two steps:

It starts with its output $y$:

$$\hat{y} = x_2 - y C^T a$$

(16)

All of these states are available for feedback. Our design task is to force the output $y$ to asymptotically track the reference output $y_r$ while keeping all the closed loop signals bounded. We employ the change of coordinates:

$$z_1 = y - y_r$$

$$z_2 = \hat{\lambda}_2 - \hat{\rho} \hat{y}_r - \alpha_i$$

(17)

(18)

Where $\hat{\rho}$ is the estimate of $\rho = 1 / b$ , our goal is to regulate $z = [z_1 \ z_2]^T$ to zero.

**Step 1:** Let the tracking error $z_1 = y - y_r$ and introduce $z_2 = \hat{\lambda}_2 - \hat{\rho} \hat{y}_r - \alpha_i$ , where $\alpha_i$ a function to be designed is.

Considering the first partial positive definite function:

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta} + \frac{b}{2\gamma} \tilde{\rho}^2 + \frac{1}{4\gamma^2} \epsilon^T P \epsilon$$

(19)

We examine the derivative of $V_1$ as:

$$V_1 = z_1 \left[ c_1 z_1 - d_1 z_1 + e_2 + (a - \hat{\rho} \hat{y}_r + \alpha_i) C \tilde{\theta} - b (\hat{\theta} + \tilde{\theta}) \hat{y}_r + \hat{b} z_2 \right] - \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} - \frac{b}{2\gamma} \tilde{\rho}^2 - \frac{1}{4\gamma^2} \epsilon^T \epsilon$$

(20)

To eliminate the unknown indefinite $\tilde{\theta}$ and $\tilde{\rho}$ terms in (20) we choose:
\[ \dot{\rho} = -\gamma \text{sgn}(b)(\dot{y} + \alpha_i) \]  

(21)

Were (21) is used as the actual update law for \( \dot{\rho} \), and \( \dot{\theta} = \Gamma r_i \), where

\[ r_i = (\omega - \dot{\rho}(\dot{y} + \alpha_i)C)z_i \]  

(22)

And \( r_i \) is the tuning function.

Substituting (21) and (22) into (20) we obtain

\[ V' \leq -c_1 \dot{z}_1^2 + b \dot{z}_1 \dot{z}_2 + \dot{\Theta}(r_i - \Gamma^{-1}\dot{\Theta}) \]  

(23)

**Step 2:** from (18) with the help of (16) we obtain

\[ \dot{z}_2 = a_2 - \beta_2 - \frac{\partial a_2}{\partial y}(\omega \dot{\Theta} + \varepsilon_2) - \frac{\partial a_2}{\partial \dot{\Theta}} \dot{\Theta} \]  

(24)

Since our system is augmented by the new state \( z_2 \), we augment the Lyapunov function (19) as:

\[ V_2 = V_1 + \frac{1}{2} \dot{z}_2^2 + \frac{1}{4d_i} \varepsilon^T P \varepsilon \]  

(25)

The derivative of \( V_2 \) satisfies

\[ \dot{V}_2 \leq -c_1 \dot{z}_1^2 - c_2 \dot{z}_2^2 - d_2 \left( z_2 \frac{\partial a_2}{\partial y} + \frac{1}{2d_2} \varepsilon_2 \right)^2 \]  

(26)

\[ \leq -c_1 \dot{z}_1^2 - c_2 \dot{z}_2^2 \]

And the control law

\[ u = \alpha_2 + \dot{\rho} \tilde{y}, \]  

(27)

### 3.2. Simulation Results of the System with Back-stepping Control Design

In this section, we depict some simulation results by using adaptive Back-stepping controller in different disturbance conditions.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance of the load parameters</td>
<td></td>
</tr>
<tr>
<td>Liapunov gains 50% L</td>
<td>( c_1=2e^{-5}, c_2=0.02 )</td>
</tr>
<tr>
<td>Observer gains 50% L</td>
<td>( c_1=1e^{-5}, c_2=0.02 )</td>
</tr>
<tr>
<td>Damping gains 50% L</td>
<td>( k_1=0.01, k_2=2 )</td>
</tr>
<tr>
<td>Adaptation gains 50% L</td>
<td>( k_1=1.6, k_2=1.6 )</td>
</tr>
<tr>
<td>Disturbance of the load parameters</td>
<td></td>
</tr>
<tr>
<td>Liapunov gains 50% R</td>
<td>( d_1=1e^{-5}, d_2=0.01 )</td>
</tr>
<tr>
<td>Observer gains 50% R</td>
<td>( d_1=1e^{-5}, d_2=0.01 )</td>
</tr>
<tr>
<td>Damping gains 50% R</td>
<td>( g_1=10, g_2=10 )</td>
</tr>
<tr>
<td>Adaptation gains 50% R</td>
<td>( g_1=1, g_2=10 )</td>
</tr>
</tbody>
</table>
Figures bellows represent the estimation of the load current and the estimation of load parameters theta1, theta2 and theta 3 for Disturbance of the load parameters of 50% L.

In order to demonstrate the adaptability and robustness of the proposed controller, an uncertain condition is considered for the simulations, where the load parameters are assumed to be 10% and 50% of uncertainties, and from these results, it is observed that the proposed
observer-based back-stepping controller is good in adapting the uncertainties and load
disturbances (refer the estimation of the load current, the adaptation is not exceeding 0.2 µs).

4. Adaptive Control

4.1. Linear Quadratic Regulator

An adaptive controller with integrator is proposed in Figure 10, it has the objective of
tracking the discrete sinusoidal \( r(k) \) reference in each sample instant \( T_s \) and also fulfil the
adaptation gains during the load changes. The augmented state variables used in the LQR
regulator are the measured capacitance voltage \( v_c(k) \), the estimated inductor current \( i_L(k) \),
the integrated tracking error \( v(k) \) and the discrete reference \( r(k) \) all through a feedback
action.

Each variable has a weighting Ricatti gains matrix \( K \) tuned in function of the estimated
parameters \( \theta(k) \), which contains the plant parameters identified by the RLS algorithm [13],
such that:

\[
K = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \end{bmatrix}
\]

(28)

K: is the Ricatti gains matrix.

Then, the closed loop system is defined as:

\[
Z(k) = \begin{bmatrix} v_c(k) & i_L(k) & v(k) & r(k) & \dot{r}(k) \end{bmatrix}^T
\]

(29)

and the LQR control is given by:

\[
u_{u\theta\theta} = -KZ(k)
\]

(30)

The system must be represented in the form:

\[
Z(k+1) = GZ(k) + Hu_{u\theta\theta}(k)
\]

(31)

4.2. Recursive Least Square Estimator

To estimate the plant parameters when the load conditions are variables, a RLS algorithm is
employed [14]. From the discrete transfer function, the difference equation of the estimated
output is written as follow:

\[
y(k) = -\theta_1 y(k-1) - \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2)
\]

(32)

We assume that the vector of parameters \( \theta \) has been defined such that the system may be
represented by:

\[
y(k) = \theta^T(k) \psi(k-1)
\]

(33)

where
\[ \Theta (k) = \begin{bmatrix} \Theta_1 & \Theta_2 & \Theta_3 & \Theta_4 \end{bmatrix} \]  

and

\[ \psi (k) = \begin{bmatrix} -y(k-1) & -y(k-2) & u(k-1) & u(k-2) \end{bmatrix} \]  

4.3. The Discrete Kalman Filter

As only the capacitor voltage is measured variable state, a Kalman filter is used to estimate the inductor current state [15]. The Kalman filter addresses the general problem in estimating the state of a discrete-time controlled process that is governed by the linear stochastic difference equation:

\[ x(k + 1) = A x(k) + B u(k) + w(k) \]  

\[ y(k) = C x(k) + v(k) \]

The variables \( w(k) \) and \( v(k) \) represent the random process and measurement noise respectively. They are assumed to be uncorrelated to each other and with normal probability distributions such that:

\[ E \left[ w(k)^T w(k) \right] = R_w > 0 \]  

\[ E \left[ v(k)^T v(k) \right] = R_v > 0 \]

\[ E \left[ w(k)^T v(k) \right] = 0 \]

5. Simulation results of ZVS Mode

The fundamental parameters of the buck-single phase full-bridge inverter for induction heating applications and controller parameters are:
Figure 11. Output Current (solid) and Reference Current (dashed) to the Abrupt Change of Load

Figure 12. Estimate Parameters $a_i$

Figure 13. Estimate Parameters $b_i$ Corresponding to the Simulation in Figure 10
To verify the tracking capability of the LQR control scheme in the closed loop system we have performed a simulation results. Figure 11 show the response of reference and the output current with LQR regulator for abrupt change of load. The load current frequency is well matched to the reference frequency assuring ZVS condition.

Figure 12 shows the applicability of the RLS algorithm which is shows the efficiency of determining the unknown parameters $\theta_1, \theta_2, \theta_3, \theta_4$.

6. Conclusion

The design of a MOSFET based class-E series-parallel resonant inverter power supply for an inductively plasma generator system has been presented. The variable load is highly inductive and requires a several kW active power at a frequency of several MHz. Based on a detailed topology investigation, a resonant circuit supplied by a voltage source class-E inverter is chosen. An analysis of the circuit and basic design rules are given. This E tuning allow strong-switching operation, but show a greater tolerance for transistor output capacitance and present waveforms approaching those of the more desirable class-F inverse, the family exhibits a trade-off between circuit complexity and performance. The tracking performance of the observer-based back-stepping controlled allowing operation of the inverter with the MOSFET switching-losses is investigated and simulation results verifying the operation of the control are shown.

A Linear Quadratic Regulator was developed to series resonant inverter, the LQR gains are calculated by minimizing a cost function. The RLS estimator identifies the plant parameters which are used to compute the LQR gains periodically for constant and variable loads. The adaptive control law has shown good results to constant and variable loads with a moderate switching frequency.

References


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His research interests in the areas of Power electronics.