An Efficient Decision Feedback Equalizer Combining the LDPC Code in Cellular Relay System

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Abstract

In this paper, a repeater system is studied which is basically used for the relay in the cellular communication. The LDPC (Low Density Parity Check) code is introduced into DFE (decision feedback equalizer) in the receiver for the BER (bit error rate) performance. For the better quality of service (QoS) and the higher convenience, the wireless repeater was considered for the relay system. In this paper, we propose an efficient equalizer algorithm for the cellular relay system, which uses the DFE combining the LDPC code. The proposed equalizer helps to compensate RF impairments and improve better performance than used independently. In addition, proposed equalizer has less iterative number of LDPC code. So, the proposed equalizer system has low complexity. Therefore, in this paper, BER performance of DFE combining the LDPC code for cellular relay system is satisfied with 4.3dB at $10^{-4}$.

Keywords: Cellular relay system, LDPC code, Decision feedback equalizer, Phase noise, Echo channel

1. Introduction

So as to meet ever increasing requirements on higher wireless access data rate and better quality of service (QoS), the wireless repeater system studied. However, when the gain of a cellular relay system is larger than the isolation between transmit and receive antennas, the echo signal that comes into the receive antenna from the transmit antenna of the repeater causes the repeater to go into echo oscillation regardless of the input signal [1-5]. In the receiver side of wireless repeater, it will receive reference signal from base station which is ideal input of wireless repeater and the other is echo signal from transmitter side of repeater which is amplified. Therefore, cellular relay system will continuously receive the interference from transmitter side of repeater. In this case, cellular relay system may be unstable due to the echo channel. Also, the phase noise effect can occur up and down converter of cellular relay system. The interference cancellation system is required in order to cancel the interference signal in cellular relay system. In this paper, we consider a multipath channel exists in echo channel and identify the unknown multipath channel by adaptive equalizer such as RLS (Recursive least square). In order to remove echo channel and phase noise, we suggest an RLS and a DFE based on LDPC code for cellular relay system. A standard approach to reducing the computational burden of the receiver is to split the detection problem into the two sub-problems equalization and decoding. Separating the equalization and decoding is suboptimal. The knowledge about the structure on the transmitted symbols imposed by the error correction code is not exploited by the equalizer. LDPC code is well known as forward error correction code which are very good performance. LDPC code was first proposed by Robert G. Gallager in 1962[6] and got attention again by Mackay and Neal in 1997 [7]. LDPC code is one of the error correction code which is closest to Shannon’s capacity. LDPC code is highly error correction performance at low SNR environment and possible fast
processing with sparse parity check matrix. However, more long sparse parity check matrix and iterative number are needed in selective fading channel. DFE combining the LDPC code can improve better performance than used independently.

2. System Model

![Diagram](image)

**Figure 1. Block Diagram of Echo Canceller and DFE combining the LDPC Code for Cellular Relay System**

Figure 1 is block diagram of echo canceller and DFE based on LDPC code for cellular relay system. In the cellular relay system, the phase noise exists in RF transmitter and RF receiver among base station, mobile station and cellular relay system respectively. And the echo channel exists between transceiver of cellular relay system. For the sake of removing interference, we adapt DFE combining the LDPC code. If the echo channel and phase noise exist, the performance of this system will get worse. In order to estimate and equalize echo channel, we use RLS algorithm. And we use DFE combining the LDPC code in order to compensate phase noise. The DFE combining the LDPC code calculates mean square error (MSE) in feedback block.

3. Echo Cancellation for Cellular Relay System

In Figure 1, the channel impulse response of echo channel is as follows.

\[ h(n) = \sum_t \alpha_t \cdot \delta(n - \tau_t) , \]

where \( \alpha \) is echo gain, \( \tau \) is delay. And the echo channel is as follows.

\[ y_F(n) = h(n) \ast x(n) + n , \]

where \( n \) is AWGN, \( x(n) \) is the output of repeater. In this paper, we estimate and equalize echo channel by RLS algorithm. The error of estimated signal is as follows.

\[ e(n) = i(n) \cdot e^{j\phi(n)} + y_F(n) \cdot e^{j\phi(n)} - y_E(n) \]
\[ = i(n) \cdot e^{j\phi(n)} + h(n) \ast x(n - k_0) \cdot e^{j\phi(n)} - w(n) \ast e(n - k_0) , \]

where \( \phi(n) \) is phase noise, \( i(n) \) is input signal of repeater, and \( k_0 \) is delay. Consequently, by producing the echo signal, we can eliminate the interference effects in receiver sides of repeater. Following Figure 1 shows the channel identifier, echo
cancellation and phase noise compensator with DFE based on LDPC code for cellular relay system.

Finally, in short, it will be as following equation (5).

\[ x(n) = \frac{G}{1 + (\hat{H} - H) \cdot G} \cdot i(n), \]  

(4)

which \( H \) is unknown multipath channel and \( \hat{H} \) is FIR adaptive equalizer.

3.1. RLS Adaptive Filter

The RLS algorithm is derived from the minimization of the sum of weighted least-square error which is expressed as follows.

\[ J_{LS}(n) = \sum_{i=1}^{n} \lambda^{n-i} e^2(n), \]  

(5)

where \( \lambda \) is the forgetting factor, which has a value more than 0 and below 1. The forgetting factor makes the current error heavier than the past error value to support filter operation in nonstationary environments. Therefore, in the least-square method, the weight of vector is optimized based on the observation, starting from the first to the current time. Therefore, RLS algorithm can be expressed as following equation.

\[ \hat{h}_i(n+1) = \hat{h}_i(n) + g(n)e(n), \]  

(6)

where the updating vector is defined as

\[ g(n) = \frac{r(n)}{1 + x^T(n)r(n)} \]  

(7)

and

\[ r(n) = \lambda^{-1} P(n-1)x(n), \]  

(8)

where \( P(n) \) is the inverse correlation matrix of input data.

\[ P(n) \equiv R^{-1}(n) = \left( \sum_{l=1}^{n} \lambda^{n-l} x(l)x^T(l) \right)^{-1} \]  

(9)

can be computed recursively as

\[ P(n) = \lambda^{-1} P(n-1) - g(n)r^T(n). \]  

(10)

4. DFE Combining the LDPC Code

In the multipath channel, we use channel coding technique by forward error correcting (FEC) in order to improve performance of receive. LDPC code is highly error correction performance at low SNR environment and possible fast processing with sparse parity check matrix. However, more long sparse parity check matrix and iterative number are needed in selective fading channel. They can be expressed as \((N, K)\).
LDPC code has a sparse parity check matrix \( H \) of \((N - K) \times N\) size. \( H \) matrix is non-systematic sparse parity check matrix.

### 4.1. LDPC Encoding

The received vector is corrupted by an error vector \( e \) as follows.

\[
\mathbf{r} = \mathbf{c} \oplus \mathbf{e} = [\mathbf{p} \quad \mathbf{m}] \oplus \mathbf{e},
\]

where \( \mathbf{p} \) is parity vector and \( \mathbf{m} \) is message vector. The parity vector and message vector are assumed to be located at the former or latter part of the code vector, respectively. The decoder is supposed to apply for this received signal vector to find the syndrome vector as

\[
\mathbf{s} = \mathbf{r}H^T = \left( [\mathbf{p} \quad \mathbf{m}] \oplus \mathbf{e} \right) \begin{bmatrix} H_1^T \\ H_2^T \end{bmatrix} = \mathbf{p}H_1^T \oplus \mathbf{m}H_2^T \oplus \mathbf{e}H^T.
\]

Noting that this syndrome vector should be zero for the non-error case of \( e=0 \),

\[
\mathbf{s} = \mathbf{p}H_1^T \oplus \mathbf{m}H_2^T = \mathbf{0}.
\]

We can write the parity vector \( \mathbf{p} \) in terms of the message vector \( \mathbf{m} \) as

\[
\mathbf{p} = \mathbf{m}H_1^T H_1^{-T}.
\]

This amounts to having the generator matrix,

\[
\mathbf{G} = \begin{bmatrix} H_2^T \quad H_1^{-T} \quad \mathbf{I} \end{bmatrix}.
\]

So that the code vector can be generated by post-multiplying the message vector \( \mathbf{m} \) with the generator matrix \( \mathbf{G} \) as

\[
\mathbf{c} = \mathbf{mG} = \begin{bmatrix} \mathbf{m}H_2^T H_1^{-T} & \mathbf{m} \end{bmatrix} = [\mathbf{p} \quad \mathbf{m}].
\]

Here, the parity-check matrix is the key to the performance of a LDPC code, and therefore the design issue of the parity-check matrix is very important.

### 4.2. LDPC Decoding

LDPC decoder calculates the probability in variable nodes and check nodes, respectively. We use SPA to process LDPC decoding because of better performance. LDPC decoder process 4 steps iteratively. Step 1 is initialization. Step 2 is updating check nodes. Step 3 is updating variable nodes. Step 4 is decision.

### 4.3. DFE combining the LDPC Code

Figure 2 shows block diagram of DFE combining LDPC code at receiver side of cellular relay system. The channel impulse response is estimated by preamble. The estimated channel \( H_p \) is as follows.
\[ H_p = \frac{Y_p}{X_p}, \]  

where \( Y_p \) is received preamble, \( X_p \) is transmitted preamble.

**Figure 2. Block Diagram of DFE Combining the LDPC Code at Receiver of Cellular Relay System**

Figure 3 is block diagram of adaptive equalizer in DFE combining the LDPC code. The ICI component is generated by phase noise. The first term of ICI is CPE term which is phase rotation factor, and the others terms are ICI components. After adaptive equalizer, we compensate phase noise effect by using phase noise compensator. In phase noise compensator, firstly, we cancel CPE component by comb type pilot, and then we compensate ICI components. After echo channel estimation and equalization, the received signal is as follows.

\[ \tilde{e}(n) = i(n) \cdot e^{j\phi(n)}. \]  

And sampled signal of \( k \)-th carrier after FFT is as follows.

\[
E_k = \sum_{n=0}^{N-1} \tilde{e}(n) \cdot e^{-j\frac{2\pi kn}{N}} = \frac{1}{N} \sum_{n=0}^{N-1} \{i(n) \cdot e^{j\phi(n)}\} \cdot e^{-j\frac{2\pi kn}{N}} \\
= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} I_l \cdot e^{j\phi(n)} \cdot e^{-j\frac{2\pi}{N} (l-k)n} \\
= I_k \left( jI_k \cdot \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(n)} + \sum_{l=0}^{N-1} I_l \cdot \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(n)} \cdot e^{j\frac{2\pi}{N} (l-k)n} \right) \\
= I_k + CPE + ICI
\]  

In (19), we don’t consider noise in order to simplify. And the effect of phase noise is divided into CPE and ICI components such (19). So as to remove CPE, we use comb type pilot. This process is as follows.

\[
CPE_k = \frac{Y_k}{X_k} = Q_0 + \frac{ICI + N_k}{X_k} = Q_0 + W_k,
\]
where \( Y_k \) is received pilot, \( X_k \) is transmitted pilot and 
\[
Q_k = \frac{1}{N_k} \sum_{n=0}^{N-1} e^{\frac{2\pi j n}{N}} \cdot e^{\frac{2\pi j (k-1)n}{N}}.
\]

And then we get average of CPE.

![Adaptive Equalizer](image)

**Figure 3. Block Diagram of Adaptive Equalizer in DFE Combining the LDPC Code**

\[
r_{rpe} = \frac{1}{N_p} \sum_{k \in S_p} CPE_k = Q_b + \frac{1}{4} \sum_{k \in S_p} W_k, \tag{21}
\]

where \( N_p \) is the number of pilot, and \( S_p \) is pilot symbol. The signal which is removed CPE is as follows by using (21).

\[
\hat{E}_k = I_k + \left( \sum_{l=0}^{N-1} I_l \left[ \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(n)} \cdot e^{j\frac{2\pi j (l-k)n}{N}} \right] \right) = I_k + \left( \sum_{l \neq k} I_l \cdot Q_{l-k} \right), \tag{22}
\]

\[
= I_k + ICI
\]

In (22), ICI component still remains. In order to compensate ICI, we use decision and correction process. \( \tilde{Q}_{l-k} \) is phase rotation value by ICI.

\[
\tilde{Q}_{l-k} = \frac{\sum_{k \in S_d} \bar{Y}_l D_d^*}{\sum_{k \in S_d} |D_d|^2}, \tag{23}
\]

where \( D_d \) is preamble, and \( S_d \) is the number of preamble. And MMSE criterion \( C_k \) is finally as follow.

\[
C_k = \frac{\tilde{Q}_{l-k}^* \cdot H_k^*}{|\tilde{Q}_{l-k}^* \cdot H_k^*|^2 + \sigma_{ph}^2}, \tag{24}
\]

where \( \sigma_{ph}^2 \) is power of ICI with calculation from pilot. In equalization process, because the effect of ICI is bigger than noise power after FFT in equalization process, we use error power from pilot symbol without dividing between phase noise and carrier frequency offset for optimization tap weighting value of equalizer.
\[
\hat{\sigma}_{ph}^2 = \frac{1}{N_p} \sum_{k \in \sigma_p} |\tilde{E}_k|^2.
\] (25)

And then phase noise compensator compensates effect of CPE and ICI that is caused by phase noise by using MMSE criterion that is updated by (24). The signal which is removed CPE pass through MMSE equalizer in order to remove effect of ICI. And final signal is as follow.

\[
\hat{I}_k = \tilde{E}_k \cdot C_k.
\] (26)

After LDPC decoder, we get MSE value of \(L_s\) which is value of before decision and \(L_d\) which is value of after decision. And in order to correct the estimated channel \(\hat{H}_p\) by preamble, we give feedback by control factor \(C_F\).

\[
\hat{L}_s = \text{norm}(L_s)
\] (27)

\[
\hat{L}_k = \sum_{k \in \sigma_s} \tilde{L}_s^* L_d
\] (28)

\[
C_F = \frac{\hat{L}_s}{\sum_{k \in \sigma_d} |\tilde{L}_s|^2}
\] (29)

\(H_p\) is the channel characteristic after channel estimator using long preamble.

\[
\hat{H}_p = H_p + C_F
\] (30)

We can improve the performance following equation with compensated channel.

\[
Y_k' = \frac{Y_k}{\hat{H}_p}
\] (31)

5. Simulation Results

<table>
<thead>
<tr>
<th>Table 1. The Simulation Parameters</th>
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<tbody>
<tr>
<td>FFT size</td>
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<tr>
<td>Cyclic Prefix</td>
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<tr>
<td>Number of Subcarriers</td>
</tr>
<tr>
<td>Modulation</td>
</tr>
<tr>
<td>Code rate</td>
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<tr>
<td>Parity check matrix size</td>
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<tr>
<td>Iteration of LDPC decoder</td>
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<tr>
<td>Channel</td>
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<tr>
<td>Phase noise</td>
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</tbody>
</table>

Figure 4 shows signal compositions of input and echo signal in time domain. Case of echo channel is \(h_1 = 1.8\delta(n-10) + 1.2\delta(n-15)\). In order to remove echo channel, we use RLS algorithm. This mean that RLS algorithm is able to eliminate the interference which is amplified transmitted echo signal.
Figure 4. Signal Compositions of Input and Echo Signal in Time Domain

Figure 5 shows BER performance of cellular relay system with DFE combining the LDPC code. The line with square marker is case of without cancellation of echo channel and phase noise in wireless repeater system. And the line with asterisk marker is case of with cancellation of echo channel by using RLS algorithm. Thus, in this result, the phase noise effects remain. And the line with circle marker is after compensation between echo channel and phase noise by RLS algorithm and DFE combining the LDPC code. As the result of Fig. 5, BER performance of DFE combining the LDPC code for cellular relay system is satisfied with 4.3dB at $10^{-4}$.
6. Conclusions

The echo channel and RF impairments such as phase noise, carrier frequency offset, and more has caused performance degradation. In order to remove echo channel and phase noise, we suggest a novel equalizer. LDPC code is well known as forward error correction codes which are very good performance. The proposed equalizer helps to compensate RF impairments and improve better performance than used independently. In addition, proposed equalizer has less iterative number of LDPC code. So, the proposed equalizer system has low complexity. Therefore, BER performance of DFE combining the LDPC code for cellular relay system is satisfied with 4.3dB at $10^{-4}$.

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References


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